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ABSTRACT

The report describes the construction, application, and theoretical implications of an econometric model depicting the effects of labor subsidies on the supply of workers in the U.S. Three papers deal with the following aspects of constructing the econometric model: (1) examination of equilibrium wages, employment, and earnings of primary and secondary workers in eight family types according to the age, sex, and education of the family head; (2) aggregation of primary individuals into reasonably homogeneous wage rate groups not too disparate in size; and (3) development of a dynamic disequilibrium model of the labor market which integrates search-turnover theory with the theory of supply and demand. Three papers deal with the results of simulations of the following programs: (1) the Work Bonus Tax Credit for all families with dependent children; (2) the Jobs and Income (JOIN) Program for low wage individuals; and (3) the Aid to Families with Dependent Children Program. Finally, three papers deal with theoretical analysis: (1) the effects of a progressive income tax on a multi-worker family; (2) the effects of changes in personal income tax parameters on the individual's decision of how to allocate time between labor and leisure; and (3) the effects of increasing payroll taxes on the labor supply. (Author/JR)

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STATE LABOR MARKET RESEARCH STUDY:
AN ECONOMETRIC ANALYSIS OF THE
EFFECTS OF LABOR SUBSIDIES

PREPARED BY

THE URBAN INSTITUTE
WASHINGTON, D.C.

DECEMBER 1974

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16. Abstracts

The objective of this research is to analyze the effects of labor subsidies on a state by state basis and, hence, for the nation as a whole. The method is first to translate the subsidy into a shift in the supply or demand for workers who are eligible for the subsidy and then to simulate with a model of the state labor market the effects of this shift on both eligible and ineligible workers in the state. The results of the study are an econometric model, program simulation, and theoretical analysis. The model, which was estimated with data from the 1970 Census 1-1000 Public Use Sample (CPUS) for 30 states or groups of states, explains equilibrium wages and employment by primary and secondary workers in eight comprehensive family types according to the demographic characteristics of the family head. The family types were chosen not on an *ad hoc* basis but using a sequential, one-way analysis of variance procedure with CPUS data from California. In addition to the static equilibrium model a dynamic disequilibrium model of the labor market was developed to explain not only wages and employment but also unemployment and labor force participation. The subsidy programs that were simulated are the Work Bonus Tax Credit, an earnings subsidy proposed by the Senate Finance Committee, and the Jobs and Income (JOIN) Program, developed by the staff of the Subcommittee on Fiscal Policy of the Joint Economic Committee. Further work was also carried out on a diffusion analysis of participation in the Aid to Families with Dependent Children (AFDC) Program. Finally, supportive theoretical analysis of the effects of the personal income tax on individual human investment and family labor supply and of the payroll tax on individual labor supply were carried out.

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State Labor Market Research Study
OVERVIEW

The objective of this research is to analyze the effect of labor subsidies by state and, hence, for the nation as a whole. The project's basic aim is the development of a theoretical and empirical underpinning for guiding Departmental approach to labor market effects of labor subsidies. States were chosen as the unit of analysis not only because labor market conditions vary across states and data on these conditions are available, but also because many subsidy programs may be implemented at the state level rather than at the national level in accordance with principles of revenue sharing and decentralization of manpower programs.

The results of the study can be conveniently partitioned into three topics:

(1) an econometric model of the state labor market;
(2) labor subsidy program simulations; and
(3) supportive theoretical analysis.

In what follows we present an overview of the model development and the results of simulations of two labor subsidy programs, and the key implications of the theoretical analysis. The remainder of the volume contains the nine component reports.

Econometric Model

We have estimated, using data from the 1970 Census 1-1000 Public Use Sample (CPUS) for 30 states or group of states, a comprehensive cross-section labor market model, which integrates the theory of human capital with the theory of supply and demand for labor. The model, described in the paper "Labor Markets, Human Capital, and the Structure of Earnings" (pp. 1.1-1.75), explains equilibrium wages, employment, and, hence, earnings by primary and secondary workers in eight family types according to the sex, age and education of the family head. Supplies per family by primary individuals (heads of families) and by secondary individuals (other family workers) in a state are, by family labor supply theory, functions of the group wage, other family income, and the demographic composition of primary and secondary individuals in the group in the state. Supplies are then aggregated in equivalent-quality units with relative group wages being measures of relative quality. Demand for equivalent-quality labor per unit of output in a state are, by the theory of industrial labor demand, a function of the numeraire wage and the industrial composition of output in the state. Supply and demand are assumed to be equal in equilibrium to hours of employment.

The family types were chosen not on an ad hoc basis but using a sequential, one-way analysis of variance procedure with CPUS data from California. This procedure, which is described in the paper "Wage Differentials, Human Capital and Demographic Characteristics" (pp. 2.1-2.20), aggregates primary individuals into reasonably homogeneous wage rate groups not too disparate in size. The result of this analysis is that we have limited the likelihood of severe heteroscedasticity which would call for weighting of the observations in estimation.

The supply, relative wage, and demand equations were estimated using a nonlinear, two-stage least squares method. The estimates indicate that demand is slightly elastic while primary and secondary supplies for all family groups are backward-bending. Predicted wages, hours and, hence, earnings for primary and secondary individuals in all family groups in a state are obtained from the estimated model by solving the nonlinear system of 16 supply equations, two relative wage equations, one demand equation, and one equilibrium identity. For all but the smallest groups and states the explanatory power of the model in 1969 is good. Nevertheless, the model is surely in a developmental stage so that it requires further refinement and validation.

In addition to the work on the static equilibrium model we have also developed a dynamic disequilibrium model of the labor market. This model, which is presented in the paper "Equilibrium and Disequilibrium in the Labor Market" (pp. 3.1-3.30), describes not only wages and employment but also unemployment, labor force, vacancies, and job stock by integrating search-turnover theory with the theory of supply and demand. One of the implications of the model is that there need not be a trade off between inflation and unemployment or a natural rate of unemployment unaffected by the rate of inflation. Rather for certain values of the parameters there is a direct relation between inflation and unemployment in the model. Therefore, an increase in aggregate demand can increase not only inflation but also unemployment. If this is true, then correspondingly both inflation and unemployment could be reduced by a decrease in the growth of aggregate demand. We have estimated a preliminary version of the model with quarterly data for the nation as a whole. While the preliminary estimates support the underlying framework of the model, further work needs to be done before we can tell whether or not there is indeed a direct relation between inflation and unemployment.

Program Simulation

Our procedure for simulating the effect of a labor subsidy program is first to translate a subsidy into a shift in the supply or demand for workers who are eligible for the subsidy in a state and then to simulate with the econometric model of state labor markets the effects of this shift on both eligible and ineligible workers in the state.

As the first implementation of this procedure we have simulated the effects of the Work Bonus Tax Credit, an earnings subsidy for all families with dependent children. This subsidy, which has been proposed by the Senate Finance Committee, is designed to act as a proportional wage subsidy at low levels of earnings and as a negative income tax at higher levels of earnings. Unlike pure wage subsidies or income subsidies it has the potential to be market neutral, since the increased labor supply from those with low earnings could be offset by the decreased supply from those with higher earnings. Our simulations, which are reported in the paper "Categorical Earnings Subsidies: Market Effects and Program Costs" (pp. 4.1-4.45), indicate that this potential would indeed be realized in the sense that neither would the benefits of the subsidy be dissipated through higher market wages nor would nonparticipants be displaced through lower wages.

To complement this study we have simulated the effects of the Jobs and Income (JOIN) Program, developed by the staff of the Subcommittee on Fiscal Policy of the Joint Economic Committee. This universal subsidy program would combine a wage subsidy for low wage individuals and public employment for very low wage individuals with an earnings tax for all individuals. In addition one parent families with pre-teenagers would be given an income subsidy. This program also has the potential to be market neutral, since the increase in labor supply from those receiving the wage subsidy could be offset by the decreased private market supply available from those in public employment and those receiving the income subsidies. Results reported in the paper "Jobs and Income (JOIN): A Labor Market Analysis" (pp. 5.1-5.52) indicate that for a particular scale of benefits this potential is fulfilled.

In addition to the simulations of proposed labor subsidies we have further developed the diffusion analysis of AFDC participation. The results of this research, which was earlier supported by ASPER-DOL and SRS-HEW, is presented in the completely revised paper "A Diffusion Analysis of Participation in the Aid to Families with Dependent Children (AFDC) Program by States" (pp. 6.1-6.25). The objective of this analysis is to explain participation in a particular government program as the result of the diffusion of information from those who are participating in the program to those who are eligible but not participating in the program. It has implications, however, for the prediction of participation and, hence, cost of government programs, in general. Typically participation in a new program grows more slowly than would be expected on the basis of eligibility but then mushrooms as information regarding the program disseminates. The diffusion model explains and predicts these differential growth rates. Moreover, it is readily amenable to empirical implementation.

Theoretical Analysis

The theoretical research that has been carried out as part of this study has been performed primarily in support of the econometric model development and the subsidy program simulations. Analyses of the interaction between labor supply and income taxes were initiated both because the existence of the income tax has significant implications for the estimation of labor supply in ways not earlier understood, and because negative income taxes, such as that embodied in JOIN, have been conceived as reforms in the existing income tax structure. Similarly, analysis of labor supply and the social security payroll tax was initiated both because the payroll tax affects labor supply in ways not previously perceived and because the Work Bonus Tax Credit was conceived as a reform in this tax.

In the paper "The Personal Income Tax and Family Labor Supply" (pp. 7.1-7.24) it is shown that labor supply effects of a progressive income tax on a multi-worker family can be analyzed as a combined wage tax and income transfer specific to each tax bracket. The wage tax equals the marginal rate paid on family income while the income transfer equals net savings from not having to pay tax at this high marginal rate on all earnings. At intervals where tax rates change the family departs from its reduced form supply equation entirely. These results limit the implications of survey research and suggest modifications in procedures for estimation and simulation of supply relationships.

The effects of changes in the parameters describing the personal income tax upon the individual's decision of how to allocate time between human capital accumulation and work effort is analyzed in the paper "Work Effort, Human Investment and the Income Tax" (pp. 8.1-8.45).

It is shown that the effects of changes in the tax parameters can be described in terms of intertemporal substitution and income effects. Given an increase in the tax rate, if the intertemporal substitution effect dominates, then the individual substitutes income in the early stages for income later in the life cycle. If the intertemporal income effect dominates, however, the individual spreads the life cycle income reduction over all stages. Similarly, given an increase in the exemption level, if the intertemporal income effect dominates, the increase in total income is spread over all stages; if the intertemporal substitution effect dominates, income just before the taxable stages is substituted for income in all other stages.

It is commonly argued that an increase in the payroll tax would decrease labor supply. The analysis presented in the paper "Labor Supply and the Social Security Payroll Tax" (pp. 9.1-9.9), implies that while for individuals with earnings below the ceiling an increase in the tax would indeed reduce their received wage rate and, thus, reduce their labor supply, for individuals with earnings above the ceiling an increase in the tax would not affect their received wage but would reduce their effective level of unearned income, and, thus, increase their labor supply. Since these two effects would tend to offset one another, an increase in the payroll tax could actually increase labor supply. The net effect can only be determined by empirical analysis of the labor market.

LABOR MARKETS, HUMAN CAPITAL, AND THE STRUCTURE OF EARNINGS

by Peter M. Greenston and C. Duncan MacRae

Abstract

A state labor market model which integrates the theory of supply and demand with the theory of human capital is developed in this paper. Families are aggregated by human capital theory into eight groups according to the demographic-educational characteristics of their head. Supply, relative wage, and demand equations are estimated with data aggregated from the 1970 Census 1-1000 State Public Use Sample for 30 states or groups of states. The estimates indicate that demand is slightly elastic while primary and secondary supplies for all family groups are backward-bending. Predicted wages, hours and, hence, earnings for primary and secondary individuals in all family groups in a state are obtained by solving the nonlinear system of equations. For all but the smallest groups and states the explanatory power of the model in 1969 is good. The model predicts, in particular, that the benefits of output growth are distributed across family groups primarily according to their labor supply response. Earnings will increase significantly more for individuals who are below the backward-bending portion of their labor supply.

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LABOR MARKETS, HUMAN CAPITAL, AND THE STRUCTURE OF EARNINGS*

by Peter M. Greenston and C. Duncan MacRae

The Urban Institute

There has been considerable discussion of reforming the current welfare system. The issues revolve around improving the distribution of income without at the same time seriously undermining work incentives for participants or displacing non-participants. Many of the proposals include work requirements for the able-bodied while extending coverage to the working poor perhaps in the form of an earnings-conditioned supplement. To deal with these issues and the implicit trade-offs policy makers need information about how proposed changes would affect work incentives and hence labor supply, and about how the changes in labor supply in conjunction with market demand determine a new market equilibrium and hence affect the distribution of income.

In this paper we specify, estimate, and evaluate a cross-section state labor market model which integrates the theory of human capital into an analysis of labor market supply and demand. The immediate result is an econometric model that predicts wage rates, employment levels, and hence earnings for sixteen demographically defined groups of individuals in each of thirty states or groups of states in the United States. The model is designed in particular to measure the wage rate and employment effects of human resource programs -- existing and proposed -- on both participants and non-participants in all states and, hence, for the nation as a whole.

In attempting to explain the distribution of earnings most investigators have either focused on wage rate differentials using demand-related factors to explain industrial or regional differences while ignoring the larger market context in which labor services are supplied and demanded, or they have focused on explaining earnings differentials without carefully distinguishing the wage rate and labor supply components.[1] In this paper, however, we employ a market model in which supply as well as demand factors account for regional wage differences.

Before presenting an overview of the model the basic assumptions which underlie the analysis should be made explicit. In the first place, the model is partial-equilibrium in nature. Hours and wage rates of groups of people are determined endogenously, while their tastes, identifying demographic characteristics, and unearned income are taken as exogenous. The level and composition of output in each labor market is also determined outside the model. Second, we estimate cross-sectional relationships using states and state aggregates as our labor markets. Workers are assumed to be mobile within a state labor market but immobile between them. Third, workers are acknowledged to offer labor of various qualities. We assume that quality differences reveal themselves in human capital stock differences and that human capital is a homogeneous and substitutable factor of production. Fourth, the existing federal and state tax structure is not explicitly incorporated into the model. Rather it is incorporated implicitly in the estimates of the coefficients of the model.

Families in each state labor market are initially partitioned into 120 mutually exclusive and completely exhaustive family types based on the demographic and education characteristics of their head. Within each family we further distinguish between family heads (primary workers) and other members (secondary workers) so that there are initially 120 primary groups and 120 secondary groups. Using a one-way sequential analysis of variance algorithm we then aggregate the primary groups into eight homgeneous human capital groups and the secondary groups into eight corresponding groups for a total of sixteen human capital groups. Representative or average primary and secondary workers in each group are defined and used as observation units in the market model. Primary and secondary worker supply equations are derived from the traditional income-leisure choice model and specified for the representative worker in each group.

By assuming a high degree of correlation between the demographic-educational characteristics of workers and their utilized stocks of human capital we can treat these characteristics as indicators of their stocks of utilized human capital. Moreover, according to human capital theory the wage rate of an individual or group of similar individuals is the product of the market rate of return to human capital and the stock of utilized human capital. Therefore, relative wage rates, the wage rate of one group vis-a-vis that of a numeraire group, are determined in equilibrium only by relative stocks of human capital and are independent of the supply and demand conditions in a particular labor market.

We express the market demand for labor in terms of the hours of the numeraire group and as a function of the numeraire wage rate to reflect the fact that the quality of labor services vary from one group to another. The hours offered by each group are weighted by an estimate of their relative stocks of human capital, as a proxy for quality differences, and then aggregated to form market demand. Equilibrium obtains when market demand is exactly satisfied by the sum of weighted hours supplied by each group. The numeraire wage rate, which is also determined in the supply-demand equilibrium, and the relative wage rates, given by relative utilized human capital, then determine the absolute levels of the other wage rates.

The body of the paper is divided into five sections. In the first section, the supply, relative wage rate and demand relationships are derived from the underlying income-leisure choice, human capital, and production function theory. The notions of family types and representative families are also introduced.

In the second section we discuss the choice of the state as the labor market area and the creation of state output data, the use of Census data to define family types in terms of demographic-education characteristics of their heads, and the aggregation of primary and secondary workers into homogeneous human capital groups. In the third section the supply, relative wage, and demand equations are specified in accord with both the theory developed in the first section and the limitations imposed by the data discussed in the second section. We also discuss the methods used to estimate the simultaneous equations model. The estimated structural equations are then presented and discussed in the fourth section and compared to results reported by other investigators. In the fifth section we evaluate the explanatory power of the model by using an iterative technique to solve the non-linear simultaneous system for the equilibrium wage rates and hours of each family type-state observation. Predicted, actual, and percentage difference values are reported by family type and by state for primary and secondary wage rates, hours per family, earnings per family and income per family. To illustrate the model's usefulness in illuminating distributional questions, we analyze the effects of exogenous output and population changes on the structure of earnings and income. We conclude with a summary of the labor market model and the simulations performed.

I. LABOR MARKET THEORY

In this section we discuss the theoretical underpinning of the determinants of wage rates and hours worked. First, labor supply equations for a primary worker and for a representative secondary worker of each family type are derived from the maximization of family utility in a static income-leisure choice model. Second, we discuss the determination in equilibrium of relative wage rates by relative stocks of utilized human capital. Third, the market demand for labor is derived from the conditions for profit maximization of firms operating in competitive markets. Hours demanded are expressed in equivalent-quality units by converting the hours of primary and secondary workers in each family type into primary hours of the numeraire family type using the appropriate human capital ratios. Intersections of market demand and aggregate supply curves then determine in each labor market the equilibrium number of equivalent-quality hours and the numeraire wage rate. The wage rates and levels of employment for the individual groups of workers then follow from the relative wage and labor supply equations.

Family Labor Supply

The decision to supply the number of hours forthcoming is seen as a family decision in which there is interdependence between the work efforts of family members. To capture this interdependence we postulate separate supply curves for the family head (primary worker) and for the other family members (secondary workers) linked together in a simultaneous equations framework. The families in a labor market area are partitioned by the demographic and educational characteristics of their heads into mutually exclusive and collectively exhaustive types. These types are referred to as family types and will be discussed in detail in subsequent sections.

Consider now a representative family of the _i_th family type living in the _j_th labor market area. We assume this family maximizes its satisfaction by achieving the optimum mix of family income and leisure consistent with its preferences for income vis-a-vis leisure and the market opportunities it faces as expressed by its budget constraint. Formally the average family is said to maximize utility

$$(1.1) \qquad U_{ij} = u_i\{(T_i^p - H_{ij}^p)/F_{ij},\ (T_i^s - H_{ij}^s)/F_{ij}, Y_{ij}/F_{ij}\}$$

subject to the family budget constraint

$$(1.2) \qquad Y_{ij}/F_{ij} = W_{ij}^p H_{ij}^p/F_{ij} + W_{ij}^s H_{ij}^s/F_{ij} + Y_{ij}^n/F_{ij}.$$

The number of families composing the type is denoted by F_{ij}. The primary worker of the family consumes $(T_i^p - H_{ij}^p)/F_{ij}$ hours of leisure per year, where H_{ij}^p is the annual number of hours devoted to working by all primary workers in the type and T_i^p is their total available time. The corresponding secondary worker consumes $(T_i^s - H_{ij}^s)/F_{ij}$ hours of leisure per year. We note that just as the family is viewed as a type average, the secondary worker in a family is really an average of the spouse, if present, and other potential workers. Family income is denoted by Y_{ij}/F_{ij}. It is the sum of primary earnings, secondary earnings, and family non-labor income: $W_{ij}^p H_{ij}^p/F_{ij}$, $W_{ij}^s H_{ij}^s/F_{ij}$, and Y_{ij}^n/F_{ij}, respectively. The market wage rates faced by primary and secondary workers of the _i_th type are denoted by W_{ij}^p and W_{ij}^s.

The maximization of family utility tells us that each member's supply of hours to the labor market depends upon his (her) wage rate which is the price of an hour's worth of leisure, the wage of the spouse, and the family's non-labor income.

If we also assume, as is commonly done, that the technical cross-substitution effect between primary and secondary family members is zero, then the effect on primary (secondary) hours of a unit change in the secondary (primary) wage rate has the same magnitude as that of a unit change in non-labor income.[2] Accordingly, other family member earnings can be lumped together with non-labor income so that the labor supply equations for primary and secondary workers can be written as:

$$(1.3a) \qquad H^p_{ij}/F_{ij} = s^p_i\{W^p_{ij},(W^s_{ij}H^s_{ij} + Y^n_{ij})/F_{ij},Z^p_{ij}\}$$

and

$$(1.3b) \qquad H^s_{ij}/F_{ij} = s^s_i\{W^s_{ij},(W^p_{ij}H^p_{ij} + Y^n_{ij})/F_{ij},Z_{ij}\},$$

where Z^p_{ij} and Z^s_{ij} are vectors of socio-demographic variables.

These equations make explicit several important features of our model. First, the labor supply response of a particular family type i to social, demographic, and economic influences is the same for a family working in any market area. Of course the magnitudes of the explanatory variables vary from one labor market to another, but the hours response to a unit change in these variables is assumed to be the same in all areas. In other words, these are family labor supply equations estimated across labor market areas. Second, we allow, for variation in labor supply behavior from one family type to another. Third, labor supplies forthcoming from primary and secondary workers are interdependent. The primary worker's hours depend upon secondary earnings and secondary worker hours depend upon primary earnings. As mentioned there are other variables (Z^p_{ij} and Z^s_{ij}) in addition to wage rates and non-labor income which affect the family's choice between employment and leisure. They are defined in the data section and discussed in the model specification section.

Having described average primary and secondary workers in family type i as facing market determined wage rates, we now turn to a discussion of the role of human capital stocks in determining the relative wage rate faced by primary workers of one family type vis-a-vis the wage rate faced by primary workers of another type, and the wage of secondary workers in a family type vis-a-vis the wage of primary workers of the same type.

Utilized Human Capital

The wage rate commanded by the family head is viewed as the product of the market's rate of return to human capital and the stock of human capital utilized by that person.[3] If stocks of human capital can be uniquely associated with membership in a particular demographic-educational group, then we can express the wage rate of a primary worker from a family of the ith type who lives and works in the jth market area as

$$W_{ij}^{P} = \omega_j K_i^{P}. \qquad (1.4)$$

The rate of return is the rental price of a unit of human capital in the market and is denoted by ω_j, and K_i^P indexes the stock of human capital. This equation expresses the fact that wage rates (of primary workers) may differ across the same family types in different market areas because of differences in the market rate of return and may also differ across family types in the same market area because human capital stocks vary over demographic groups.

Because of occupational discrimination by race and sex certain groups in our society earn less than the market rate of return on the stock of human capital that they possess.[4] We view this as underutilization of their human capital and do not attempt to explain it any more than we explain the distribution of human capital stocks across groups. In our discussion of wage rate determination, therefore, we are referring to utilized rather than actual human capital.

In equilibrium human capital stocks and supply-demand conditions, as reflected in the market rate of return, determine the absolute market wage rate. Relative market wage rates, however, are determined only by relative stocks of human capital:

(1.5) $$W_{ij}^{p}/W_{1j}^{p} = K_{i}^{p}/K_{1}^{p},$$

for the ith primary group relative to the first primary group in the jth labor market. This relation follows directly from (1.4). It does so because we are assuming that there is one market where all groups can sell their human capital with equal ease and where they receive the same rate of return on the marginal unit. In this sense human capital is homogeneous and perfectly substitutable.

Substitutability can be illustrated in this context by considering a situation in which relative wage rates are out of equilibrium as given by (1.5). Suppose the ith group's relative wage rate exceeds its relative stock of human capital. An employer will observe that an additional dollar spent on his wage bill will purchase more units of human capital relative to its cost if it is spent on obtaining services from group 1 rather than group i. By the homogeneity assumption he views the services provided as identical and, consequently, proceeds to purchase extra services from group 1. This increases W_{1j}^{p} relative to W_{ij}^{p} and pushes wage rates towards equality with relative stocks of utilized human capital.

If the original premise of this discussion -- that demographic and educational characteristics uniquely determine the stock of human capital -- is not fulfilled, then relative wage rates will not be constant across market areas. The closer the premise is to being fulfilled, the less will be the variation in relative wage rates. We note that for any demographic group, variation in the utilization rate of its human capital across market areas will also produce variation in the corresponding relative wage rates.

As already mentioned, family types are defined by the demographic characteristics of the family head. Thus, secondary workers of a given family type are grouped according to the head's characteristics. There is a presumption, therefore, that demographic similarities of secondary workers within a given family type are greater than similarities among secondary workers of different family types. The reader is reminded that the secondary worker is an average of spouse and dependents so that the presumed intra-group similarities also imply family compositional similarity.

Accordingly, we assume a high degree of correlation but not equality between the stock of human capital of the head and that of the secondary workers so that secondary workers of the *i*th family type can be though of as homogeneous in their stocks of human capital. This allows us to write a wage equation for secondary workers analogous to that for primary workers:

$$W^{s}_{ij} = \omega_j K^{s}_{i}. \tag{1.6}$$

The equilibrium market wage confronting a representative secondary worker of the *i*th family type working in the *j*th labor market area is the product of a market rate of return and her(his) stock of utilized human capital.

Therefore, wage rates of secondary relative to primary workers of a given family type are determined independently of the particular market by the corresponding relative stocks of human capital in equilibrium:

$$W^{s}_{ij}/W^{p}_{ij} = K^{s}_{i}/K^{p}_{i}. \tag{1.7}$$

This relationship, just like (1.5), is contigent on an association between demographic characteristics of the primary worker and his (her) stock of human capital and on a high correlation between primary and secondary worker utilized human capital stocks.

We consider next the demand for labor of different qualities and the role of relative stocks of human capital in the aggregation of labor inputs to obtain market demand.

Market Labor Demand

In each market area the demand for labor in equivalent-quality units is derived from the aggregate demand for output. We employ a constant elasticity of substitution production function to relate factor inputs to aggregate output. The production function characterizing the jth market area is

$$Q_j = \alpha_j(\beta_j C_j^{-\rho} + (1-\beta_j)L_j^{-\rho})^{-\mu/\rho}, \tag{1.8}$$

where Q is real value added, C is an index of aggregate capital services, and L is an index of aggregate labor services. The elasticity of substitution σ may be written as $\sigma = 1/(1+\rho)$. The returns to scale are measured by the value of the parameter μ.

The existence of consistent indexes of aggregate capital and labor services is intimately related to estimates of the partial elasticities of substitution among the components of the index.[5] In our model we assume that the elasticity of substitution between the different types of labor is infinite within any market area, while no substitution is possible across labor market areas.[6] Relative stocks of human capital, which in equilibrium equal relative wage rates, are used to weight the labor inputs of each human capital group. Using primary workers of the first family type as a numeraire, the aggregate labor index is:

(1.9) $$L_j = \sum_i (\kappa_i^p/\kappa_1^p)H_{ij}^p + \sum_i (\kappa_i^s/\kappa_i^p)(\kappa_i^p/\kappa_1^p)H_{ij}^s.$$

In effect, the index translates the hours of each primary and secondary group into the equivalent numeraire group number and aggregates them. The result is a measure of equivalent-quality hours. It is based on previous research which has applied relative wage rates to obtain a measure of labor input in constant-quality units for estimation of an inter-regional production function,[7] and to adjust an historical measure of labor hours for changes in the quality of labor services.[8]

The demand for labor is derived from the production funtion (1.8) under the assumption of perfect competition among employers: free entry and exit, and the inability of any employer to perceive an influence on factor input and output prices from his actions. In particular we assume that competitive, profit-maximizing employers alter their input mix until each factor's marginal product equals its cost and returns to scale at the margin are constant (μ=1). The demand function for aggregate labor may then be written as:

(1.10) $$L_j = (\beta_j\mu)^{1/(1+\rho)}Q_jW_{1j}^{-1/(1+\rho)}$$

where W_{1j} is the real price of L_j. Since output is assumed to be given exogenously, we may rewrite (1.10) as

(1.11) $$L_j/Q_j = d_j\{W_{1j}, I_j\},$$

where I_j captures industrial differences in the demand for labor. Demand for equivalent-quality labor per dollar of output in the jth market area is a function of the numeraire group wage rate. Moreover, demand will differ from one market area to another corresponding to differences in industrial composition.

Equilibrium in the jth labor market is achieved when the demand and supply of equivalent-quality hours are equal, as expressed by (1.9). This completes the theoretical development of the market model. To review: wages and hours of each family type are jointly determined in each market area. For each family type there are primary and secondary supply functions ((1.3a) and (1.3b)) and relative human capital equations ((1.5) and (1.7)) for the representative family. The market demand function is expressed in equivalent-quality, i.e. numeraire group, hours ((1.11)). Market equilibrium jointly determines primary and secondary hours for each family type, the numeraire group wage rate, and by construction -- via relative human capital stocks -- the values of the remaining wage rates. The level and composition of output and demographic characteristics and unearned income by family type are taken as exogenous.

II. LABOR MARKET DATA

In this section we first discuss the choice of states as labor market areas and the measurement of state output using data published by the Bureau of Economic Analysis (BEA), U.S. Department of Commerce. With the State Public Use Sample of the 1970 Census as the data source, labor supply data are considered next: family membership and types are defined, and the measurement of economic variables for representative workers is described. Finally, using family types as building blocks, the formation of human capital groups to represent homogeneous units of labor supply is described.

Labor Market Areas

We use the State Public Use Sample (1 in 1000) of the 1970 Census as our basic source. The labor market area chosen is the state. In selecting this unit the advantages and disadvantages of alternatives were considered. To analyze the distribution of wage rates, hours, and hence earnings by family type in the U.S. we require a set of mutually exclusive and exhaustive units. States are obviously exclusive and exhaustive units, but they may contain several distinct labor market areas, and labor market areas may even cross state boundaries. A prime alternative to the state would be the functional economic area (FEA) — a primary place of economic activity and commuting surrounded by the rest of the area for which it is the trading and labor market center. These areas have been delineated by the BEA. There are 173 mutually exclusive and completely exhaustive areas.

We chose not to use the FEA for several reasons. Most importantly, it would involve use of the County Group Public Use Sample and would require use of the 1 in 100 sample for smaller areas, greatly increasing data processing requirements. Secondly, states are for many purposes appropriate units for analyzing the effects of human resource programs -- a task for which this model has been designed -- because such programs are defined, within limits, and administered on a state level. However, the fact that we do aggregate some of the less populous states lessens this advantage of states vis-a-vis FEA's.

State have been aggregated to reduce sampling error and to create a set of observations compatible with the Current Population Survey Public Use Sample so that our results can be tested with an alternative set of data.[9] The following aggregations have been created: Alabama-Mississippi; Alaska-Hawaii-Washington; Arizona-Colorado-New Mexico; Arkansas-Oklahoma; Delaware-Virginia; Idaho-Montana-Nevada-Utah-Wyoming; Iowa-Minnesota; Kansas-Nebraska-North Dakota-South Dakota; Maine-Massachusetts-New Hampshire-Rhode Island-Vermont; Michigan-Wisconsin; North Carolina-South Carolina. In all, 32 states have been aggregated into 11 larger groups, while 18 states and the District of Columbia stand by themselves, making a total of 30 labor market observations.

Non-farm output estimates by state are constructed from state personal income estimates.[10] The August issue of the Survey of Current Business publishes estimates of national and personal income by major source or industrial sector. In the July issue estimates of national total income by industry division are published. In order to derive an estimate of the total income originating in a given sector in a state we assume that the ratio of total to personal income originating in any state equals the ratio of total to personal income originating in that sector for the nation. Estimates of total income are derived in this manner for each state or state aggregate for these sectors: mining; contract construction; manufacturing; wholesale and retail trade; finance, insurance, and real estate; transportation, communications, and public utilities; and services.

Estimates of total income originating in the government sector by state are handled analogously. Here we assume that the ratio of total income to the nonmilitary government payroll in any state equals the ratio of income originating in all governments in the nation to the wages and salaries paid by all governments in the nation.

State output is the sum of the estimated contributions to value added from each sector. It should be stressed that our output measures do not include depreciation or indirect business taxes. They correspond to national income at factor prices and are, therefore, the most satisfactory measure for estimating derived demand functions for factors of production.

In the demand equation we denote the state output estimates for 1969 by NOUT. Variation in industrial composition across states is captured by NOUTI, which denotes the percentage of output originating in the industrial sector (mining, contract construction, manufacturing, transportation, communication and public utilities).

Finally, it should be noted that wages, income and output are measured in nominal terms because a satisfactory regional price index does not currently exist.[11]

Census Public Use Sample

We now turn to the definitions of family members and family types with Census data. The Census is a household survey. The head of the primary family in the household and any unrelated (to the head) individuals, roomers, boarders, and lodgers are treated as primary workers in our model. All other people in the household -- i.e. other members of the primary family and members of any subfamily -- are treated as secondary workers. We exclude all people living in group quarters except for those in rooming and boarding houses, tourist homes, and communes.[12] Of those living in households, we exclude from the labor force people less than sixteen years of age, those employed by the military, and those who earn their living on farms (Census Occupation Codes 801-846).

In each state or state aggregate primary workers are partitioned into 120 groups by race, age, sex, and education. Race is white and non-white. The age categories are defined by five ranges which intuitively correspond to different periods of the working life cycle: 16-21, 21-35, 36-54, 65 and over. The education categories represent six levels of schooling: less than 9 years, 9-11 years, high school graduation, some college, college graduation, and graduate education. Lastly, sex separates male-headed from female-headed families. As discussed, secondary workers are partitioned by the demographic characteristics of their primary workers. There are 2x5x6x2 family types defined, although we note that in the 1970 State Public Use Sample not all 120 types exist in all thirty states and groups of states.

Annual hours are estimated for each group by multiplying the number of weeks worked in 1969 by the average number of hours worked in the Census reference week in 1970.[13] Although this calculation may produce considerable error for any single worker, we believe that our use of groups rather than individuals precludes significant bias across groups: hours of the representative primary (secondary) worker of a given family type are obtained by summing the hours of all such primary (secondary) workers and dividing by the total number of families of that type. There may be, however, a seasonal bias if the Census reference week is not representative of the entire year.

Annual earnings are composed of wages and self-employed non-farm income. Earnings of the representative primary (secondary) worker is the sum over all such workers divided by the number of families of the type considered. Since the Census does not report wage rates, we divide earnings by hours to obtain an annual hourly wage rate of the representative worker.

Non-labor income is a family rather than a primary or secondary worker variable. It includes, social security, unemployment compensation, pension receipts, rental and dividend income, but excludes public assistance receipts. Social security, unemployment compensation, pension receipts, and public assistance receipts may all reflect the person's employment level as well as help to determine it. Public assistance, however, is to be distinguished from the other three forms of income transfer by the manner in which receipts vary continuously with earnings. The effect of the other transfers is captured in part by the age variable. Non-labor income for the representative family is obtained by summing non-labor income over all families and dividing by the number of families of this type.

In addition to the economic variables, socio-demographic information for the individuals comprising each family type is tabulated. Before describing these variables, we turn to a discussion of the statistical and computational problems inherent in the use of 240 human capital groups and the resultant aggregation that was performed.

Human Capital Aggregation

Not only are some of the 240 human capital group cells in each state empty, but there are a large number composed of only a few individuals. This is not surprising considering that, for example, non-white primaries comprise approximately ten percent of the population but are partitioned into sixty cells; or female-heads which are approximately ten percent of all heads and also partitioned into sixty cells. Furthermore, the youngest and oldest age cells, as well as the higher education cells, are comprised of small numbers of workers. The associated high sampling errors for these cells would likely swamp the true (population) values so that supply functions for many of the family types could not be estimated. Even pooling the types and correcting for heteroscedasticity error variance would not likely be sufficient.

Accordingly, the 120 family types were used as building blocks to form larger groups of workers reasonably homogeneous in their amounts of utilized human capital. To do this we turned to the records of individual primary workers in California and used as an indicator of utilized human capital the individual's annual average hourly wage rate. A one-way, sequential analysis of variance algorithm--Automatic Interaction Dectector -- was employed to form those aggregate groups which best explain the wage rate variation by maximizing inter-group and minimizing intra-group variation.[14] This technique was chosen in preference to linear regression because we did not wish to impose any a priori restrictions on the relationship between human capital and the demographic characteristics used as predictor variables. The explanatory variables are the same race, age, sex, and education variables which generate the 120 family types. A breakdown of the sample -- number of workers and average wage rates -- by these variables is presented in Table 1.

Any variance or total sums-of-squares can be definitionally separated into two parts, an inter-group or between group sums of squares (BSS) -- the so-called explained variation given the groups -- and the sum of the intra-group sums of squares. The algorithm calculates the variance of the wage rate for each possible dichotomization of the group under scrutiny by each explanatory variable, splitting that group on the variable which accounts for the most variation in the wage rate, i.e. the one which gives the largest BSS. For example, in the analysis of California primary workers, the entire sample, was initially split by the education variable, separating those workers with at least college degrees (average wage rate of $6.94) from those with less schooling ($4.70). Continued splitting occurs on those groups which meet a minimum variation requirement -- otherwise they are considered homogeneous -- if the split reduces the unexplained variation by a specified proportion of the original parent sample variation, and to keep sampling error in bounds -- if the split results in offspring meeting a minimum size requirement. With the set of predictor variables used the maximum number of final groups is 120. We note that the splitting stopped far short of this maximum.

Before describing the final groups it is instructive to describe the branching or splitting as it occurred. Recalling that the entire sample initially split on education into those with 0-15 years of schooling and those with 16 and over, the less educated group next split on sex. In the ensuing rounds, female primary workers split first on schooling (0-8 years versus 9-15 years) and then the more educated group split on age (16-35 years olds versus 36-over), while males went

TABLE 1

Composition of California Primary Workers by Sex, Education, Age, and Race with Average Hourly Wage Rates 1970 Census State Public Use Sample

	Observations		Av. Hourly
	Number	Proportion	Wage Rate
Sex			
Male	3794	.827	$ 5.48
Female	796	.173	3.30
Education			
0-8 years of school	526	.115	$ 4.37
9-11 years of school	714	.156	4.45
High school graduate	1543	.336	4.72
Some college (13-15 years)	981	.214	5.03
College graduates	383	.083	6.49
17-over years of school	443	.097	7.34
Age			
16-21 years old	161	.035	$ 2.68
22-35 years old	1572	.342	4.42
36-54 years old	2010	.438	5.62
55-64 years old	669	.146	5.27
65-over	178	.039	6.88
Race			
White	4179	.910	$ 5.21
Non-white	411	.090	4.05
Total	4590	1.000	$ 5.11

through a similar but lengthier series of splits. The race variable was used only once to dichotomize middle-aged (36-64) males with less than high school degrees. The implication is that in explaining the total variation in wage rates across a large diverse sample -- such as all California primary workers--education, age, and sex differences are relatively more important vis-a-vis race by themselves, together with the fact that nonwhites comprised only nine percent of the sample and hence could not account for a significant proportion of the total variation unless their wage rates were extremely low--which they are not: an average hourly wage of $3.82 compared to $4.88 for middle aged males without high school degrees.

Returning to the college educated branch, there are splits only on age and schooling, indicating that there are relatively few women in our sample with 16 or more years of schooling, so that sex differences do not account for much of the total group variation. This does not imply that male-female wage differences are not present (see below), just that in this branch the effect of sex on wage rates may be swamped by other characteristics.

The algorithm produced thirteen final groups. Using a priori judgement with a view to having groups not too dissimilar in size, several of the final groups were collapsed so that we finally arrived at eight reasonably homogeneous human capital groups of primary workers. These groups are listed in Table 2 along with their alphabetic identifiers (which will be used throughout the paper). For each group the Table also reports the number of primary workers and their proportion of the total state sample, their average hourly wage rate, and their implied units of utilized human capital, using Group 4 M-O-H as a numeraire.

The group with the highest average hourly wage rate are college graduates who are 36 years and older. Younger college graduates of both sexes had wage rates approximately the same as older male high school graduates and below those of older males with some college, indicating that experience (as measured by age) does sub-

TABLE 2

Human Capital Groups by Demographic Characteristics, California Primary Workers
1970 Census Public Use Sample

	Observations		Average Hourly Wage Rate	Units of Utilized Human Capital
	Number	Proportion		
1. Male and Female, age 35-over College graduates (M/F-O-C)	513	.112	$ 7.79	1.37
2. Male and Female, age 16-35 College graduates (M/F-Y-C)	313	.068	5.56	.98
3. Males, age 36-over, School 0-11 years (M-O-NH)	790	.172	5.00	.88
4. Males, age 36-over, High school graduate (M-O-H)	742	.162	5.67	1.00
5. Males, age 36-over, Some college (M-O-SC)	420	.092	6.17	1.09
6. Males, age 16-35, School 0-15 years (M-Y-NC)	1127	.245	4.28	.75
7. Females, age 36-over, School 0-15 years (F-O-NC)	392	.085	3.50	.62
8. Females, age 16-35 School 0-15 years (F-Y-NC)	293	.064	2.83	.50
Total Sample	4590	1.000	$ 5.11	

stitute for education. Moreover, the sex composition difference between Groups M/F-Y-C, and M-O-H and M-O-SC is a slight factor: younger male college graduates had wage rates of $5.78, slightly higher than older male high school graduates and still below older males with some college. The lower wage rates of female primary workers can be inferred from a comparison of Groups M-Y-NC and F-Y-NC, and F-O-NC with an average of M-O-NH, M-O-H and M-O-NC. Moreover, among college graduates, wage rates of older females average $4.71 compared to $8.18 for their male counterparts, and those of younger females average $4.43 compared to $5.78 for younger males. The striking feature is that the human capital of college graduate female primary workers fails to grow at anything near the male rate over the life cycle. Other detail not shown indicates a similar phenomenon for females vis-a-vis males with less than college degrees. Among the younger females, the difference between those with 0-8 years of schooling and those with some college is only $0.19 per hour compared to $1.28 for males; among the older females it is $0.67 compared to $1.17 for males. In sum, average wage rates correspond closely to educational levels although there is a trade-off between schooling and experience. Average wage rates of female primary workers are less than corresponding male workers and the pay-off to more schooling is also lower for females.

The analysis of variance technique has been used to delineate eight groupings of primary workers in California which are reasonably homogeneous in wage rates and, therefore, in utilized human capital. This structure is applied to each state to create sixteen groups of workers -- eight primary and eight secondary -- from the 120 family types. Accordingly, there are sixteen labor supply observations in each state.

In addition, to the economic variables, socio-demographic information for the individual is extracted and the associated human capital group variables are built up from individual characteristics. Appearing in the labor supply equations are: RACEP, RACES; AGEP, AGES; SEXP, SEXS; EDUP, EDUS; SPOUSE; DPENDP, DPENDS; and URBAN. The P and S suffixes denote primary and secondary workers respectively. For each group in each state: RACE is the proportion of white workers; AGE is the proportion of prime-age workers (22-54 years old); SEX is the proportion of males; EDU is the proportion of high school graduates; SPOUSE is the proportion of families with spouse present in the household; DPENDP is the proportion of families in which there are financially dependent children (those less than 18 years of age), while DPENDS denotes the proportion of families with adult secondary workers who also have dependent children; and URBAN is the proportion of families living in urban places. We note that since the characteristics of the primary worker define the family type, SEXP, and EDUP take on only 0 to 1 values for some of the groups.

III. STRUCTURAL EQUATIONS SPECIFICATION

In this section we specify the relative human capital, supply, and demand equations used to predict wages and hours for primary and secondary workers in each of sixteen human capital groups in each of thirty states and state aggregates. We present a rationale for the variables which have been included in the equations and discuss the methods employed to estimate the simultaneous equations model.

Relative Wage Equations

The reader may recall that in equilibrium and under the assumptions that we have made, relative primary wage rates (any primary group vis-a-vis the numeraire group) and relative secondary wage rates (any secondary group vis-a-vis its primary) are equal to their corresponding relative stocks of human capital. To estimate the relative stocks of human capital for each group we have chosen an analysis of variance-regression model in which the logarithm of the relative wage is the dependent variable and the explanatory variables are demographic variables.[15] The primary relative wage rate equation is

$$(3.1) \quad \log(W^{p}_{ij}/W^{p}_{4j}) = a_1 D_{1j} + \dots + a_3 D_{3j} + a_5 D_{5j} + \dots + a_8 D_{8j} + \gamma_{ij}$$

where M-O-H is the numeraire group, and the D's are binary variables with one for each age-sex-education interaction. The error term is denoted by γ_{ij}.

Relative primary wage rates for each group are estimated over the thirty state observations and the predicted values are taken as estimates of equilibrium relative stocks of human capital. We denote them, following (1.5), by $\widehat{K^{p}_{i}/K^{p}_{1}}$, for $i = 1, \dots, 3, 5, \dots 8$.

Specification of the relative secondary wage rate equation follows analogously:

$$(3.2)\ \log(W^{s}_{ij}/W^{p}_{ij}) = b_1D_{1j} + \ldots + b_3D_{3j} + b_4D_{4j} + \ldots + b_8D_{8j} + \delta_{ij}.$$

Relative secondary wage rates for each group are estimated over the thirty state observations and the predicted values are taken as estimates of the equilibrium ratio of primary to secondary stocks of human capital. Following (1.7) we denote them by $\widehat{K^{s}_{i}/K^{p}_{i}}$, for i = 1,..., 8.

In addition to the binary age-sex-education variables we also introduce a south/non-south dummy variable NS to test for regional variation in relative wage rates. If, in the first place, there is differential discrimination across states confronting one or several human capital groups the proportion of utilized to actual human capital will vary accordingly and some of the variation may be explainable by a simple dummy variable. Second, there has been much discussion of the variation in the quality of schooling across the country and, particularly, that the quality is lower in the South. If the quality of schooling is not uniformly inferior across groups in the South then its presence should be reflected in regional variation in relative wage rates. The southern state observations are: Alabama-Mississippi, Arkansas-Oklahoma, Delaware-Virginia; District of Columbia, Florida, Georgia, Kentucky, Louisiana, Maryland, North Carolina-South Carolina, Tennessee, Texas, and West Virginia.

Labor Supply Equation

The primary supply equation in its feedback formulation is given by

$$(3.3)\quad H^p_{ij}/F_{ij} = c_{0i} + c_{1i}W^p_{ij} + c_{2i}(W^p_{ij})^2 + c_{3i}(W^s_{ij}H^s_{ij} + Y^n_{ij})/F_{ij}$$
$$+ c_{4i}\ \mathrm{RACEP}_{ij} + c_{5i}\ \mathrm{AGEP}_{ij} + c_{6i}\ \mathrm{SEXP}_{ij} + c_{7i}\ \mathrm{EDUP}_{ij}$$
$$+ c_{8i}\ \mathrm{URBAN}_{ij} + c_{9i}\ \mathrm{SPOUSE}_{ij} + c_{10i}\ \mathrm{DPENDP}_{ij} + \varepsilon_{ij},$$

where ε_{ij} is a random error term and the socio-demographic variables have been defined in the previous section. There are eight primary human capital groups (index i) in each of thirty states (index j). In our estimation work we take as the null hypothesis a simpler, recursive formulation -- primary hours supplied are independent of secondary earnings — and examine whether the data support the more complex feedback behavior. In addition to our preference for simpler over complex formulations, it has also been conjectured by other researchers that the dependence of the primary worker upon secondary earnings is small relative to the dependence of the secondary worker upon primary worker earnings.[16] The argument is that the existence of positive secondary earnings may reflect the fact that the head of the family is not working as well as being a determinant of the number of primary hours worked.

We experimented with regressions for each group separately and with stacking several groups in a regression--the extreme case being all eight groups in one regression of 240 observations. In separate regressions for certain groups note that SEXP_{ij} and EDUP_{ij} are constant across states and consequently excluded. In a stacked regression we are assuming the same wage rate and income response behavior from one group to another but are allowing for different demographic characteristics between groups to shift the labor supply curve.

Given the Census definition of head of family we expect the labor supply response of people in male-headed families to be different from that of people in female-headed families. Male-headed and female-headed families are structurally different; in the former there may be a spouse, while in the latter by definition there is no spouse. When there is a spouse present the family has an additional factor of production and the primary worker enjoys greater flexibility in allocating his time between work, non-market work, and leisure. The female head becomes responsible for both earning a living and taking care of the home and children so that her labor supply response to market wage rates is influenced by a host of limiting considerations.

The sex of the family head also determines the composition of its secondary workers. Female-headed families do not have an adult spouse as part of their secondary worker labor force. In these families the supply of secondary hours is composed of dependents and other individuals related to the head. We conjecture that the compositional difference may yield different labor supply response behavior on the part of secondary workers.

We have also used "permanent" or potential wage rates instead of actual wage rates in the supply equations. It has been hypothesized that the family's labor supply decision revolves around its perception of potential income over the intermediate term horizon rather than current earnings. The labor supply behavior observed is strongly influenced by expectations which themselves are shaped by economic institutions and demographic-education factors.[17] The model was also estimated with actual wage rates but gave inferior estimates.

The basis for our application of human capital theory to the labor market model is the assumption that differences in potential wage rates are primarily attributable to differences in productivity. As a first approximation, we use the combinations of demographic characteristics employed in the definition of human capital groups as proxies for productivity differences. Accordingly, we take the estimated relative stock of human capital as a measure of average relative productivity of groups, and construct potential wage rates as follows:[18]

(3.4) $$\hat{W}^p_{ij} = (K^p_i/\hat{K}^p_4)W^p_{4j} .$$

In equilibrium the representative primary worker (of the _i_th family type and _j_th state) can be thought of as having expectations of a wage rate proportional to that of the numeraire group, the constant of proportionality being a measure of his(her) relative productivity. We also note that the concept of potential wage rates gives us a method for imputing a wage rate to a group which has reported zero hours and earnings and consequently, for whom the actual wage rate is undefined.

We seek to explain the variation in the supply of hours from the representative primary worker of each family type over the state observations. _A priori_ expectations are for increased supply in response to increased wage rates ($c_{1i}>0$) though backward-bending phenomena may be observed ($c_{2i}\leq 0$). There is a presumption that primary hours supplied will vary inversely with the sum of secondary earnings and family non-labor income ($c_{3i}<0$); this assumes that the primary worker's consumption of leisure would increase as family income increases. Two variables are introduced to account for the influence of marital status and the presence of children. The presence of a spouse or dependents may entail greater financial responsibilities and may alter the family head's preference for work over leisure; we would expect $c_{9i}>0$ and $c_{10i}>0$. Geographical location may also influence the income-leisure choice. In highly urbanized areas, the alternative of leisure may be less compelling because of a higher opportunity cost of leisure, the external influence of others in determining one's own consumption patterns and because of a possible increased cost of leisure activities; we would expect $c_{8i}>0$.

The white/non-white variable reflects any differences in taste preferences for work versus leisure, including those resulting from past occupational discrimination. The use of a prime/non-prime age variable reflects several considerations. Individual work preferences vary over the life cycle. The difficulty of borrowing for consumption in early adult life may require greater work effort during those early years than in later years. Older workers may choose to curtail their work effort in order to maintain their health or at least their ability to enjoy leisure hours. The sex variable accounts for differences in work-leisure preferences resulting from the more limited opportunities for work of female heads as well as the presence of programs such as AFDC which primarily affect work effort of female heads. A high school graduate versus non-graduate variable reflects different taste preferences for work which may arise in part from the fact that graduates are likely to have more opportunities for employment. It should be noted that a logarithmic formulation of primary supply was also estimated but found generally inferior in terms of signs and significance levels to the quadratic formulation.

The supply curve of a representative secondary worker is estimated using a quadratic formulation:

$$
\begin{aligned}
(3.5)\qquad H^{s}_{ij}/F_{ij} &= d_{0i} + d_{1i}W^{s}_{ij} + d_{2i}(W^{s}_{ij})^{2} + d_{3i}(W^{p}_{ij}H^{p}_{ij} + Y^{n}_{ij})/F_{ij} \\
&+ d_{4i}\ \mathrm{RACES}_{ij} + d_{5i}\ \mathrm{AGES}_{ij} + d_{6i}\ \mathrm{SEXS}_{ij} + d_{7i}\ \mathrm{EDUS}_{ij} \\
&+ d_{8i}\ \mathrm{URBAN}_{ij} + d_{9i}\ \mathrm{SPOUSE}_{ij} + d_{10i}\ \mathrm{DPENDS}_{ij} + \eta_{ij},
\end{aligned}
$$

where η_{ij} is a random error term. We also experimented unsuccessfully with a logarithmic formulation. As with the primary supply equation we ran regressions on groups separately and stacked.

Secondary workers are expected to increase their work effort as wages increase ($d_{1i}>0$) though supply may bend backward ($d_{2i}\leq 0$). Secondary hours also are expected to vary inversely with other family income ($d_{3i}<0$), though a positive sign may indicate a preference for market work by the secondary worker (housewife) when family income is sufficient to either mechanize the home work or hire a substitute. The proportion of white, prime age, male, high school graduate secondary workers in the state for a given group are variables which control for differential tastes for work versus leisure, in the same way that RACEP, AGEP, SEXP, EDUP do in the primary supply equation. The proportion of families with spouse present accounts for differences in family composition. Finally, the presence of dependent children in a family is surmised to affect the work-leisure choice of the spouse; the direction is indeterminate ($d_{10i}\gtrless 0$), depending on the spouse's hourly wage rate relative to child care costs and the value of house (non-market) work.

Just as in the case of primary workers we take the estimated relative stock of human capital as a measure of average relative productivity of groups and construct potential wage rates:

$$\hat{W}^{s}_{ij} = (\widehat{K^{s}_{i}/K^{p}_{i}})\hat{W}^{p}_{ij} . \tag{3.6}$$

The representative secondary worker is expected to recieve a wage rate proportional to the potential wage rate of his(her) primary worker, the constant of proportionality being a measure of relative productivity.

Labor Demand Equation

We express the market demand for labor services in equivalent-quality hours (i.e. units of Group M-O-H hours) and posit that the demand per dollar of product output in a state (NOUT) is explained by the prevailing wage rate for numeraire group workers and the industrial composition in that state (NOUTI). Following (11) the equation is

$$\text{(3.7)} \qquad \log(H_j^d/NOUT_j) = \log(e_0) + e_1\log(W_{4j}^p) + e_2 NOUTI_j + \Theta_j ,$$

where Θ_j is a random error term. Total equivalent-quality hours, H_j^d, are derived by (1.9) using the weights estimated in (3.1) and (3.2):

$$\text{(3.8)} \qquad H_j^d = \sum_i (\widehat{K_i^p/K_4^p}) H_{ij}^p + \sum_i (\widehat{K_i^s/K_i^p})(\widehat{K_i^p/K_4^p}) H_{ij}^s .$$

An estimate of the elasticity of demand for equivalent-quality hours with respect to the numeraire wage rate is given by e_1. An estimate of the effect on the demand for labor of a shift in industrial composition is given by e_2. A negative coefficient is consistent with the greater labor intensity of the service sector.

We turn now to a discussion of the methods employed to estimate the supply and demand equations. The simultaneous equations are estimated using an instrumental variables method. Moreover, since the model is nonlinear we augment the exogenous variables with instruments which incorporate information regarding the specific form of nonlinearity in each equation. Accordingly, for the demand equation, (3.7), we augment the instruments with the logarithm of the fitted value of W_4^p based on a regression of all the exogenous variables, which are the other instruments for the equation. For the supply equations, we augment the instruments with the square of the fitted wage rate and the product of the fitted values of H^s and W^s in the primary equation, and with the square of the fitted value of W^s and the product of the fitted values of H^p and W^p in the secondary supply equation.[19]

In the model, there are two exogenous variables, NOUT and NOUTI, determining equivalent-quality labor demand. Moreover, for each family type, there are nine exogenous variables: F, Y^n, URBAN, SPOUSE, DPENDP(DPENDS), RACEP(RACES), AGEP(AGES), SEXP(SEXS), and EDUP(EDUS) determining primary (secondary) worker labor supply. Therefore, if all the exogenous variables in the model were used in the estimation of the demand and supply equations, there would be 112 instruments (14 variables times eight groups) in addition to the two output variables and the fitted instruments. Needless to say, this would present a severe degrees of freedom problem in estimation since we have observations from only 30 states or groups of states. To circumvent the problem we have selected the most important exogenous variables for use as instruments in each equation. For the primary (secondary) supply equations, we have chosen as instruments the two output variables, the nine exogenous variables corresponding to the group for which supply is being estimated, and the corresponding fitted instruments as described above. For the demand equation we have chosen not only the output variables and the logarithm of the fitted wage, but also the exogenous variables for the numeraire group, since demand is measured in terms of hours supplied by the head as a function of his/her wage rate.

IV. ESTIMATED STRUCTURAL EQUATIONS

In this section we present the estimates of the relative wage, the labor supply and demand equations. The estimates are discussed and compared with those obtained from similar labor supply models and data bases by other investigators.

Relative Wage Equations

It may be recalled that a major implication of our model of (absolute) wage rate determination is that in equilibrium relative wage rates are determined by relative stocks of utilized human capital and therefore are constant across state labor markets, although we do test for regional (South vs. non-South) differences. The estimated coefficients with standard errors in parentheses are reported below:

$$(4.1) \qquad \log(W^p_{ij}/W^p_{4j}) = \underset{(.022)}{.399}\,D_1 + \underset{(.022)}{.069}\,D_2 - \underset{(.022)}{.185}\,D_3 + \underset{(.022)}{.123}\,D_5 - \underset{(.022)}{.254}\,D_6$$

$$- \underset{(.022)}{.553}\,D_7 - \underset{(.022)}{.585}\,D_8 + \underset{(.015)}{.054}\,NS\,,$$

$$R^2 = .91, \quad S.S.R. = 2.43, \quad S.E.E. = .11, \quad NO.OBS. = 210,$$

where R^2 is the coefficient of determination, S.S.R. is the sum of squared residuals, and S.E.E. is the standard error of the estimate. The coefficients can be interpreted as the percentage deviation of the wage rate of the group in question from the wage rate of the numeraire group when the absolute difference is not large. The south/non-south dummy is significant and indicates primary relative wage rates are higher in the non-south states. We note that the standard errors are identical because the binary variables are independent.

The high proportion of variance explained as well as the low standard error of the estimate is strong confirmation of the reasonableness of assuming that relative wage rates are constant across states for the human capital groups that have been delineated.

In Table 3 we present the estimated relative primary wage rates (or human capital stocks) along with the associated multiplicative standard errors in parentheses for each group. The relative primary wage equation corroborates over all states the disparity between the wage rates of male and female primary workers, and the higher wage rates of more educated people with a trade-off between education and experience which was first observed in the formation of the human capital groups by looking at California primary workers. Indeed, the relative wage rates reported in Table 3 not only reflect the same human capital ordering in California (see Table 1), but the ratios are also similar in size, -- especially the non-college graduate groups. This similarity and the high explanatory power of the relative wage equation supports the use and results of the one way sequential analysis of variance procedure in forming homogeneous human capital groups from California data and applying that structure to the other states.

We turn now to the estimation of the relative secondary wage rate or human capital equation. If the assumption of high correlation between the human capital stocks of secondary workers and their associated primary workers holds and if primary worker relative human capital stocks are indeed constant, then the ratio of secondary to primary human capital stocks should be approximately constant across states for each group. To estimate this ratio we regress the logarithm of secondary to primary relative wage rates against a set of 8 binary variables representing age-sex-education combinations. We also

TABLE 3

RELATIVE WAGE RATE OR HUMAN CAPITAL STOCK ESTIMATES BY HUMAN CAPITAL GROUP FOR PRIMARY AND SECONDARY WORKERS

(multiplicative standard errors in parenthesis)

	HUMAN CAPITAL GROUP			PRIMARY WORKERS W_i^p/W_4^p		SECONDARY WORKERS W_i^s/W_i^p	
	SEX	AGE	ED	Non-South	South	Non-South	South
1	M/F	O	C	1.57 (1.02)	1.49 (1.02)	.52 (1.03)	.54 (1.04)
2	M/F	Y	C	1.13 "	1.07 "	.83 "	.86 "
3	M	O	NH	.88 "	.83 "	.68 "	.71 "
4	M	O	H	1.00	1.00	.60 "	.63 "
5	M	O	SC	1.19 "	1.13 "	.57 "	.59 "
6	M	Y	NC	.82 "	.78 "	.80 "	.84 "
7	F	O	NC	.61 "	.58 "	1.03 "	1.08 "
8	F	Y	NC	.58 "	.56 "	.92 (1.04)	.96 "

M = male head
F = female head

O = age 36-over
Y = age 16-35

C = college graduate
SC = some college
H = high school graduate
NC = non-college graduate
NH = non-high school graduate

introduce a dummy to test for a regional difference in relative wage rates. The estimated coefficients with standard errors in parentheses are reported below:

$$(4.2) \qquad \log(W^{s}_{ij}/W^{p}_{ij}) = \underset{(.035)}{-.617}\, D_1 - \underset{(.035)}{.145}\, D_2 - \underset{(.035)}{.345}\, D_3 - \underset{(.035)}{.458}\, D_4 - \underset{(.035)}{.520}\, D_5$$

$$- \underset{(.035)}{.176}\, D_6 + \underset{(.035)}{.074}\, D_7 - \underset{(.037)}{.037}\, D_8 - \underset{(.023)}{.046}\, NS\ ,$$

$$R^2 = .63, \quad S.S.R. = 7.36, \quad S.E.E. = .180, \quad NO.OBS. = 237.$$

The south/non-south dummy is negatively signed and significantly different from zero, indicating that relative secondary wage rates are greater in the south. We note, however, that relative secondary wage rates are higher in the south by approximately the same percentage that relative primary wage rates are lower in the south, so that secondary wage rates relative to the numeraire group display small south/non-south differences. In Table 3 the estimated ratio of secondary to primary relative wage rates along with the multiplicative standard error is presented.

A priori we would not expect secondary wage rates to exceed primary wage rates in the male-headed groups, while that possibility cannot be ruled out among female-headed families. In general we would expect that among the higher human capital groups there is apt to be a greater relative difference between the human capital of primary and secondary workers than among the lower human capital groups because utilized human capital is not evenly distributed. The results support these *a priori* notions.

Judging from the summary statistics, relative secondary wage rates display more variability across states than do their primary counterparts. Nevertheless, the assumption of approximate constancy is not unreasonable with the goodness of fit obtained. The greater variability is not surprising in view of the fact that human capital groups have been defined in terms of the primary worker's characteristics.

Labor Supply Equations

As already mentioned we experimented with separate supply equations for each human capital group and with stacks of equations. For primary workers we found that separate equations (except for the M/F-O-C and M/F-Y-C groups) gave results most in accord with our a priori notions. Evidently there are significant wage rate and income response differences among groups because stacking groups which gave reasonable estimates separately usually produced inferior results. The one exception was the college graduate groups for which we did not find significant response differences. Beginning with a preference for the simpler, recursive formulation we found no evidence for rejecting this in favor of the feedback formulation.

In Table 4 the primary supply coefficients and standard errors of the quadratic recursive formulation are reported for each human capital group. All the supply curves bend backwards, at rates ranging from \$2.33 per hour for Group F-Y-NC to \$7.83 per hour for Groups M/F-O-C and M/F-Y-C. The income term coefficients have the theorectically expected sign in four equations, two cases of which are statistically significant. In the female-headed family groups and the young, male, non-college graduate group the coefficients indicate that labor supply increases (leisure decreases) in response to an increase in unearned income. Although statistically significance cannot be attached to the coefficients, the response is certainly plausible for the younger groups in which the heads have strong job commitments and hence preferences for earnings relative to leisure. A similar argument could be made

TABLE 4

PRIMARY SUPPLY EQUATIONS
Estimated Coefficients and Standard Errors

	GROUP	C	W^P	$(W^P)^2$	Y^n/F	RACEP	AGEP	SEXP	EDUP	URBAN	SPOUSE	DPENDP	R^2	S.E.E.	No. Obs.
1-2.	M/F-O-C & M/F-Y-C	.487 (.88)	.094 (.23)	-.006 (.02)	-.008 (.03)	.008 (.43)	.206 (.26)	-.103 (.43)	—	.199 (.26)	.517 (.49)	.573 (.42)	.53	.112	60
3.	M-O-NH	- .613 (1.33)	.395 (.59)	-.054 (.07)	-.061 (.10)	.094 (.26)	2.78*** (.83)	—	—	.350* (.19)	- .135 (.91)	- .712 (.97)	.82	.082	30
4.	M-O-H	-2.13 (1.57)	.783 (.51)	-.091* (.05)	-.158*** (.04)	-.010 (.15)	.689* (.38)	—	—	.428 (.26)	1.81*** (.52)	.132 (.33)	.76	.057	30
5.	M-O-SC	-2.21 (4.89)	.695 (1.30)	-.058 (.11)	-.095** (.05)	.637 (.41)	.779 (.73)	—	—	-.202 (.33)	1.23 (1.69)	.133 (.57)	.57	.133	30
6.	M-Y-NC	- .092 (1.39)	.457 (.55)	-.062 (.07)	.232 (.25)	-.214 (.29)	-.011 (.95)	—	-.152 (.37)	.320* (.18)	.338 (.63)	1.22* (.70)	.54	.073	30
7.	F-O-NC	- .366 (1.24)	.403 (.88)	-.058 (.15)	.201 (.13)	-.235 (.28)	1.08** (.52)	—	-.165 (.36)	.002 (.23)	—	.111 (.85)	.63	.085	30
8.	F-Y-NC	-1.37 (3.35)	1.25 (2.67)	-.268 (.47)	.075 (.30)	-.163 (.32)	.797 (.58)	—	.602 (.53)	.149 (.53)	—	- .112 (.46)	.28	.153	30

Note: Asterisks denote significance at the 1% (***), 5% (**), and 10% (*) levels for t tests on all parameters.

for the group of older female primary workers on the grounds of family-head responsibilities in a family without substantial secondary earnings.

Several of the socio-demographic variables played significant roles in four of the supply equations. Not surprisingly, the prime-age variable, AGEP, was significant only in the 36-over years old groups: M-O-NH, M-O-H, F-O-NC. The direction of its effect suggests that labor supply tends to be greater for peopole of prime age vis-a-vis people over 55 years old. The influence of marital status (SPOUSE) was significant in only one of the male-headed family groups (M-O-H), and the presence of dependent children (DPENDP) was significant only for the younger, male group (M-Y-NC). Geographic location (URBAN) was a significant influence in two equations; in these cases living in urban areas is associated with greater labor supply. No significant association between race and labor supply was found. Finally, in the three groups characterized by 0-15 years of schooling, those with high school diplomas did not have significantly different labor supplies. The coefficient of determination is greater than 0.50 for all the groups except for F-Y-NC. Further evidence of the good fit is provided by the standard error of the estimate which ranges from 57 to only 153 hours compared to average labor supplies of 670 to 1820 hours per year for the groups.

We now turn to the estimates of secondary worker supply equations. In contrast to the primary supply equations, more reasonable supply parameter estimates for secondary workers were obtained by stacking the groups in three sets -- college graduates, male-headed, and female-headed family non-college graduate -- rather than estimating separate equations for each group. Evidently, the additional demographic-education variation introduced in stacking and similarity of labor supply response behavior of the separate groups were factors in causing the better fit. The compositional difference between secondary workers in families headed by males vis-a-vis females provided a natural partition among the non-college graduates and was found to give better estimates than the other partitions tried.

In Table 5 the secondary supply parameter estimates and standard errors are presented. Response to wage rates is backward bending and statistically significant for both male and female-headed family non-college groups but not for the college graduate groups. Other family income is significant only for the male, non-college groups and opposite in effect to that expected. This may indicate a preference by the housewife for market work when husband's earnings are sufficient either to mechanize the home work or to hire a substitute. This finding is also consistent with the increasing labor force participation of women in the last decade. At least one socio-demographic variable was significant in each equation. They reflect, in large part, the compositional difference among secondary workers between male and female-headed families. The goodness of fit, as measured by the R^2 and S.E.E. summary statistics, matches that for the primary equations.

We have estimated supply equations for primary and secondary workers partitioned into homogeneous human capital groups. The units of observation have been viewed as average or representative workers of each group. In order to get some indication of how our estimates compare with those of other investigators we calculated the total income elasticity (TIE) and compensated substitution elasticity (CSE) for the average worker in each group.[20]

Precise comparisons with other research, however, are not possible because model specification, data base, and estimating techniques vary considerably. A major difference is our use of groups or average workers rather than single individuals. Nevertheless there are other aspects of similarity. With regard

TABLE 5

SECONDARY SUPPLY EQUATIONS
Estimated Coefficients and Standard Errors

GROUP	C	W^s	$(W^s)^2$	$(W^pH^p+Y^n)/F$	RACES	AGES	SEXS	EDUS	URBAN	SPOUSE	DPENDS	R^2	S.E.E.	No. Obs.
1-2. M/F-O-C, & M/F-Y-C	.127 (.85)	- .012 (.51)	-.008 (.06)	-.031 (.02)	.094 (.23)	.227 (.45)	.272 (.39)	-.474* (.28)	.440** (.20)	.814*** (.29)	- .450 (.29)	.47	.076	60
3-6. M-O-NH, & M-O-H, & M-O-SC, & M-Y-NC	-2.46** (1.23)	1.64** (.79)	-.297** (.13)	.032* (.019)	- .312** (.13)	.389 (.33)	1.62*** (.36)	-.250 (.23)	.077 (.15)	.698* (.42)	- .063 (.22)	.57	.120	120
7-8. F-O-NC, & F-Y-NC	-2.07** (1.01)	1.29* (.72)	-.197 (.13)	-.029 (.08)	.012 (.09)	.309** (.15)	.098 (.09)	.307*** (.11)	-.093 (.21)	----	1.42** (.56)	.58	.084	57

to data sample, this research is similar to that by Ashenfelter and Heckman; Boskin; Cohen, Rea, and Lerman; and Garfinkel in that the sample is not stratified by income level.[21] They focus, however, only on prime age males. If one compares dependent variables our model is similar to Hall's who also included all people in the labor force in his data sample.[22] Other investigators have either chosen a two-stage procedure in which the labor force participation decision and the hours of work decision are separated, or they have restricted the sample to those actually working.[23] Like Hall, Boskin, and Kalachek and Raines,[24] we explain wage rates by demographic characteristics in a first stage and derive a set of potential wage rates rather than use the observed values in the supply equation estimation.

The TIE is defined as the percentage change in labor supply with respect to the percentage change in "total" income. It should be negative if leisure is not an inferior good and not smaller than -1 if income is not inferior. It is the elasticity implied in the supply equation estimation weighted by the ratio of earnings to the income variable used (in the estimation). Hence, if unearned income is the variable used, as it is in our primary supply equation, then we effectively inflate the estimated elasticity by $(W^pH^p/F)/(Y^n/F)$ to obtain the family-head TIE. In the secondary supply equation the income variable used is $W^pH^p+Y^n$, and to obtain the TIE we effectively deflate the estimated elasticity by $(W^sH^s/F)/((W^pH^p+Y^n)/F)$. The TIE does not depend on the size of the income components used to measure the income variable and a comparison of income elasticities is thus facilitated. The CSE is defined as the wage elasticity minus the TIE and should be positive according to the assumptions underlying the income-leisure choice model.

For primary workers, the CSE and TIE estimates for the college graduate groups and the older, male-headed family groups were of the theoretically correct sign and were similar in magnitude to those values reported by other investigators of prime-age male labor supply. For the younger, male group and the female-headed family groups, however, the CSE's were negative, resulting from a combination of positive TIE's and negative wage rate elasticities at the mean wage rate on the supply curve. Similarly for secondary workers the CSE's were negative for all but the female-headed family groups. Thus, where comparisons can be made our results accord with the most recently available empirical evidence.

Labor Demand Equation

The demand for total equivalent-quality hours is significantly responsive to market rates but apparently not to differences in industrial structure. The estimated relationship is

$$(4.2)\qquad \log(H^d/NOUT) = \underset{(.20)}{6.675^{***}} - \underset{(.13)}{1.049^{***}}\log W_4^p - \underset{(.13)}{.013}\ NOUTI$$

$$R^2 = .75,\ S.E.E. = .064,\ NO.OBS. = 30.$$

The estimated demand elasticity with respect to the wage rate of 1.05 is in agreement with other studies which appear to be converging on an estimated value of unity for the elasticity of substitution.[25]

The empirical result that the industrial composition of output (NOUTI) does not have a statistically significant effect on the demand for labor may at first blush seem counter-intuitive. It is generally felt that the service sector is more labor-intensive than the non-service sector. This does not mean, however, that the service sector employs more labor in equivalent-quality units than does the industrial sector. In general, the former has lower wages and by implication lower quality labor than the latter. Therefore, more people can be employed per unit of output in the service sector without it being more labor intensive than the industrial sector.

V. COMPLETE MODEL SIMULATIONS

In this section we evaluate the model by comparing the actual and predicted values of the endogenous variables by state and human capital group. We next analyze the effects of exogenous changes in output and population upon the structure of earnings and income by groups.

Model Solution

Our simultaneous equation model of the labor market is composed of a demand equation for equivalent-quality hours (within which is imbedded the equilibrium condition), primary supply and relative wage equations, and secondary supply and relative wage equations. Endogenous variables are annual hours and wage rates for 16 age, sex, and education groups.

To predict with the model, we solve this nonlinear system of equations using an iterative solution technique. The recursive formulation of the primary supply equation allows the substitution of the primary into the secondary supply equations. Secondary hours can then be summed and the simulation reduced to one excess demand equation in one unknown, the numeraire group wage rate. Because the supply equations are backward-bending, however, there is the possibility of multiple solutions (equilibria) or no solution to the model. In the case of more than one solution we report the solution which is closest to the actual data. The alternative solutions, when they occurred, were at wage rates far outside the realm of observation and, thus, not economically meaningful. There were three states in which there was no equilibrium solution: Connecticut, Illinois and New York. In these cases we chose the wage rate which minimized the excess labor demand.

Model Evaluation

We report the endogenous variables separately by state and by human capital group. We do not break out groups within a state or states within a group. These cells are, of course, the building blocks of the model but space constraints prohibit reporting them here. Consequently we present for each state (and the nation) and for each group average wage rate, hours, and earnings for primary and secondary workers, and earnings, unearned income and total income for the family.

In Tables 6, 7, and 8 the actual, predicted, and percentage error data are presented by states and for the nation in 1969. Likewise, Tables 9, 10, and 11 report the same information by group. For primary and secondary workers averages are calculated as follows: wage rates are weighted by hours, while both hours per family, earnings per family, and income per family are weighted by number of families. For the state tables the weighting is done over groups; for the group tables the weighting is done over states. The national averages are weighted over the states, the weights being the state's share of the national total. For a particular group, wage rates, hours per family, earnings per family, unearned income per family, and income per family are weighted and summed over states to form national averages, the weights being the state's share in the national total of hours or families, as appropriate.

One striking feature of Table 6 is that for each state the average number of primary hours worked by all families is considerably less than the full-time equivalent of 2000 hours per annum. There are a number of reasons for the low average which Table 9 allows us to identify. (The

TABLE 6

ACTUAL 1969 VALUES

STATE AVERAGES

STATE	WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	YN/F	Y/F
ALA-MISS	3.48	2.32	1367	576	4766.	1339.	6104.	697.	6801.
ALK-HAW-WASH	5.16	3.59	1383	502	7163.	1802.	8965.	1057.	10023.
ARIZ-COLO-NMEX	4.36	2.90	1375	470	6019.	1366.	7386.	1169.	6555.
ARK-OKLA	3.60	2.52	1336	528	4812.	1330.	6142.	915.	7057.
CALIFORNIA	5.22	3.49	1364	441	7117.	1540.	8658.	1213.	9871.
CONNECTICUT	5.14	3.41	1510	594	7759.	2025.	9783.	1452.	11235.
DELA-VA	4.19	2.90	1503	603	6294.	1752.	8046.	973.	9020.
D.C.	4.39	3.82	1358	504	5963.	1931.	7894.	1119.	9013.
FLORIDA	4.29	2.72	1229	546	5278.	1486.	6764.	1748.	8511.
GEORGIA	3.93	2.76	1511	664	5945.	1837.	7782.	790.	8572.
ID-MT-NV-UT-WY	4.39	2.60	1459	508	6407.	1325.	7732.	1003.	8735.
ILLINOIS	5.02	3.41	1438	565	7221.	1927.	9148.	986.	10134.
INDIANA	4.52	2.96	1492	586	6748.	1734.	8482.	927.	9409.
IOWA-MINN	4.32	2.99	1442	526	6240.	1575.	7815.	991.	8806.
KAN-NEB-ND-SD	4.14	2.69	1434	506	5938.	1365.	7303.	980.	8283.
KENTUCKY	3.92	2.83	1306	456	5126.	1290.	6417.	798.	7215.
LOUISIANA	3.99	2.75	1362	476	5433.	1311.	6744.	628.	7371.
ME-MA-NH-RI-VT	4.49	3.15	1445	633	6482.	1999.	8482.	1135.	9616.
MARYLAND	4.99	3.32	1549	654	7736.	2170.	9906.	965.	10871.
MICH-WISC	5.06	3.41	1412	528	7152.	1800.	8952.	994.	9945.
MISSOURI	4.45	2.95	1341	521	5966.	1541.	7507.	1064.	6571.
NEW JERSEY	5.16	3.36	1478	644	7629.	2164.	9792.	1161.	10954.
NEW YORK	5.05	3.65	1386	521	6995.	1902.	8897.	1187.	10084.
NO CAR-SO CAR	3.43	2.50	1508	730	5169.	1826.	6995.	741.	7736.
OHIO	4.77	3.24	1454	513	6939.	1664.	8604.	1035.	9639.
OREGON	4.48	3.10	1316	426	5894.	1321.	7214.	1090.	8304.
PENNSYLVANIA	4.48	2.94	1363	576	6107.	1697.	7804.	996.	8799.
TENNESSEE	3.79	2.65	1373	627	5203.	1662.	6865.	723.	7588.
TEXAS	4.07	2.66	1503	546	6115.	1452.	7567.	982.	8548.
WEST VIRGINIA	3.95	2.51	1261	450	4983.	1127.	6110.	772.	6882.
NATION	4.58	3.09	1411	545	6462.	1687.	8149.	1052.	9201.

TABLE 7

PREDICTED 1969 VALUES

STATE AVERAGES

STATE	WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	YN/F	Y/F
ALA-MISS	3.45	2.44	1360	547	4703.	1335.	6038.	697.	6735.
ALK-HAW-WASH	4.83	3.18	1445	533	6976.	1697.	8674.	1057.	9731.
ARIZ-COLO-NMEX	4.05	2.73	1436	502	5820.	1372.	7192.	1169.	8361.
ARK-OKLA	3.49	2.46	1322	487	4614.	1201.	5814.	915.	6730.
CALIFORNIA	4.81	3.20	1451	512	6982.	1643.	8625.	1213.	9838.
CONNECTICUT	6.12	3.86	1372	367	8396.	1418.	9814.	1452.	11265.
DELA-VA	4.20	2.94	1460	595	6214.	1753.	7967.	973.	8940.
D.C.	4.34	3.21	1347	488	5852.	1568.	7419.	1119.	8538.
FLORIDA	3.78	2.65	1272	534	4815.	1415.	6230.	1748.	7977.
GEORGIA	4.03	2.87	1461	600	5892.	1723.	7615.	790.	8405.
ID-MT-NV-UT-WY	4.01	2.67	1481	510	5948.	1362.	7310.	1003.	8313.
ILLINOIS	5.77	3.79	1353	419	7811.	1590.	9401.	986.	10387.
INDIANA	4.80	3.22	1457	554	6993.	1787.	8780.	927.	9707.
IOWA-MINN	4.60	3.04	1375	525	6322.	1599.	7922.	991.	8913.
KAN-NEB-ND-SD	4.28	2.86	1347	504	5768.	1444.	7211.	980.	8191.
KENTUCKY	3.69	2.61	1342	570	4957.	1489.	6446.	798.	7245.
LOUISIANA	3.66	2.58	1443	567	5285.	1462.	6747.	628.	7374.
ME-MA-NH-RI-VT	4.65	3.10	1420	578	6609.	1793.	8402.	1135.	9536.
MARYLAND	4.82	3.35	1536	591	7397.	1981.	9378.	965.	10343.
MICH-WISC	4.94	3.27	1466	565	7238.	1851.	9089.	994.	10083.
MISSOURI	4.36	2.97	1361	507	5936.	1507.	7443.	1064.	8507.
NEW JERSEY	5.52	3.61	1448	529	7992.	1911.	9903.	1161.	11065.
NEW YORK	5.87	3.83	1301	369	7643.	1492.	9135.	1187.	10322.
NO CAR-SO CAR	3.82	2.73	1471	614	5621.	1678.	7299.	741.	8040.
OHIO	4.99	3.32	1445	539	7212.	1791.	9003.	1035.	10038.
OREGON	4.50	3.05	1319	455	5935.	1388.	7323.	1090.	8413.
PENNSYLVANIA	4.73	3.14	1369	574	6475.	1800.	8275.	996.	9271.
TENNESSEE	3.81	2.72	1392	609.	5299.	1656.	6955.	723.	7679.
TEXAS	4.02	2.83	1467	579	5905.	1644.	7548.	982.	8530.
WEST VIRGINIA	3.64	2.52	1264	534	4608.	1348.	5956.	772.	6728.
NATION	4.68	3.12	1403	522	6568.	1629.	8198.	1052.	9249.

1.50

TABLE 8

PERCENTAGE ERROR 1969 VALUES

STATE AVERAGES

STATE	WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	Y/F
ALA-MISS	-.82	5.11	-.49	-5.11	-1.31	-.27	-1.06	-.97
ALK-HAW-WASH	-6.76	-11.30	4.45	6.19	-2.61	-5.81	-3.25	-2.91
ARIZ-COLO-NMEX	-7.40	-5.95	4.43	6.73	-3.30	.38	-2.62	-2.27
ARK-OKLA	-3.10	-2.15	-1.04	-7.74	-4.11	-9.72	-5.33	-4.64
CALIFORNIA	-7.78	-8.26	6.38	16.24	-1.90	6.64	-.38	-.33
CONNECTICUT	19.10	13.21	-9.14	-38.15	8.21	-29.98	.31	.27
DELA-VA	.23	1.35	-1.50	-1.27	-1.28	.05	-.99	-.68
D.C.	-1.05	-16.10	-.84	-3.23	-1.87	-18.81	-6.02	-5.27
FLORIDA	-11.83	-2.62	3.48	-2.21	-6.77	-4.77	-7.89	-6.27
GEORGIA	2.50	3.81	-3.32	-9.62	-.90	-6.18	-2.15	-1.95
ID-MT-NV-UT-WY	-8.56	2.62	1.52	.21	-7.17	2.83	-5.46	-4.83
ILLINOIS	15.02	11.18	-5.96	-25.77	8.16	-17.47	2.76	2.49
INDIANA	6.07	8.97	-2.31	-5.41	3.62	3.07	3.51	3.16
IOWA-MINN	6.26	1.68	-4.64	-.13	1.33	1.56	1.37	1.22
KAN-NEB-ND-SD	3.42	6.18	-6.08	-.42	-2.86	5.74	-1.25	-1.11
KENTUCKY	-5.92	-7.68	2.78	25.03	-3.31	15.42	.46	.41
LOUISIANA	-8.17	-6.29	5.93	19.02	-2.72	11.54	.05	.04
ME-MA-NH-RI-VT	3.70	-1.82	-1.69	-8.67	1.95	-10.33	-.95	-.83
MARYLAND	-3.53	1.01	-.88	-9.64	-4.38	-8.73	-5.33	-4.86
MICH-WISC	-2.50	-3.92	3.79	7.08	1.20	2.88	1.54	1.38
MISSOURI	-2.01	.52	1.52	-2.67	-.52	-2.17	-.86	-.75
NEW JERSEY	6.91	7.37	-2.01	-17.73	4.76	-11.67	1.13	1.01
NEW YORK	16.32	4.81	-6.07	-25.15	9.26	-21.55	2.68	2.36
NO CAR-SO CAR	11.43	9.40	-2.42	-16.00	8.73	-8.10	4.34	3.92
OHIO	4.59	2.37	-.64	5.12	3.92	7.61	4.64	4.14
OREGON	.45	-1.47	.24	6.70	.70	5.13	1.51	1.31
PENNSYLVANIA	5.55	6.53	.45	-.43	6.03	6.07	6.04	5.36
TENNESSEE	.49	2.49	1.35	-2.79	1.85	-.37	1.31	1.19
TEXAS	-1.01	6.66	-2.45	6.13	-3.44	13.20	-.24	-.22
WEST VIRGINIA	-7.75	.77	.24	18.69	-7.53	19.60	-2.52	-2.24
NATION	2.20	.73	-.54	-4.12	1.65	-3.42	.60	.53

TABLE 9

ACTUAL 1969 VALUES

HUMAN CAPITAL GROUP AVERAGES

	GROUP TYPE			WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	YN/F	Y/F
	SEX	AGE	ED									
1.	M/F	O	C	7.51	3.89	1615	451	12138.	1755.	13893.	2295.	16188.
2.	M/F	Y	C	5.31	4.40	1727	350	9172.	1541.	10713.	411.	11124.
3.	M	O	NH	4.13	2.82	1284	732	5309.	2069.	7378.	1139.	8517.
4.	M	O	H	4.87	2.99	1783	826	8686.	2470.	11156.	1047.	12202.
5.	M	O	SC	5.66	3.34	1774	728	10083.	2435.	12518.	1726.	14244.
6.	M	Y	NC	3.85	3.14	1820	472	7013.	1483.	8497.	181.	8677.
7.	F	O	NC	2.90	3.02	670	283	1945.	857.	2802.	1400.	4202.
8.	F	Y	NC	2.85	2.84	948	80	2702.	228.	2930.	311.	3240.
9.	ALL	TYPES		4.56	3.09	1411	545	6462.	1687.	8149.	1052.	9201.

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TABLE 10

PREDICTED 1969 VALUES

HUMAN CAPITAL GROUP AVERAGES

	GROUP TYPE			WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	YN/F	Y/F
	SEX	AGE	ED									
1.	M/F	O	C	7.67	3.92	1625	434	12472.	1704.	14175.	2295.	16470.
2.	M/F	Y	C	5.53	4.50	1732	339	9576.	1529.	11106.	411.	11517.
3.	M	O	NH	4.24	2.88	1269	711	5378.	2052.	7430.	1139.	8569.
4.	M	O	H	4.94	3.01	1767	775	8740.	2338.	11078.	1047.	12125.
5.	M	O	SC	5.76	3.25	1763	716	10197.	2326.	12522.	1726.	14249.
6.	M	Y	NC	3.93	3.10	1811	450	7114.	1396.	8510.	181.	6691.
7.	F	O	NC	2.96	3.06	677	254	2007.	780.	2786.	1400.	4187.
8.	F	Y	NC	2.83	2.84	916	112	2590.	319.	2909.	311.	3220.
9.	ALL	TYPES		4.68	3.12	1403	522	6568.	1629.	8198.	1052.	9249.

TABLE 11

PERCENTAGE ERROR 1969 VALUES

HUMAN CAPITAL GROUP AVERAGES

	GROUP TYPE SEX	 AGE	 ED	WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	Y/F
1.	M/F	O	C	2.12	.90	.61	-3.77	2.74	-2.91	2.03	1.74
2.	M/F	Y	C	4.08	2.40	.31	-3.05	4.40	-.73	3.67	3.53
3.	M	O	NH	2.47	2.10	-1.15	-2.84	1.29	-.80	.70	.61
4.	M	O	H	1.51	.88	-.87	-6.17	.63	-5.35	-.69	-.63
5.	M	O	SC	1.77	-2.86	-.63	-1.67	1.13	-4.48	.04	.33
6.	M	Y	NC	1.94	-1.34	-.50	-4.59	1.43	-5.87	.16	.15
7.	F	O	NC	2.18	1.41	.96	-10.24	3.16	-6.97	-.55	-.37
8.	F	Y	NC	-.77	-.14	-3.39	40.23	-4.13	40.03	-.70	-.64
9.	ALL	TYPES		2.20	.73	-.54	-4.12	1.65	-3.42	.60	.53

reader is referred to Table 3 for identification of the groups.) In the first place, not all heads of households necessarily work. This is particularly true since we include in the group of primary individuals many who are only marginally in the work force. While they can work, they may not choose to work at the existing market wage. For example, we do not exclude those over age 65 or those participating in social welfare programs without effective work requirements. The primary hours forthcoming from the non- college graduate female-headed family groups are also quite low. Perhaps these people as a group have a high AFDC participation rate. Second, not all heads of households choose to work all year round. Also, many individuals may work all year round but choose to satisfy their supply through part-time rather than full-time work. Finally, the Census reference week may not be representative of the year as a whole and constructing a measure of annual hours by multiplying weeks worked last year by hours worked during the reference week could lead to the lower levels calculated. We tend to discount this reason for all groups, however, because several groups are, in fact, working near the full-time equivalent.

In all states but three an equilibrium solution was found and a glance at Table 8 reveals that the percentage deviations between the actual state averages and those predicted by the model are small for a majority of the states. In general we can say that for primary workers the wage rate errors exceed the hours errors, and vice-versa for secondary workers, so that the secondary hours predictions are noticeably worse than the primary hours predictions. Moreover, secondary earnings errors exceed the primary earnings errors. We note, however, that family earnings errors are almost always smaller than their components because wage rate and hours errors offset each other as do primary and secondary earnings errors in many states. Striking examples are: Connecticut, Illinois, and California. Furthermore, when states are aggregated their errors dramatically offset each other, producing quite small errors for the nation as a whole: wage rate,hours, and earnings errors less than five percent for secondary workers and less than three percent for primary workers.

From a group perspective (Table 11) the percentage errors are smaller than from a state perspective, though we observe the same pattern: the model does a relatively better job with primary hours and secondary wage rates, and primary and secondary wage rates, and primary and secondary worker earnings errors offset each other to produce family earnings estimates with errors of less than one percent on average.

Exogenous Output and Population Changes

To derive from the model the effects of exogenous changes in national output and population upon the distribution of income in each state labor market, we introduce shifts in the supply and demand curves and then solve the model for the new equilibrium values. An exogenous expansion in national output or income is represented by a proportionate increase in state output and a proportionate increase in unearned income so that Y^n/F is also increased. All other exogenous variables are unchanged. Second, the effects of national population change are explored by an across the board increase in the number of families with all other exogenous variables being unchanged.

Before analyzing these cases in detail a few general remarks about the results may be helpful. By looking at averages we are observing a mixture of compositional effects (across groups in a given state, and across states for a given group) and individual family or market effects. The reader will recall that in forming these averages, wage rates are weighted by hours, while hours per family are weighted by families. The family weights will not change, but the hours weights may change since hours are endogenously determined. Hence, wage rates and earnings per family will be affected by compositional changes, the former directly and the latter indirectly through wage rate changes. We have found, however, that the compositional effects tend to support rather than oppose the individual effects. While we do not report the group-state detail, the averages are representative of the components in that the qualitative changes are reflected in the averages.

A one percent increase in state output (NOUT) will shift the market demand curve to the right by exactly one percent due to the constant returns to scale property of the underlying production function. The associated one percent increase in unearned income per family (Y^n/F) shifts five of the primary supply surves to the left and three to the right, and shifts four of the secondary curves to the left and four to the right. All the righward shifts in supply are by less than 0.50 percent so that together the demand and supply shifts produce excess demand at the old equilibrium wage rate. Market forces increase the primary and secondary wage rates to new equilibria which are 1.10 and 0.94 percent, respectively, above the old values on average for all states (although the new wage rates are slightly below the original ones in the disequilibrium states—

the result of comparing disequilibrium positions) as indicated in Table 12. The wage rate increases vary considerably across states. In some of them the increases in wage rates have absorbed more than the original excess demand inasmuch as hours per family actually decreased in almost half of the states as suppliers moved up the backward-bending portion of their labor supply curves. Over the nation primary and secondary worker hours per family have fallen by 0.19 and 0.80 percent, respectively, in response to the exogenous demand shift. Wage rate changes dominated hours changes so that for primary workers earnings per family increased in all except the disequilibrium states, while secondary earnings per family increased in all but three other states. Family earnings increases exceeded 0.90 percent for all but the disequilibrium states (virtually unchanged) and New Jersey (0.67 percent increase).

In Table 13 the percentage changes in response to the output-unearned income exogenous increase are reported from the group perspective. Primary worker earnings registered increases in all but the F-Y-NC group, in which a large hours decline outweighted the wage rate increase. The sizes of the earnings increases varied, from 1.75 percent down to 0.54 percent. From a glance at the wage rate and hours components we observe that groups are ranked in the same order on both H^P/F and E^P/F. The groups rank in descending order on E^P/F increases as follows: F-O-NC, M/F-Y-C, M/F-O-C,M-O-NH, M-Y-NC, M-O-SC, M-O-H, and F-Y-NC. Primary workers in the top two groups increased hours worked, whereas the others all moved up the backward-bending portion of their supply curves. In general the groups are bunched into four sets: older female heads are on top, while younger female heads are at the bottom; between them are the

TABLE 12

PERCENTAGE OUTPUT - UNEARNED INCOME MULTIPLIERS

STATE	WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	YN/F	Y/F
ALA-MISS	.68	.65	.15	.79	.83	1.45	.97	1.00	.97
ALK-HAW-WASH	1.21	1.13	-.17	-.57	1.04	.55	.94	1.00	.95
ARIZ-COLO-NMEX	.80	.72	.07	.59	.87	1.32	.96	1.00	.96
ARK-OKLA	.68	.64	.14	.88	.82	1.52	.97	1.00	.97
CALIFORNIA	1.24	1.16	-.19	-.71	1.06	.43	.94	1.00	.95
CONNECTICUT	-.19	-.15	.08	.67	-.10	.52	-.01	1.00	.12
DELA-VA	.92	.87	.05	.09	.95	.97	.95	1.00	.96
D.C.	1.09	1.00	-.03	-.48	1.06	.51	.95	1.00	.95
FLORIDA	.74	.71	.10	.63	.85	1.35	.96	1.00	.97
GEORGIA	.88	.84	.04	.21	.93	1.05	.96	1.00	.96
ID-MT-NV-UT-WY	.78	.72	.08	.65	.86	1.38	.96	1.00	.97
ILLINOIS	-.20	-.17	.12	.64	-.08	.47	.01	1.00	.10
INDIANA	1.50	1.43	-.35	-1.36	1.15	.05	.93	1.00	.93
IOWA-MINN	1.08	1.02	-.10	-.23	.99	.79	.95	1.00	.95
KAN-NEB-ND-SD	.87	.81	.04	.33	.91	1.14	.96	1.00	.96
KENTUCKY	.75	.72	.11	.56	.86	1.31	.96	1.00	.97
LOUISIANA	.74	.70	.11	.65	.85	1.36	.96	1.00	.97
ME-MA-NH-RI-VT	1.10	1.05	-.10	-.32	1.00	.73	.94	1.00	.95
MARYLAND	1.40	1.34	-.20	-1.38	1.20	-.06	.93	1.00	.94
MICH-WISC	1.51	1.42	-.35	-1.38	1.16	.02	.93	1.00	.93
MISSOURI	.98	.93	-.04	.05	.94	.98	.95	1.00	.96
NEW JERSEY	7.04	6.66	-3.30	-16.76	3.51	-11.22	.67	1.00	.70
NEW YORK	-.11	-.09	.06	.31	-.05	.22	-.01	1.00	.11
NO CAR-SO CAR	.82	.79	.07	.39	.89	1.18	.96	1.00	.96
OHIO	1.68	1.60	-.44	-1.92	1.23	-.35	.92	1.00	.93
OREGON	1.06	.99	-.09	-.14	.97	.84	.95	1.00	.95
PENNSYLVANIA	1.27	1.22	-.22	-.67	1.05	.54	.94	1.00	.94
TENNESSEE	.80	.77	.08	.43	.88	1.21	.96	1.00	.96
TEXAS	.84	.80	.07	.33	.91	1.12	.96	1.00	.96
WEST VIRGINIA	.70	.69	.13	.73	.83	1.43	.95	1.00	.97
NATION	1.10	.94	-.19	-.80	.91	.13	.75	1.00	.78

TABLE 13

PERCENTAGE OUTPUT - UNEARNED INCOME MULTIPLIERS

	GROUP TYPE			WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	XN/F	Y/F
	SEX	AGE	ED									
1.	M/F	O	C	1.09	1.06	-.05	-1.25	1.04	-.20	.89	1.00	.91
2.	M/F	Y	C	1.14	1.05	.07	-1.82	1.21	-.80	.93	1.00	.94
3.	M	O	NH	1.12	1.01	-.35	-.44	.76	.57	.71	1.00	.75
4.	M	O	H	1.11	1,03	-.57	-.65	.54	.42	.51	1.00	.55
5.	M	O	SC	1.10	.94	-.09	-1.51	1.01	-.58	.71	1.00	.75
6.	M	Y	NC	1.08	.80	-.09	-1.91	.99	-1.12	.64	1.00	.65
7.	F	O	NC	1.10	.94	.65	.09	1.75	1.02	1.55	1.00	1.36
8.	F	Y	NC	.99	.58	-1.01	5.44	-.03	6.05	.63	1.00	.67
9.	ALL TYPES			1.10	.94	-.19	-.80	.91	.13	.75	1.00	.78

college graduate, the older less educated male, and the younger male groups on the upper side, and the older more educated males on the lower side.

Relative to their primary workers, there was greater variation in wage rate changes and larger hours changes among secondary workers. In fact, hours decreases exceeded wage rate increases for four of the groups so that their earnings fell. (Note what appears to be a spuriously large increase in secondary hours for the F-Y-NC group resulting from a large estimated wage elasticity due to a small average number of secondary hours in this group). Nevertheless, the secondary contribution is relatively small and the prevailing family earnings pattern changes are described by the primary worker earnings component. In sum, one percent expansion of output and unearned income is associated with family earnings increases ranging from 0.51 to 1.55 percent and averaging 0.75 percent over all the groups.

To examine the effects of population growth, we next introduced a one percent increase in the number of families (Tables 14 and 15). The resultant changes are mirror images of the case just considered because constant returns to scale in demand from increases in output is matched by constant returns to scale in supply from increases in the number of families. Since supply is in terms of hours per family a one percent increase in the number of families by itself will increase total hours by one percent. Moreover, aggregate supply is further increased from those groups in which H/F increases in response to the implied fall in Y^n/F. However, for those groups with positive income elasticities a fall in Y^n/F will also cause H/F

TABLE 14

PERCENTAGE FAMILY MULTIPLIERS

STATE	WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	YN/F	Y/F
ALA-MISS	-.68	-.64	-.14	-.81	-.82	-1.44	-.96	-.99	-.96
ALK-HAW-WASH	-1.14	-1.05	.13	.43	-1.01	-.63	-.93	-.99	-.94
ARIZ-COLO-NMEX	-.79	-.70	-.08	-.60	-.87	-1.30	-.95	-.99	-.95
ARK-OKLA	-.67	-.62	-.14	-.89	-.81	-1.51	-.96	-.99	-.96
CALIFORNIA	-1.17	-1.08	.15	.53	-1.02	-.55	-.93	-.99	-.94
CONNECTICUT	-.79	-.70	.46	3.18	-.33	2.46	.08	-.99	-.06
DELA-VA	-.91	-.85	-.03	-.14	-.93	-.99	-.95	-.99	-.95
D.C.	-1.06	-.96	.04	.35	-1.02	-.61	-.94	-.99	-.94
FLORIDA	-.73	-.67	-.12	-.63	-.85	-1.30	-.95	-.99	-.96
GEORGIA	-.87	-.82	-.04	-.25	-.91	-1.07	-.95	-.99	-.95
ID-MT-NV-UT-WY	-.77	-.70	-.09	-.67	-.86	-1.37	-.95	-.99	-.96
ILLINOIS	-.29	-.26	.17	1.01	-.12	.75	.03	-.99	-.07
INDIANA	-1.37	-1.29	.28	1.03	-1.09	-.27	-.92	-.99	-.93
IOWA-MINN	-1.04	-.97	.08	.12	-.96	-.85	-.94	-.99	-.94
KAN-NEB-ND-SD	-.86	-.79	-.03	-.39	-.89	-1.17	-.95	-.99	-.95
KENTUCKY	-.74	-.71	-.10	-.60	-.85	-1.31	-.95	-.99	-.96
LOUISIANA	-.74	-.68	-.11	-.67	-.84	-1.35	-.95	-.99	-.96
ME-MA-NH-RI-VT	-1.07	-1.00	.09	.21	-.98	-.79	-.94	-.99	-.94
MARYLAND	-1.31	-1.24	.17	1.12	-1.14	-.14	-.93	-.99	-.93
MICH-WISC	-1.38	-1.29	.29	1.07	-1.10	-.23	-.92	-.99	-.93
MISSOURI	-.96	-.89	.03	-.13	-.92	-1.02	-.94	-.99	-.95
NEW JERSEY	-2.54	-2.40	.97	4.70	-1.60	2.19	-.87	-.99	-.88
NEW YORK	-.35	-.32	.21	1.24	-.14	.92	.03	-.99	-.09
NO CAR-SO CAR	-.81	-.77	-.07	-.42	-.88	-1.19	-.95	-.99	-.95
OHIO	-1.50	-1.41	.36	1.46	-1.15	.02	-.92	-.99	-.92
OREGON	-1.01	-.93	.06	.04	-.95	-.89	-.94	-.99	-.95
PENNSYLVANIA	-1.19	-1.14	.19	.49	-1.01	-.65	-.93	-.99	-.94
TENNESSEE	-.79	-.75	-.08	-.46	-.87	-1.21	-.95	-.99	-.95
TEXAS	-.83	-.77	-.07	-.36	-.90	-1.13	-.95	-.99	-.95
WEST VIRGINIA	-.70	-.67	-.13	-.74	-.82	-1.41	-.96	-.99	-.96
NATION	-.97	-.85	.13	.48	-.84	-.37	-.75	-.99	-.78

TABLE 15

PERCENTAGE FAMILY MULTIPLIERS

	GROUP TYPE SEX	AGE	ED	WP	WS	HP/F	HS/F	EP/F	ES/F	E/F	YN/F	Y/F
1.	M/F	O	C	-.96	-.94	.02	1.04	-.93	.08	-.81	-.99	-.64
2.	M/F	Y	C	-.98	-.94	-.07	1.56	-1.05	.61	-.82	-.99	-.83
3.	M	O	NH	-.96	-.90	.21	.20	-.76	-.70	-.74	-.99	-.77
4.	M	O	H	-.97	-.95	.35	.38	-.63	-.58	-.62	-.99	-.65
5.	M	O	SC	-.96	-.87	-.06	1.09	-1.02	.21	-.79	-.99	-.81
6.	M	Y	NC	-.96	-.80	.08	1.40	-.88	.59	-.64	-.99	-.65
7.	F	O	NC	-.95	-.84	-.24	-.64	-1.18	-1.47	-1.26	-.99	-1.17
8.	F	Y	NC	-.91	-.39	.87	-5.39	-.04	-5.77	-.67	-.99	-.70
9.	ALL TYPES			-.97	-.85	.13	.48	-.84	-.37	-.75	-.99	-.78

1.63

to decline. Nevertheless, since the largest positive income elasticity is significantly less than unity, the exogenous changes do yield excess supply at the old equilibrium wage rates. In response to this excess supply, both primary and secondary wage rates fall by more than would otherwise be necessary in many states to compensate for the increase in hours per family which occurs in half of the states as suppliers move down the backward-bending portion of their supply curves. Primary wage rate decreases outweigh the hours per family increases so that primary earnings fall in every state, while secondary earnings fall in all but the three disequilibrium states and Ohio. The relatively large secondary earnings increase exceeds the primary earnings decrease in the disequilibrium states so that family earnings rise in these three states but fall in others.

From a group perspective, the excess supply associated with the one percent increase in the number of families is taken up by similarly sized primary wage rate decreases of slightly less than one percent and hours changes that vary--three groups being on the upward-sloping portion and five on the backward-bending portion of their supply curves--from -0.24 to 0.87 percent. Accordingly, primary workers in these five groups register smaller earnings decreases than workers in the other groups. In fact, a ranking of the groups in descending order on E^P/F decreases is almost identical (only the third and fourth positions are interchanged) to the ranking on E^P/F increases caused by the exogenous output shift. Thus, family heads which fare relatively better when output increases are also likely to fare relatively worse when population increases. Moreover, we note that for primary workers in all but one of the groups the earnings decrease associated with an increase in population is smaller than the earnings increase associated with the increase in output.

The decrease in secondary wage rates vary across the groups from -0.39 to -0.95 percent while hours per family increase for all but the female-headed family group secondaries who find themselves below the backward-bending wage rate. Secondary earnings fall for only four of the groups, but family earnings fall for all of the groups as the primary earnings changes dominate. In sum, a one percent increase in the number of families results in excess labor supply which is absorbed, on average, by a combination of 0.97 and 0.85 percent decreases in primary and secondary wage rates, respectively, and hours per family increases of 0.13 and 0.48 percent for primary and secondary workers resulting from the increase in demand caused by the wage rate decreases. The wage rate changes outweigh the hours increase and family earnings decline, on average, by 0.75 percent.

VI. CONCLUSIONS

In this paper we have presented an econometric labor market model which integrates the theory of human capital with the theory of supply and demand. The model explains primary and secondary wage rates, hours and hence, earnings in eight mutually exclusive and completely exhaustive family types across the thirty states or groups of states in the United States. In this model we hypothesize that the average wage rate for a group in a state is the product of the average stock of utilized human capital in the group and the market rate of return on human capital in the state. Relative wages in a state are then determined by the distribution of human capital across groups, while absolute wage rates and employment are determined by the supply and demand for equivalent-quality labor in the state. We also establish an empirical correspondence between the distribution of human capital and the distribution of workers by demographic characteristics - age, education, and sex - so that human capital groups are defined in terms of these characteristics.

The model was estimated primarily with data aggregated from the 1970 Census 1-1000 Public Use Sample. Relative wage equations relating primary workers across human capital groups and relative primary and secondary workers within groups were estimated by an ordinary least squares method. The supply equations for primary and secondary workers and the demand equation for equivalent-quality hours were estimated by an instrumental variables method with augmented non-linear instruments.

The estimated relative wage equations indicate a large explained variance in primary wage rates between human capital groups and a small unexplained variance across states within the same group. We found that primary wage rates relative to the wage rates of a numeraire group (male heads with high school education, age 36 and over) are significantly lower in the South. However, we also found that secondary wages relative to primary wages were high in the South. The estimated demand equation describes a unitary elastic demand for labor in response to variations in market wages. The primary and secondary supply equations yield significant backward-bending labor supplies. Other family income and demographic characteristics were also found to have a significant effect on labor supply.

Predictions are obtained from the model by solving for each state or group of states the nonlinear, simultaneous equation system consisting of a supply equation for primary and secondary workers in every human capital group, relative wage equations relating the wage rates in these groups, a demand equation for all labor, and an equilibrium identity equating supply and demand for hours of equivalent-quality. We have examined the ability of the model to explain primary and secondary wages, hours, and earnings by state and, hence, for the nation as a whole averaged over all groups and by group averaged over all states. The ability of the model to explain the state averages in 1969 was very good; for the nation it was excellent.

We also explored the effects of exogenous increases in output and population upon equilibrium wage rates, hours, and earnings. In both cases in response to the exogenous changes, we observed greater variation of hours across human capital groups than variation of wage rates. Although hours changes displayed more variation, the wage rate change was larger and thereby determined the direction of the change in earnings. The differential effects were determined by the curvature of the supply curve around the backward-bending wage rate and whether the group was moving along the upward-sloping or backward-bending portion of the curve. Finally, we observed that those family types which benefited the most from an increase in output(namely, the older female-headed group; both college graduate groups; the older, less educated male-headed group; and the younger male-headed group) were the ones to experience the largest decrease in earnings when population was increased exogenously. In this case earnings per family registered the smallest decreases for the younger female-headed group, and the older male-headed groups with high school and some college. Correspondingly, these two groups also benefited least from output increases. Therefore, those who benefit (suffer) the most from output (population) increases would suffer (benefit) the most from output (population) decreases and vice-versa.

FOOTNOTES

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1. Fuchs [14], Gallaway [15], Scully [34], and Segal [35] examine regional wage rate differences. Griliches and Mason [19], Hanoch [22], Hansen, Weisbrod, and Scanlon [23], and Hanushek [24] examine individual earnings differentials. Hanushek examines earnings relationships over labor market areas and finds large differences in the returns to human capital and that much of the difference in regional earnings can be attributed to structural differences in earnings functions. We would attribute these differences not to differences in the structure of human capital and the supply and demand for labor, but instead to differences in the level of unearned income and in the level and composition of families and output.

2. For the derivation of the supply equations from the maximization of family utility, see Cohen, Rea, and Lerman [10], Kosters [28] or Rea [32].

   Zero cross-substitution effects is an assumption that if the utility of the family is held constant by compensatory changes, then the demand for leisure by the primary (secondary) worker is independent of the demand for leisure by the secondary (primary) workers. This assumption is made by Rea [32, pp. 7-12] in his derivation of the supply equations and by Kosters [28, pp. 11-17] in a study of the effects of the income tax on labor supply. He assumes that the income compensated component of the (substitution) effect of the wife's wage rate on the husband's labor supply is small. In [29, p. 308] he proves that the smaller is the ratio of secondary to primary earnings, the smaller the bias in calculating the substitution effect ignoring the cross-substitution term.

3. The seminal piece is Becker [2]. For a survey of the distribution of earnings literature from a human capital viewpoint, see Mincer [30].

4. To be sure, past discrimination has inhibited the accumulation of human capital with the outcome that current stocks are lower than they would have been otherwise. But there is also evidence which suggests that occupational discrimination in conjunction with supply and demand for labor plays a large role in keeping wage rates low. For an empirical study see Bergman [3].

5. Berndt and Christensen [4] relate the equality conditions on the Allen partial elasticities of substitution (AES) to separability restrictions on a function. Since Solow and Hulten related separability to conditions for the existence of an aggregate index, Berndt's theorems relate restrictions on the AES to aggregation conditions.

6. In a production function study using national data over the period 1929-68, Berndt and Christensen [4] find that capital equipment and structures can be aggregated to form a consistent index of capital, but that it is not possible to form a consistent aggregate index of blue and white collar labor, or blue collar labor and capital, or white collar labor and capital.

   Bowles [8] and Psacharopoulos and Hinchliffe [31], however, find high, though not infinite, elasticities of substitution between different types of educated labor in a cross country comparison. Bowles has proposed incorporating the estimate of the elasticity of substitution into the weighting procedure for forming the index.

   We believe that tests for consistent aggregate indices depend on the level of aggregation of the components, the data sample, and the production function employed to do the test.

7. Griliches [18] applies relative wage rates to obtain a measure of labor input in constant quality units when estimating an interregional production function.

8. Denison [12] also uses relative wage rates to adjust a historical measure of labor hours for changes in the quality of labor service. Schwartzman [33] employs an hourly earnings index to estimate the contribution of education to the change in the quality of labor.

9. Sampling error is a potentially serious problem in our work because of the way we have defined family types. We rely on the aggregation of family types into human capital groups to keep sampling error manageable.

10. The purpose of the procedure is to obtain the value of income originating in a sector in a given state from the value of personal income received in the state by employees in that sector. This is done to reflect the unreported capital component excluded from estimates of personal income.

11. See Crandall, MacRae, and Yap [12] for a discussion of the regional price index problem.

12. Those living in group quarters who are excluded comprise patients and inmates, people living on army bases, and in college dormitories.

13. See Fuchs [14, p. 4 and Appendix B] who calculates annual hours in the same manner.

14. The discussion which follows is an overview of research by Greenston and Riordan [17]. A description of the Automatic Interaction Detector algorithm is found in Sonquist and Morgan, The Direction of Interaction Effects, Monograph No. 35, Survey Research Center, Institute for Social Research, University of Michigan, 1964.

15. Hall [21, pp. 112-113] also uses an analysis of variance model to explain wage rates with demographic-education variables.

16. For a discussion of the issue and some empirical evidence see Bowen and Finegan [7, p. 30 and p. 70] and a comment on another paper by Hansen [6, p. 595]. A theoretical discussion of the issue in terms of a family labor supply model is given by Gronau [20].

17. Potential wage rates are discussed and estimated by Kalachek and Raines [27, pp. 160-1, and pp. 182-5]. These researchers also refer to work by Mincer for support of the role of "permanent" rather than current earnings in supply function estimation. Kalachek and Raines emphasize the role of the industrial environment in shaping supply response: "Most manufacturing firms, for instance, require the same number of manhours per year from their production employees, regardless of rank or earnings. The low-level semiskilled worker normally labors as long as the senior level semiskilled worker, though he may earn substantially less per hour. Experience and seniority are the prerequisites for advancement in wages, and they can be obtained only by working with reasonable competence for the required number of manhours per year. The labor supply response of the semiskilled worker who fulfills the company's manhours expectations cannot be attributed then to his current wage, but rather to the average expected wage discounted over his planning horizon."

18. Kalachek and Raines estimate potential wages from equations fitted by a multiple regression analysis using a subset of their population in which sex-race-age-education interactions and location are the regressors. Our technique is similar in that demographic-education information as it determines relative human capital stocks (through a variance - regression model) is used to estimate potential wage rates.

19. Fisher [13, pp. 30-33] suggests the method of augmentation which we have followed.

20. See Cain and Watts [9, pp. 330-337] for definitions of the total income elasticity.

21. Cain and Watts [9] present a comparison of labor supply estimates obtained by various investigators with emphasis on their implications for income maintenance policy. The discussion here draws on that comparison. It involves research by Ashenfelter and Heckman [1], Boskin [5], Cohen, Rea, and Lerman [10], and Garfinkel [16], Hall [21], Kalachek and Raines [27], and Hill [25].

22. See Hall [21].

23. Cain and Watts [9, pp. 348-352] discuss the choice of the dependent variable.

24. See Kalachek and Raines [27].

25. Jorgenson [26] surveys the empirical work.

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WAGE DIFFERENTIALS, HUMAN CAPITAL, AND DEMOGRAPHIC CHARACTERISTICS

by Peter M. Greenston and Dale P. Riordan

Abstract

A sequential one-way analysis of variance procedure (Automatic Interaction Detector) is applied to the explanation of individual wage rates as a function of demographic and educational characteristics with data from the 1970 1-1000 State Public Use Sample for California. One of the results of this analysis is that race is not as important a predictor in explaining average hourly wage rates as are other demographic variables, nor is sex among higher educated individuals. Age seems to be a stronger predictor of wage rates among individuals with a B.A. or advanced degree than is sex. Race may be manifesting its effects, however, through other variables such as education. When an explanation is sought of the variation in wage rates of an entire population, rather than an analysis of wage rate differentials between two specific demographic groups, racial or sexual differences may not account for a substantial amount of the total variation.

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2.1

WAGE DIFFERENTIALS, HUMAN CAPITAL, AND DEMOGRAPHIC CHARACTERISTICS*

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In recent years, much research has been reported which analyzes the source of wage rate and earnings differentials within and across markets. The purpose of this paper is to apply human capital theory to an analysis of wage rate differentials within a labor market, using demographic characteristics as proxies for human capital. Although this is not novel in analyzing wages, our methodology is somewhat different in that we use an analysis of variance technique, rather than multiple regression, to explain wage rate differentials. This allows us to focus on the interactions of a set of demographic characteristics in determining wage rates.

Previous research on wage rate differentials has consistently employed multiple regression techniques, often with no specification of interaction effects between the independent variables. Fuchs, for instance, analyzes wage differentials between the South and Non-South in 1959, by race, age, sex and education. His specification, however, is a simple linear one, without interaction effects.[1] Blinder utilizes the same technique in decomposing wage differentials between males and females, and blacks and whites, by regressing wage rates on age, education, race, parent's income, etc.[2] The implication is that one would add the coefficients of education and work experience to get the combined effect of these two variables, but Thurow has shown that when education and work experience are allowed to interact, the combined effect is approximately four times as large as the sum of the separate effects.[3] Hence, previous estimates of the wage differential between blacks and whites where blacks have less formal education and less continuous work experience may be seriously biased downward.

An analysis of variance technique similar to ours has been used by Hall and by Boskin,[4] but they have imposed some rather stringent restrictions on the interaction of their explanatory variables. For instance, in explaining the wage differential between two race-sex groups, the wage profile by age is not allowed to vary with education. In other words, if we compare thirty year old white females and black females with a high school degree, we would obtain the same wage ratio if we looked at thirty year old white and black females with a B.A. Our technique, however, would impose no such restriction on the wage rate ratio.

A sizeable body of literature also exists in which human capital theory is used to explain earnings differentials. Hansen, Weisbrod and Scanlon regressed yearly earnings on several independent variables: years of schooling completed, Armed Forces Qualification Test percentile, training, and years of work experience, among others.[5] Their sample consists of 17-25 year old males who were rejected for military service, but no information is given as to how many annual hours these individuals worked. Griliches and Mason propose a similar specification, though they restrict their sample to males 21-34 years old, working full time.[6] Whereas analyzing wage rate differentials using human capital theory is a sound approach, using it to analyze earnings differentials is not. Since earnings is the product of a wage rate and hours supplied to the market, an earnings function is actually a reduced-form of the labor supply system. For example, suppose two individuals in a labor market have the same demographic characteristics (say, white females with a

B.A. in economics) and same wage rate. However, one woman has more of a taste for leisure and hence works only half as much as her counterpart. Using human capital theory to explain this differential would imply that the woman with higher earnings has more human capital, when, in fact, they both have the same amount, the difference in earnings being due to different utility functions. Therefore, human capital theory alone is not appropriate in studying earnings differentials. Human capital theory is essentially one of relative wages, not earnings, and although this distinction would not be crucial if everyone worked approximately the same number of hours, the diversity of hours offered in the market by demographic groups suggests that the error may be quite significant.

In this paper, Section I describes the human capital model, and Section II describes the algorithm used to detect the interaction of the demographic characteristics. Section III describes the data base, and Sections IV and V report results and conclusions.

I. HUMAN CAPITAL THEORY

According to the theory of human capital, wage rates are the product of a market rate of return and an effective stock of human capital:

$$(1) \qquad W_{ij} = \omega_j K_i \gamma_i \ ,$$

where W_{ij} is the wage rate of the i^{th} individual or group of "identical" individuals in the j^{th} labor market, ω_j is the market rate of return, K_i is the individual or group average human capital stock, and γ_i is the utilization rate of the individual's or group's human capital in the market place. Wage rates can differ across individuals or groups, therefore, because they may work in different markets or have different human capital stocks and utilization rates. In equilibrium in any given market, however, relative wage rates are determined only by relative utilized human capital stocks. For example, consider the n^{th} individual or group in the j^{th} state. The relative wage rate between the i^{th} and n^{th} individuals or groups is expressed by

$$(2) \qquad \frac{W_{ij}}{W_{nj}} = \frac{\omega_i K_i \gamma_i}{\omega_j K_n \gamma_n} = \frac{K_i \gamma_i}{K_n \gamma_n} .$$

In this paper we attempt to find those combinations of demographic-education characteristics by which individuals can be grouped so as to form the most homogeneous groups in terms of human capital. Since in equilibrium relative stocks of utilized human capital determine relative wage rates, the search for homogeneous groups can be carried out by finding those groups which maximize our ability to explain wage rate differentials.

The rate of utilization, γ, is conceptually formed as:

$$(3) \qquad \gamma = (1-t_i-t_\ell),$$

where t_i is a proportion of the amount of work time spent in on-the-job training or investment, and t_ℓ is the leisure component expressed as a proportion of work time.[7] Since training can be either specific, general, or some combination of the two, individuals in the same market with the same stock of human capital and the same utilization rate, can have different wage rates if one individual receives only completely specific training while the other receives completely general training, since a firm will incur the cost of specific training.[8] In our analysis we focus on utilized human capital, assuming that "identical" individuals have the same utilization rates. Discrimination, by race or sex, can be viewed as affecting the utilization rates of groups. We make no attempt to explain the variation in utilization rates among groups, just as we do not explain the distribution of human capital.

As a proxy for human capital, we propose the use of race, age, sex, and education characteristics. _A priori_, we believe that race accounts for the differential work and educational opportunities that have been available to non-whites because of a variety of factors, among them discrimination. Age represents on-the-job training and work experience, and thus directly influences the utilization rate. Sex is a variable representing the more limited opportunities for education and selection of jobs available to females vis-a-vis males. Education is a variable denoting formal training.

II. ANALYSIS OF VARIANCE MODEL

In what follows we describe the algorithm used to partition the sample into homogeneous groups so as to maximize wage rate differentials between groups and minimize that within each group. An analysis of variance technique is employed because of the importance of allowing complete interaction among the demographic-education characteristics in forming homogeneous groups.

The total variance (TSS) of individual wage rates within a market can be separated into two parts, an inter-group sums of squares of the explained variation given the groups (hereafter called the BSS), and the sum of intra-group sums of squares or the unexplained portion (USS). Consider $n_1+n_2+ \ldots + n_m$ wage rate observations, W_{ij}, which have been classified into m groups. The total variance can be expressed as follows:

$$\text{(4)} \qquad TSS = \sum_{i=1}^{m}\sum_{j=1}^{n_i}(W_{ij} - \bar{\bar{W}})^2 = \sum_{i=1}^{m} n_i(\bar{W}_i - \bar{\bar{W}})^2 + \sum_{i=1}^{m}\sum_{j=1}^{n_i}(W_{ij} - \bar{W}_i)^2 \equiv BSS + USS,$$

where $\bar{\bar{W}}$ is the mean of the parent group or the grand mean, $\bar{W}_i$ is the mean of each sub-group, and n_i is the number of observations in each of the m groups. The first term is the total sums of squared deviations of the m group means about the grand mean and the second term is the sum of the variation within each group.

To take an example, consider a partition of the observations based on sex: $m = 2$. Equation 4 is rewritten:

$$\text{(5)} \qquad \sum_{i=1}^{2}\sum_{j=1}^{n_i}(W_{ij} - \bar{\bar{W}})^2 = n_1(\bar{W}_1 - \bar{\bar{W}})^2 + n_2(\bar{W}_2 - \bar{\bar{W}})^2 + \sum_{j=1}^{n_1}(W_{1j} - \bar{W}_1)^2 + \sum_{j=1}^{n_2}(W_{2j} - \bar{W}_2)^2$$

or

$$\text{(6)} \qquad n_1(\bar{W}_1 - \bar{\bar{W}})^2 + n_2(\bar{W}_2 - \bar{\bar{W}})^2 = \sum_{i=1}^{2}\sum_{j=1}^{n_i}(W_{ij} - \bar{\bar{W}})^2 - \left|\sum_{j=1}^{n_1}(W_{1j} - \bar{W}_1)^2 + \sum_{j=1}^{n_2}(W_{2j} - \bar{W}_2)^2\right.$$

If such a classification happened to produce perfectly homogeneous groups -- i.e., the wages of all males are the same, and the wages of all females are the same so that the last two terms are zero -- then the total variance would be accounted for solely by the intergroup mean differences. If this partition did not produce perfectly homogeneous groups, then introduction of another demographic variable (like education) might reduce the unexplained variance.

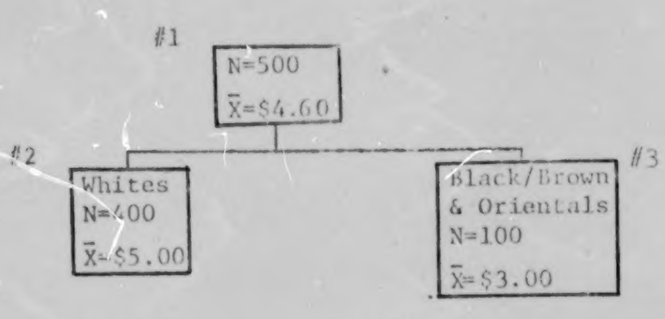
The problem is essentially one of selecting groups using demographic-education characteristics so that the variance in wage rates within these groups is minimized. A number of reasonable constraints have been imposed so that the algorithm will generate meaningful, prominent groups. First, we require that each group account for at least one percent of the total variance. Second, we ensure that the predictor variable (i.e., the demographic characteristics) which is used to split a group reduces the unexplained variance by at least a minimal amount. This is done by requiring that the between-sums-of-squares, BSS, for the proposed partition be at least a specified proportion of the original total sums of squares. We also establish a minimum group size to keep sampling error in check, and limit the number of groups generated. Within these constraints, we then maximize the left-hand side of (6), thus minimizing the unexplained variation in wage rates.

The computer algorithm used is the Automatic Interaction Detection (AID) program.[9] This algorithm uses a non-symmetrical branching process to form human capital groups by partitioning the sample (using demographic-education predictor variables) to best explain the variation in hourly wage rates.

To demonstrate the branching algorithm, suppose we have 500 individuals in our parent group, and we have two demographic predictor variables: race (white, black/brown, and oriental) and education (highest year of schooling completed: grades 1-8, grades 9-12, grades 13-16, and 17 years or more). Accordingly, we can divide the parent sample into three categories based on race, or into four categories based on education. For each predictor, the categories are put into ascending order based on the mean wage rate of the observations in each category.[10] Suppose the ordering is oriental, black/

brown, white. Then a trial dichotomization of the observations on the race variable occurs in which "oriental" observations are separated from the others. The amount of variation (i.e., the BSS) that this partition accounts for is calculated. Another trial dichotomization is performed on orientals and black/brown versus whites. The same sequence of ordering and trial dichotomizations are performed using the education predictor variable. There will be two trial dichotomizations on the race variable and three on the education variable. The parent sample will now be split by the dichotomization (partition) with the largest BSS. Suppose this is on the race variable, where whites as one group are separated from the black/browns and orientals. Each new group is then verified to see whether it accounts for at least some specified amount of the original total variance. If it doesn't, we conclude that the parent group was fairly homogeneous, and need not have been split.

FIGURE 1



For example, suppose both groups in Figure 1 meet the above criterion. The next split is contemplated on the group with the largest amount of variance. If this is group #2, we calculate the BSS for all the possible dichotomizations of the N = 400 observations with respect to the education variable, and select the next split according to the largest BSS. If group #3 has the largest amount of variation, we calculate the BSS between black/browns and orientals and also for the dichotomies of the N = 100 observations on the education variable.

We can also restrict the type of split made by requiring a predictor variable to be monotonic. This means that categories of this variable must be partitioned into contiguous sets. We did impose the continuity restriction on the education variable because we believe that average wage rates increase monotonically with schooling completed. We did not, however, impose it on the age variable because, a priori, we expect some young individuals who are accumulating human capital to have similar average hourly wage rates as some older individuals whose human capital is already depreciating.

In summary, AID forms groups by calculating the amount of variation explained (BSS) for each dichotomization of the group of each predictor variable, splitting the parent group on that predictor variable which accounts for the most variation of the dependent variable, i.e., the one with the largest BSS. The next split is contemplated on the group with the most variation, again calculating the appropriate between-sums-of-squares. If the variation within a group is not significant as defined by the user, it is considered homogeneous and it becomes a final group. For those groups that are candidates for further splitting, it is also required that the reduction in unexplained variation from splitting on a predictor be some minimal proportion of the TSS. If there is no predictor satisfying this condition, then the group will not split any further. The process terminates when there are no groups capable of being split.

III. CENSUS PUBLIC USE SAMPLE

To ensure we are observing the effects of human capital, and not those of supply and demand, we must control for market effects; hence, we draw observations from only one market. We use states to delimit labor market areas, but we do recognize that a state may contain several labor markets or a labor market may span several states.

We use data for the State of California from the State Public Use Sample (1/1000) of the 1970 Census. We choose California because it is the largest state in the nation and hopefully representative of the nation. Our dependent variable is the average hourly wage rate of primary workers,[11] formed by dividing annual earnings of each individual by the product of hours worked in the Census reference week (March, 1970) and the weeks worked last year (1969). Only those individuals who reported earnings and hours are included in the sample. Our predictor variables are:

(a) Race - 2 categories: White; Oriental, Nonwhite and Others

(b) Age - 5 categories: 16-21; 22-35; 36-54; 55-64; 65 and over

(c) Sex - a binary variable

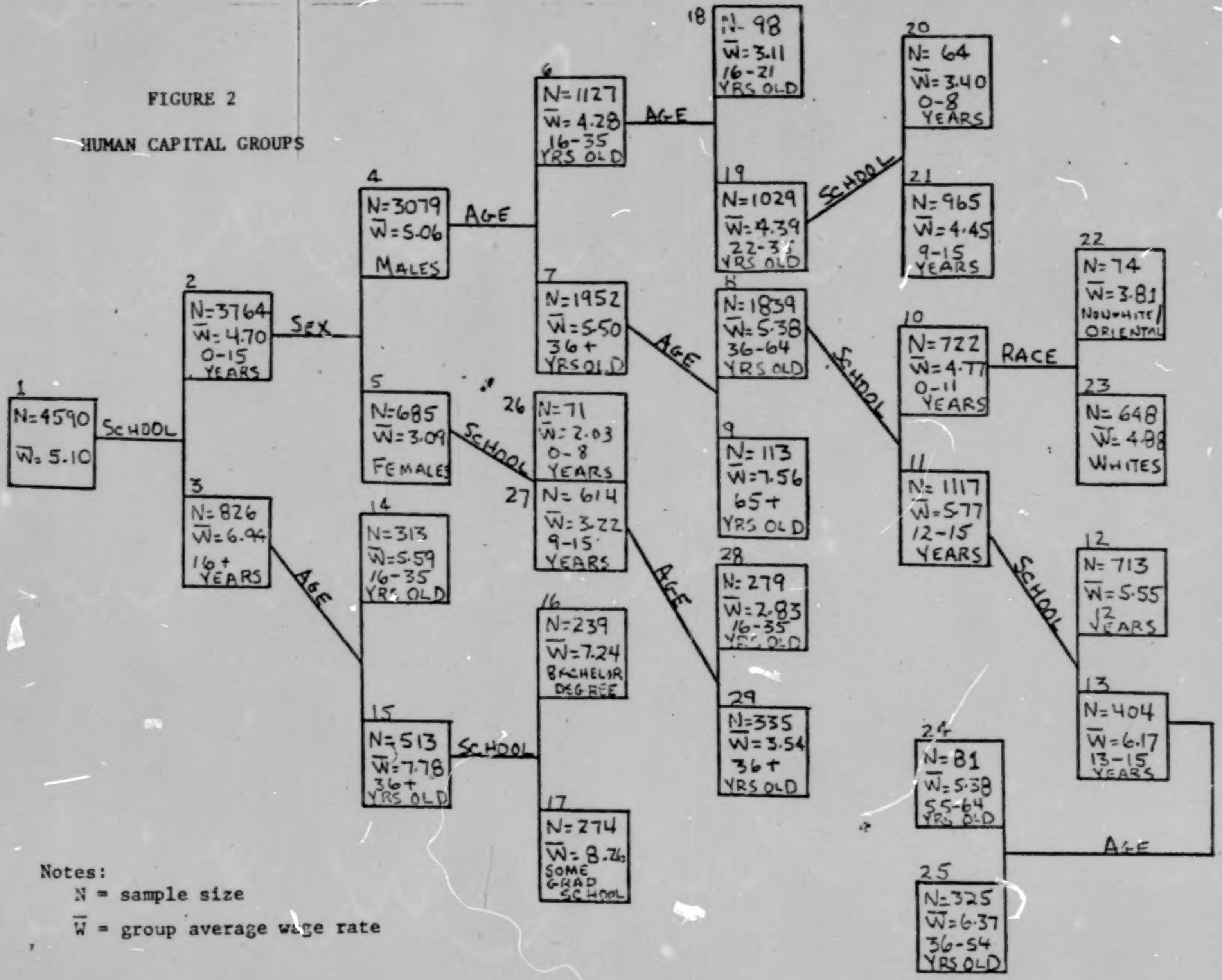
(d) Education - 6 categories by years of schooling completed: 0-8; 9-11; 12; 13-15; 16; 17 or more.

IV. HOMOGENEOUS HUMAN CAPITAL GROUPS

The homogeneous human capital groups produced by the AID algorithm are shown in Figure 2. In each box is the number of observations (N), the average wage rate ($\bar{W}$), and other relevant demographic information for that group. The number on top indicates the order in which AID did its splitting. For example, group 4 was split before groups 3 or 5 because its variation was larger. In describing the results, however, we do not follow the sequence in which the groups were formed.

FIGURE 2

HUMAN CAPITAL GROUPS


1 N=4590 $\overline{W}$=5.10
SCHOOL
2 N=3764 $\overline{W}$=4.70 0-15 YEARS
3 N=826 $\overline{W}$=6.94 16+ YEARS
SEX
4 N=3079 $\overline{W}$=5.06 MALES
5 N=685 $\overline{W}$=3.09 FEMALES
AGE
14 N=313 $\overline{W}$=5.59 16-35 YRS OLD
15 N=513 $\overline{W}$=7.78 36+ YRS OLD
AGE
6 N=1127 $\overline{W}$=4.28 16-35 YRS OLD
7 N=1952 $\overline{W}$=5.50 36+ YRS OLD
SCHOOL
26 N=71 $\overline{W}$=2.03 0-8 YEARS
27 N=614 $\overline{W}$=3.22 9-15 YEARS
SCHOOL
16 N=239 $\overline{W}$=7.24 BACHELOR DEGREE
17 N=274 $\overline{W}$=8.26 SOME GRAD SCHOOL
AGE
18 N=98 $\overline{W}$=3.11 16-21 YRS OLD
19 N=1029 $\overline{W}$=4.39 22-35 YRS OLD
AGE
8 N=1839 $\overline{W}$=5.38 36-64 YRS OLD
9 N=113 $\overline{W}$=7.56 65+ YRS OLD
AGE
28 N=279 $\overline{W}$=2.83 16-35 YRS OLD
29 N=335 $\overline{W}$=3.54 36+ YRS OLD
SCHOOL
20 N=64 $\overline{W}$=3.40 0-8 YEARS
21 N=965 $\overline{W}$=4.45 9-15 YEARS
SCHOOL
10 N=722 $\overline{W}$=4.77 0-11 YEARS
11 N=1117 $\overline{W}$=5.77 12-15 YEARS
RACE
22 N=74 $\overline{W}$=3.81 NON-WHITE/ ORIENTAL
23 N=648 $\overline{W}$=4.88 WHITES
SCHOOL
12 N=713 $\overline{W}$=5.55 12 YEARS
13 N=404 $\overline{W}$=6.17 13-15 YEARS
AGE
24 N=81 $\overline{W}$=5.38 55-64 YRS OLD
25 N=325 $\overline{W}$=6.37 36-54 YRS OLD

Notes:
N = sample size
$\overline{W}$ = group average wage rate

The schooling variable splits the parent sample into those with a bachelors degree or more from those without a college degree. Hereafter, we shall refer to subsequent splits of the less educated group (#2) as being in the upper segment, and those of the more educated group (#3) as being in the lower segment.

The next split in the upper segment is on sex, females again dividing on the schooling and age variables. The males (#4) proceed to split on a series of age-schooling divisions, a finding in concurrence with previous work. One exceptional result, however, is that the race variable does not become a significant predictor of differences in the average hourly wage rate until rather far into the upper segment (groups #22 and 23). It is rather interesting to note, however, that race is a significant predictor only for middle-aged males with less than a high school degree.[12] Apparently, the effects of discrimination, as revealed through a smaller utilized stock of human capital, are most visible in this demographic group. Most studies have found race to be of prime importance in explaining wage rate differentials. Our results do not imply, however, that if we selected two individuals with similar demographic characteristics except for race that race would not be an important factor. Rather, they imply that race by itself does not account for a large enough proportion of total wage rate variation in the observation set relative to that accounted for by education, age and sex. One reason for this is that nonwhites comprise a small proportion of the sample and hence could not account for a high proportion of the variance unless their wage rates were extreme outliers -- which they apparently are not.

The lower segment splits only on age and schooling, indicating either that extreme differences in average hourly wage rates do not exist for the more educated females vis-a-vis males, or that there are relatively few women in our sample with 16 or more years of formal schooling, so that wage rate differences between the sexes do not account for much of the total group variation. Again, we are not implying that a male-female (holding other demographics constant) wage-rate polarity does not exist, just that the effect of sex on wage rates may be swamped by other characteristics.[13]

According to our human capital model, relative wage rates in equilibrium are proportional to relative utilized stocks of human capital. In Table I, we present the human capital groups that resulted from AID, (with some groups in Figure 2 being reaggregated). They are ranked from highest to lowest according to the average hourly wage rate of each group. We also present the implied utilized human capital units for each group using group (a) as the numeraire.

Age and education are the key variables in explaining differences in utilized human capital. Presumably because of schooling, those in group (c), who are otherwise demographically similar to those in (a), have 12 percent less utilized human capital than those in (a). Education also explains why those in group (f) have 10 percent less utilized human capital than those in (d). Due to age, individuals in (j) have a 25 percent higher wage rate than individuals in group (l), while race accounts for those in (g) having 28 percent more utilized human capital than those in (i). Overall, our results accord with those found in previous research, except for the role of race.

Hall and Boskin have used analysis of variance techniques in constructing wage equations for their labor supply models. Hall's method is somewhat more restrictive than ours, however, in that within each race-sex group, he

TABLE 1

HUMAN CAPITAL GROUPS BY DEMOGRAPHIC CHARACTERISTICS

Group Identifier*	Average Hourly Wage Rate	Utilized Human Capital Units (Group (a) = 1.00)
a	$ 8.26	1.00
b	7.56	.92
c	7.24	.88
d	6.17	.75
e	5.59	.68
f	5.55	.67
g	4.88	.59
h	4.39	.53
i	3.81	.46
j	3.54	.43
k	3.11	.38
l	2.83	.34
m	2.03	.24

*(a) All individuals, 36 years old or more, one or more years of graduate school.
(b) Males, 65 years old or more, 0-15 years of school.
(c) All individuals, 36 years old or more, college graduate.
(d) Males, 36-64 years old, some college.
(e) All individuals, under 35 years old, college graduate or one or more years of graduate school.
(f) Males, 36-64 years old, high school graduate.
(g) White males, 36-64 years old, 0-11 years of schooling.
(h) Males, 22-35 years old, 0-15 years of school.
(i) Males, nonwhite and oriental, 0-11 years of school.
(j) Females, 36 years old or more, 9-15 years of school.
(k) Males, 16-21 years old, 0-15 years of school.
(l) Females, 16-35 years old, 9-15 years of school.
(m) Females, 0-8 years of school.

assumes that the effect of education, on average, is independent. For example, Hall finds that white males, 25-34 years old, with 12 years of education, have an hourly wage rate which is approximately 25 percent higher than that of black males in the same age and education category. By implication, if one compared the same two race-sex-age groups, but this time those with a college degree, the white males would still have an hourly wage rate 25 percent higher than black males.[14] Our results, on the contrary, suggest an interaction between education and age. Comparing white males with oriental nonwhite males, age 36-64, we find that the ratio of their wage rates is 1.00 for those with 16 or more years of schooling, 1.00 for those with a high-school degree or some college, but 1.28 for those with 11 years or less of schooling. Since Boskin's method parallels Hall's, it is subject to the same restrictions.

Other research is even more restrictive than Hall's or Boskin's in not allowing for interaction of any demographic characteristics. Blinder's analysis suggests that age accounts for five percent of the white-black male wage differential in his sample, while education accounts for 20 percent. This suggests that a white male who is older and has more education than a black male should have a 25 percent higher hourly wage rate, but Thurow has shown that for his education and experience variables, the combined effect is approximately four times as large as the sum of the separate effects.[15] Although we would not necessarily expect the same 4:1 ratio, it seems safe to conjecture that the combined effect would be greater than 25 percent.

V. CONCLUSIONS

Several points emerge from our research. First, we have used human capital theory to explain relative wages, not earnings, since earnings are a function of a wage and hours supplied to the market, and therefore cannot be analyzed by human capital theory alone. Second, we have utilized an analysis of variance technique to explain the variation in average hourly wage rates rather than multiple regression. Using the former technique allows for complete interaction of the independent variables, revealing insights into the joint effects of the independent variables on the dependent variable. Third, we have found that race is not as important a predictor in explaining average hourly wage rates as our other demographic variables, nor is sex among higher educated individuals. The former may be due to the fact that race manifests its effects through other variables such as education. For instance, if most black males in a certain age group had less education than same-aged white males, the split might take place on the education variable rather than the race variable. Similarly, age may be a stronger predictor of wage rates among individuals with a B.A. or advanced degree than is sex. We do not infer from these results that racial or sexual discrimination is of small consequence, but that when we seek to explain the variation in wage rates of an entire population, rather than analyze wage rate differentials of two specific demographic groups, racial or sexual differences in some cases may not account for a substantial amount of the total variation.

FOOTNOTES

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1. See Fuchs [4].

2. See Blinder [2]. Scully [13] implicitly takes some interactions into account in forming a human capital variable.

3. See Thurow [18].

4. See Hall [7] and Boskin [3].

5. See Hansen, Weisbrod and Scanlon [9].

6. See Griliches and Mason [6]. Becker [1], Hanoch [8] and Hanushek [10] also use human capital theory to explain earnings differentials.

7. In a strict dynamic sense, we would have to allow for a feedback of t_i on K, since on-the-job-training is a capital-forming activity.

8. General training is defined as training that increases the marginal product of an individual to all firms, while specific training raises an individual's marginal product only in the firm providing the training. Hence, a firm could be expected to provide specific, but not general, training since it can capture the returns to investment in the former case but not the latter. For a complete discussion, see Becker [1]. Of course, a sound argument can be made that relatively little training is firm-specific.

9. For a complete discussion of the algorithm, see Sonquist and Morgan [16]. For an application of AID to the income distribution problem, see Smith and Morgan [15].

10. When the categories are ordered by their mean values, the variable is said to be "free". If we impose a monotonic relationship between the dependent variable and a predictor variable, then the categories of the predictor variable are ordered contiguously rather than by their category means.

11. We exclude from the potential labor force those individuals under 16 years of age, and those currently in the military service. We also exclude 16 and 17 year olds attending school.

12. Hall [7] obtains a similar result.

13. Contrary to our results, Kreps [11] finds that women generally have lower wage rates than men, the difference being more pronounced for those over 35 years old.

14. See Hall [7, p. 115].

15. See Thurow [18].

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EQUILIBRIUM AND DISEQUILIBRIUM IN THE LABOR MARKET

by C. Duncan MacRae

Abstract

A dynamic disequilibrium model of the labor market is presented and analyzed in this paper. This model integrates the classical theory of supply and demand with the modern theory of job search and labor turnover. One of the implications of the model is that there need not be a trade off between inflation and unemployment or a natural rate of unemployment unaffected by the rate of inflation. Rather for certain values of the parameters there is a direct relation between inflation and unemployment in the model. Therefore, an increase (decrease) in aggregate demand can increase (decrease) not only inflation but also unemployment.

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EQUILIBRIUM AND DISEQUILIBRIUM IN THE LABOR MARKET*

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The traditional classical analysis of wages and employment is in terms of the demand and supply of labor. Since both firms' demand and households' supply is a function of the market wage rate, employment and wages are then determined in static equilibrium by the equality of supply and demand. In this equilibrium analysis either there is no role for unemployment, or it is explained as the result of disequilibrium in which supply exceeds demand.

The modern explanation of the wage and employment dynamics underlying the Phillips relation, however, is based on job search and labor turnover behavior.[1] The change in wage rates is determined by the balance between wage offers and reservation wages which are assumed to be functions of job vacancy and unemployment rates. Given the turnover rate, the change in employment is then the result of unemployed people matching with vacant jobs. For alternative levels of job stock and labor force, these two dynamic relations imply a Phillips relation between wage change and the unemployment rate, where the job stock and labor force are determined by demand and supply. But when the supply and demand for labor are equal, the level of employment is less than that desired by both households and firms.

The purpose of this paper is to develop a model of the labor market which integrates the classical theory of supply and demand with the modern theory of job search and labor turnover. In the classical tradition, the levels of employment desired by firms and households are determined by wages, output, and population. Therefore, in equilibrium, when demand, supply, and hence employment, are equal, wages and employment are determined in the traditional manner. When employment and demand are not equal, however, firms are assumed to adjust their search effort as measured by vacant jobs. For example, if desired employment by firms is greater than actual employment, then firms increase the number of vacancies to hire more people. Similarly, if actual employment and labor supply are not equal, households alter their job search as represented by the level of unemployment. In the modern tradition, wage and employment dynamics are functions of the vacancy and unemployment rates. Therefore, the levels of unemployment and vacancies necessary to maintain supply-demand equilibrium are determined by search-turnover behavior. Job stock and labor force then follow, by definition, from job vacancies, employment, and unemployment.

The paper begins with a complete statement of the labor market model. A comparative statics analysis of the effects of output and population changes is presented. An analysis of the labor market in a state of steady growth is then performed. Finally, dynamic disequilibrium behavior in the market is examined by determining the conditions under which the model is stable.

I. LABOR MARKET MODEL

In this section a model of the labor market is presented. This model explains wages, employment, job vacancies, unemployment, hence, job stock and labor force by integrating the classical theory of supply and demand with the modern concepts of job search and labor turnover.

As in the classical analysis of wage and employment determination, the desired demand, D, for labor is determined by the real wage, W, and the level of output, Q.

$$D = d(W,Q), \tag{1.1}$$

where the form of (1.1) is derived from the theory of the firm.[2] The lower is W and the higher is Q, then the higher is D.

If the level of employment desired by firms is not equal to the actual level of employment, then firms are assumed to adjust their level of job vacancies, V, according to

$$\text{(1.2)} \qquad d\ln V/dt = v\{D/E\},$$

where $d\ln V/dt$ is the proportionate rate of change in vacancies. If D exceeds E so that firms desire to employ more people than they are currently employing, then they increase their search effort as represented by the number of job openings (V) in their personnel offices. If firms would like to employ less people, then they decrease the number of job slots which are authorized but not filled. Note that they do not adjust employment directly; they only affect it indirectly by adjusting vacancies.[3] In the first case they increase the probability of some person finding one of their jobs. In the second case they allow normal labor turnover to decrease their level of employment. Note also that firms are indifferent to the level of vacancies (V); they are only concerned about the relation between their desired level of employment (D) and the actual level of employment (E).[4] Maintaining vacant jobs is only a means by which they attempt to maintain equality between D and E.

In general the level of employment desired by firms is not observable, but it is a function of wages and output, which are observable. Therefore, we can substitute the labor demand relation (1.1) into the vacancy change equation (1.2) to obtain vacancy dynamics as a function of wages, output, and employment:

$$\text{(1.3)} \qquad d\ln V/dt = v\{d\{W,Q\}/E\}.$$

For a given level of output the relation between dlnV/dt, W, and E can be seen in Figure 1. The locus of all (E,W) points for which dlnV/dt is zero is nothing but the classical demand curve. These are the combinations of employment and wage for which D=E so that vacancies are neither increased nor decreased. All points below this curve, therefore, are cases in which D>E so that firms increase vacancies. Similarly, for all employment-wage combinations above the curve, vacancies are decreasing. Loci of constant vacancy change are thus similar in shape to the classical demand curve. The same dlnV/dt would be generated either by low E and high W, hence low D, or by high E and low W, hence high D.

By definition the stock of jobs, J, is the sum of jobs which are filled (E) and jobs which are not filled (V):

$$\text{(1.4)} \qquad J = E + V.$$

Therefore, the job stock is determined both by employment, which firms cannot control directly, and by vacancies, which firms do control. If vacancies do not change, J will reflect employment behavior; however, if employment does not change, it will reflect vacancy behavior. Thus job stock behavior is a by-product of employment and vacancy dynamics instead of being a determinant of these dynamics.

Just as in the classical model the demand for labor is a function of the real wage and output so also is the desired supply of labor, S, a function of the real wage and the level of population, P:

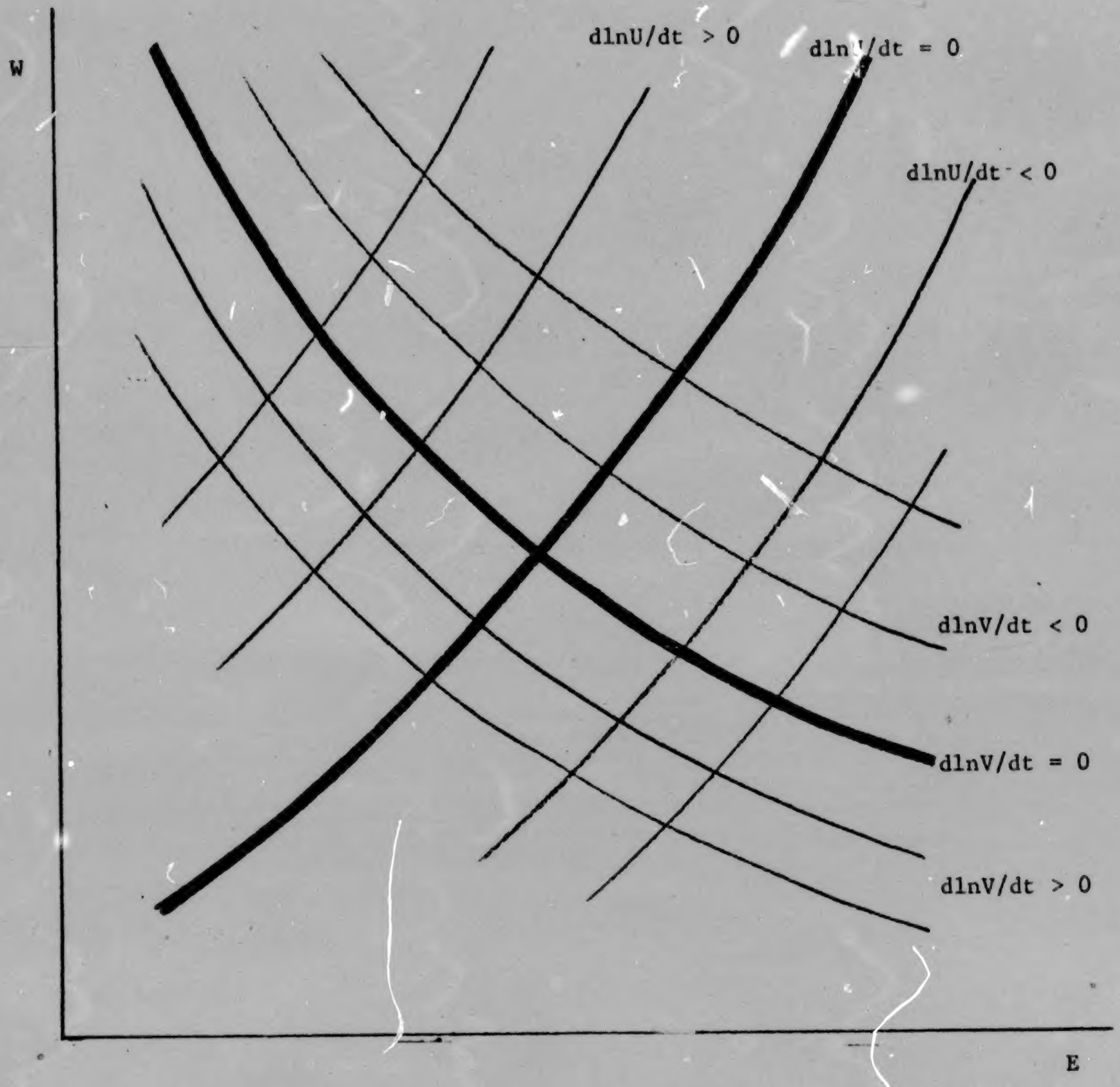
W
E
dlnU/dt > 0
dlnU/dt = 0
dlnU/dt < 0
dlnV/dt < 0
dlnV/dt = 0
dlnV/dt > 0

Figure 1
Vacancy and Unemployment Dynamics

$$(1.5) \qquad S = s\{W,P\},$$

where the form of (1.5) is determined by the theory of the household.[5] In general, the higher are W and P, then the higher is S. If a normal income effect dominates the substitution effect, however, so that labor supply is backward-bending, then an increase in the wage can actually decrease labor supply.

If families find that their current level of employment (E) does not match their desired level of employment (S), then they alter their search behavior as represented by the level of unemployment, U:

$$(1.6) \qquad d\ln U/dt = u\{S/E\},$$

where $d\ln U/dt$ is the proportionate rate of change in U. If S exceeds E, representing the fact that people would like to work more than they are currently working, then they enter into search for additional work. The higher is the number of people searching for jobs who do not have jobs, the higher is the level of unemployment. If they would like to work less than they are working, then they retire from the search, which was necessary to maintain their level of employment. The lower is the number of people searching for jobs, the lower is unemployment. Therefore, just as firms control only their job vacancies, so also households control only their unemployment by deciding whether or not to search for a job: to increase employment, they increase search effort; to maintain employment, they maintain the level of search; and they reduce their employment by retiring from the search for a job.

Just as we substituted the labor demand relation into the vacancy change equation, we can now substitute the labor supply relation (1.5) into the unemployment change equation (1.6) to obtain an observable relation between unemployment dynamics, wages, population, and employment:

$$(1.7) \qquad d\ln U/dt = u\{s(W,P)/E\}.$$

This relation is described in Figure 1 for a given level of population. Indeed the classical supply curve is the set of employment-wage combinations for which $d\ln U/dt = 0$. Therefore, for all points to the right of the supply curve, unemployment is decreasing, and for all points to the left of the curve, unemployment is increasing. Again as in the case of vacancy dynamics, the loci of (E,W) points for which $d\ln U/dt$ is constant are similar in shape to the supply curve. A given rate of change in U can be maintained either with low employment and low wage, hence low supply or with high employment and high wage, hence high supply.

Labor force, L, is of course, the sum of people who have jobs (E) and people who do not but are looking for jobs (U):

$$(1.8) \qquad L = E + U$$

It is the result rather than the determinant of employment dynamics and unemployment dynamics.[6] Moreover, labor force behavior is uniquely determined neither by employment behavior nor by unemployment behavior. An increase in labor force can be accompanied by a decrease in employment if unemployment increases. Similarly, a decrease in L can occur when U increases if E decreases. What happens to labor force will all depend on the relative magnitude of the two dynamics.

As we have seen, neither firms nor households alone determine employment dynamics. Rather they are determined by the process of job search and labor turnover. By definition the proportionate rate of change in employment, dlnE/dt, is equal to the difference between the accessions rate, A/E, and the turnover rate, T/E:

(1.9) $$dlnE/dt = A/E - T/E,$$

where A consists of new hires and other accessions and T encompasses quits, dismissals and other separations.

Accessions are the result of vacant jobs finding unemployed people and vice-versa. Thus the accession rate is a function of the unemployment rate, U/E, and the vacancy rate, V/E, with employment as the base:

(1.10) $$A/E = a\{U/E, V/E\}.$$

For a given rate of search by firms as measured by V/E, the higher is the rate of search by households, as measured by U/E, the higher will be the accessions rate. Similarly for given U/E, the higher is V/E, then the higher will be A/E.

Turnover is also a function of the unemployment rate and the vacancy rate:

(1.11) $$T/E = t\{U/E, V/E\}.$$

The tighter is the labor market as measured by the vacancy-unemployment ratio, V/U, the higher will be the probability of an unemployed person finding a vacant job but the lower will be the probability of a vacant job finding an unemployed person. Therefore, members of households will be more likely to quit to find better jobs and managers of firms will be less likely to dismiss someone to find better employees.

Since both accessions and turnover are functions of the unemployment and vacancy rates, we can substitute (1.10) and (1.11) into (1.9) and rewrite it as

$$\text{(1.12)} \qquad d\ln E/dt = e\{U/E, V/E\},$$

where the function e is the difference between the functions a and t. The relation between dlnE/dt, U/E, and V/E is illustrated in Figure 2. It depicts the balance between the accessions rate and the turnover rate. In general, the effect of vacancies and unemployment on the turnover rate will tend to cancel out, since quits increase as dismissals decrease, and vice-versa. Their effect on accessions, however, will be multiplicative. Therefore, a given dlnE/dt can be maintained with either a high U/E and low V/E or vice-versa. The higher are both the vacancy rate and the unemployment rate, the higher will be the algebraic difference between accessions and separations. But there is one locus of (V/E,U/E) points for which they cancel out so that dlnE/dt = 0. For all points to the right of this curve employment is increasing; for all points to the left, E is decreasing.

It should be noted that while the vacancy and unemployment rates have been expressed with employment as a base, there is a one-to-one correspondence between U/E and the traditional measure U/L. This can be seen from:

$$\text{(1.13)} \qquad U/L = (U/E)/[1 + (U/E)].$$

The higher is U/E, then the higher is U/L and vice-versa. Similarly the common measure of the vacancy rate:

$$\text{(1.14)} \qquad V/J = (V/E)/[1 + (V/E)].$$

Therefore, there is also a direct relation between V/E and V/J.

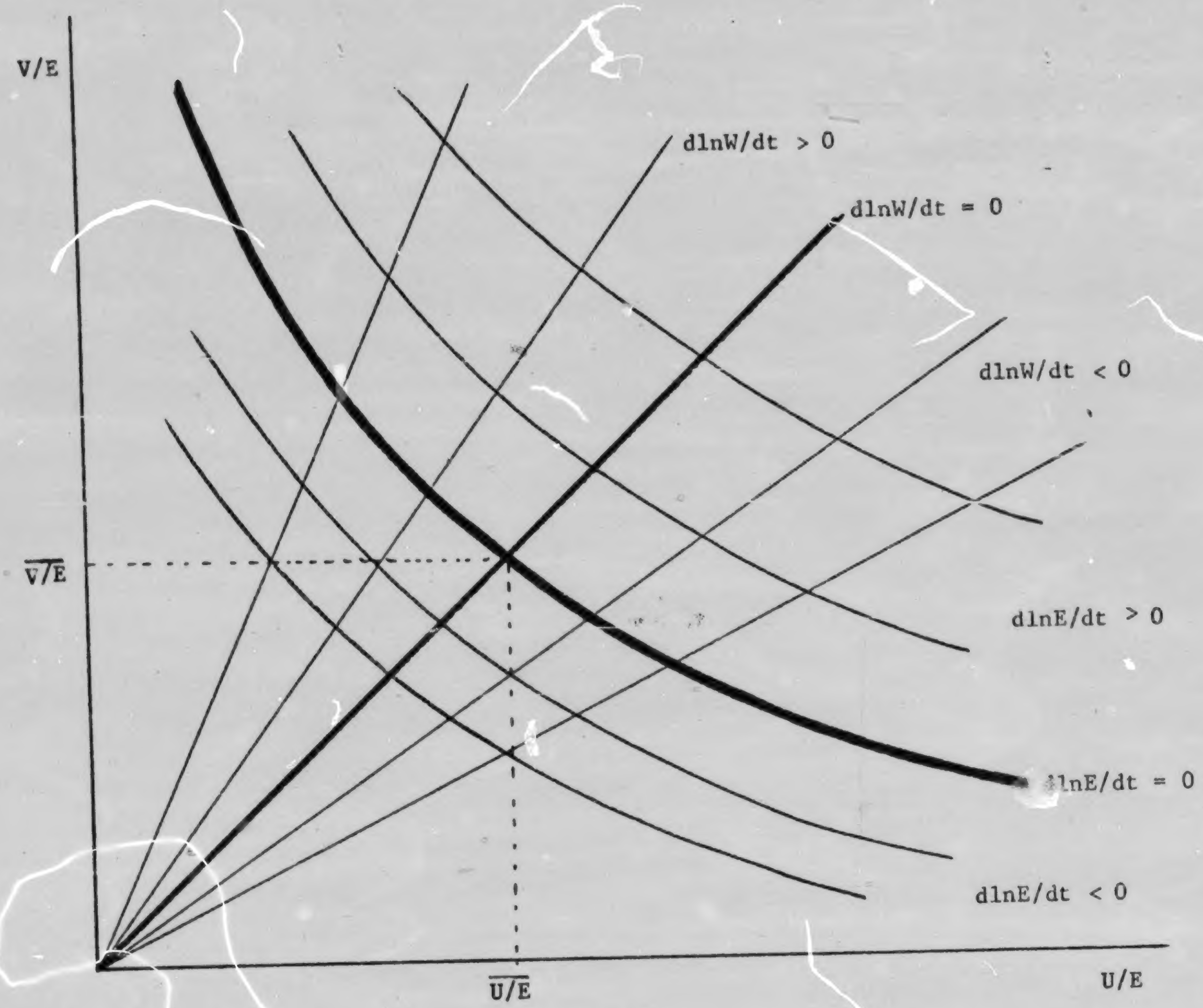
V/E
dlnW/dt > 0
dlnW/dt = 0
dlnW/dt < 0
$\overline{V/E}$
dlnE/dt > 0
dlnE/dt = 0
dlnE/dt < 0
$\overline{U/E}$
U/E

Figure 2
Employment and Wage Dynamics

Just as neither firms nor households alone determine employment so also does neither alone determine wages. They are, instead, determined competitively in the process of job search and labor turnover. The more employed people there are searching for jobs, in general, the longer it will take them to find jobs and the lower will become their reservation wage. Similarly, the more vacant jobs there are looking for people to fill them, the longer it will take for these jobs to be filled, and, hence, the higher will become the wages offered for these jobs. Therefore, the rate of wage change is determined by the balance between the vacancy rate and the unemployment rate:

$$d\ln W/dt = w\{V/E, U/E\}, \qquad (1.15)$$

where $d\ln W/dt$ is the proportionate rate of change in the market wage.[7]

The wage dynamics relation is depicted in Figure 2. It describes the balance between increasing wage offers and decreasing reservation wages. The higher is the vacancy rate relative to the unemployment rate, the higher is the pressure on wages to increase, the lower is the V/U ratio, then the lower is the pressure. In general, there will be a locus of (U/E,V/E) points for which the pressures on wages balance out so that $d\ln W/dt = 0$. Above this curve, wages are increasing, below it they are decreasing. Note, however, that this curve does not necessarily correspond to equality between the vacancy and unemployment rates. These rates are only proxies for the durations of search by firms and households. How the durations are translated into movements in wage offers, reservation wages, and hence, real wages depends on the particular participants in the labor market. A low V/U ratio could be associated with either increasing or decreasing wages. Nevertheless, we do know that the lower is V/U the lower will be the increase, be it positive or negative.

The model is now complete. Given the evolution of output (Q) and population (P) over time and the initial levels of vacancies (V), unemployment (U), employment (E), and wages (W), then the evolution of V, U, E, and W is determined by (1.3), (1.7), (1.12), and (1.15). Job stock (J) and labor force (L) are then by-products of (1.4) and (1.8).

II. COMPARATIVE STATICS

In this section the labor market model is solved in static equilibrium. The effects of marginal changes in output and population on the equilibrium values of wages, employment, unemployment, vacancies, labor force, and job stock are then determined.

In equilibrium the actual level of employment (E) is equal to both the level desired by firms (D) and the level desired by households (S). Therefore firms have no motivation to alter their number of vacancies (V). Similarly, households see no reason to vary their level of search, as represented by the level of unemployment (U). Employment and wages are thus determined as in the classical analysis by the equality of demand and supply. This can be seen algebraically by noting that $d\ln V/dt = 0$ in (1.3) and $d\ln U/dt = 0$ in (1.7) together yield a relation between equilibrium employment, $\bar{E}$, and wage $\bar{W}$, given by

$$d\{\bar{W},Q\} - \bar{E} = 0 \qquad (2.1)$$

and

$$s\{\bar{W},P\} - \bar{E} = 0. \qquad (2.2)$$

The determination of $\bar{E}$ and $\bar{W}$ is illustrated in Figure 1. The locus of employment-wage combinations corresponding to no vacancy change is the demand curve, while the locus corresponding to no unemployment change is the supply curve. Thus wages and employment are determined in equilibrium solely by the intersection of demand and supply. Job search and labor turnover play no role in their determination. Hence, equilibrium employment and wages are not influenced by vacancies and unemployment. Only a change in output (Q) or population (P) will alter $\bar{E}$ and $\bar{W}$.

Equality of demand and supply does not mean that there are no vacant jobs or that there are no unemployed people. Rather because of labor turnover, there is a level of job search, hence, vacancies and unemployment that is necessary to maintain equilibrium employment and wages. If both V and U are too low, then E will decrease as separations exceed accessions so that the actual level of employment will be less than the level desired by both firms and households. If vacancies and unemployment are too high, however, then employment will exceed both demand and supply. Alternatively, if V is too high relative to U, then competition by firms will drive wages up so that the level of employment desired by households exceeds actual employment but the level desired by firms is less than E. If unemployment is too high relative to vacancies, however, then competition by households will drive wages down so that E is less than S but D is greater than E. Thus, there is an equilibrium level of vacancies, $\bar{V}$, and unemployment, $\bar{U}$, that is required to maintain $\bar{E}$ and $\bar{W}$.

To see how equilibrium vacancies and unemployment are determined let us turn our attention now to employment and wage dynamics. As can readily be seen from (1.12) and (1.15) the vacancy rate in equilibrium, $\overline{V/E}$, and the employment rate, $\overline{U/E}$, are determined solely by search-turnover behavior. When employment and wages are constant, as they are in static equilibrium, then vacancy and unemployment rates are given implicitly by

(2.3) $e\{\overline{U/E}, \overline{V/E}\} = 0$

and

(2.4) $w\{\overline{V/E}, \overline{U/E}\} = 0$.

The determination of $\overline{V/E}$ and $\overline{U/E}$ is portrayed in Figure 2. They are given by the intersection of the locus of (U/E, V/E) points for which $d\ln E/dt = 0$ with the locus for which $d\ln W/dt = 0$. For a given labor market tightness as measured by the vacancy-unemployment ratio, the greater is the tendency for employers to lay off or for employees to quit, the higher will be the equilibrium vacancy and unemployment rates; the more efficient, however, is the process by which vacant jobs match with unemployed people, then the lower will be these rates. Again for a given V-U ratio, the more reluctant are firms to increase their wage offers, the higher will be $\overline{V/E}$ and the lower will be $\overline{U/E}$; the more reluctant are households to decrease their reservation wages, however, then the lower will be the equilibrium vacancy rate and the higher will be the unemployment rate.

Once $\overline{V/E}$ and $\overline{U/E}$ are given the determination of $\overline{V}$ and $\overline{U}$ follows immediately since $\overline{E}$ is already determined by supply and demand. The higher is $\overline{E}$, then the higher will be $\overline{V}$ and $\overline{U}$. Similarly, from (1.4) and (1.8) it follows that the higher are equilibrium vacancies and unemployment, the higher will be equilibrium job stock, $\overline{J}$, and labor force, $\overline{L}$. Thus, in static equilibrium there is a dichotomy between wages and employment on the one hand and vacancy and unemployment rates on the other hand. The former are determined by supply and demand; the latter are given by job search and labor turnover.

The equilibrium dichotomy is made particularly apparent when we consider the effects of output and population changes on labor market equilibrium. This can be done by differentiating (2.1) and (2.2) logarithmically and rearranging terms to obtain

$$\text{(2.5)} \qquad d\ln E = [\sigma_W/(\sigma_W-\delta_W)]\,d\ln Q + [-\delta_W/(\sigma_W-\delta_W)]\,d\ln P$$

and

$$\text{(2.6)} \qquad d\ln W = [\delta_Q/(\sigma_W-\delta_W)]\,d\ln Q - [\sigma_P/(\sigma_W-\delta_W)]\,d\ln P,$$

where $\delta_W(\sigma_W)$ is the wage elasticity of demand (supply), δ_Q is the output elasticity of demand, and σ_P is the population elasticity of supply. In general, $\delta_W<0$ and $\sigma_W>0$, but σ_W can be negative if supply is backward-bending. Both $\delta_W>0$ and $\sigma_P>0$, and in the case of constant returns to scale in production and households responding on a per capita basis, $\delta_Q=\sigma_P=1$. Thus, as in the classical analysis, the effects of proportionate changes in output (dlogQ) and population (dlogP) on wages and employment depend only on the wage, output and population elasticities of demand and supply. The "natural" vacancy and unemployment rates are unaffected by these changes. The only effect is to alter $\bar{E}$, hence, $\bar{V}$ and $\bar{U}$ in the same proportion.

While output and population changes will not affect the unemployment rate in equilibrium, they will affect the labor force participation rate, L/P. Since both $\bar{E}$ and $\bar{U}$ change proportionately it follows from (1.8) that equilibrium labor force, $\bar{L}$, will change in like proportion. Thus, output changes will generate a direct relation in equilibrium between employment and the labor force participation rate. The higher is $\bar{E}$, then the higher will be $\overline{L/P}$. This relation follows tautologically from the fact that there is an equilibrium natural rate of unemployment; it does not depend on a "discouraged worker" hypothesis.[8] If changes in labor demand do not affect the equilibrium unemployment rate, then labor force participation must necessarily rise and fall with changes in labor demand. Of course, if changes in employment are brought about by changes in population, the relation between $\bar{E}$ and $\overline{L/P}$ will be modified. Nevertheless, unless labor supply is population elastic, employment and the labor force participation rate will move together. Finally, it should be noted that in equilibrium there is no relation between the unemployment rate and the labor force participation rate. A shift in demand or supply will not influence the natural rate of unemployment.

III. THE STEADY STATE

In this section the character of a labor market growing at a constant rate is examined. In particular the effects of changes in the rate of growth of output and population on steady state values are analyzed.

In general the market is not in equilibrium. Growth in aggregate demand increases labor demand so that employment is less than that desired by firms. Similarly, population growth increases labor supply so that employment is also less than that desired by households. Both firms and households then increase vacancies and unemployment so that employment also grows.

A case of disequilibrium which is of particular interest is a state of steady growth. This is a state in which vacancies, unemployment, employment, hence job stock and labor force, all grow at the same proportionate rate. Differentiating (1.1) logarithmically with respect to time we obtain the proportionate rate of growth in labor demand:

$$\text{(3.1)} \qquad d\ln D/dt = \delta_W d\ln W/dt + \delta_Q d\ln Q/dt.$$

Similarly, the rate of growth in labor supply is obtained from (1.5):

(3.2) $$d\ln S/dt = \sigma_W d\ln W/dt + \sigma_P d\ln P/dt.$$

Since $d\ln D/dt = d\ln S/dt = d\ln E/dt$, it then follows that employment and wage change are simultaneously determined from (3.1) and (3.2) by the rates of growth in output and population:

(3.3) $$d\ln E/dt = d\ln P/dt + (\sigma_W/(\sigma_W-\delta_W))(d\ln Q/dt-d\ln P/dt)$$

and

(3.4) $$d\ln W/dt = (\delta_Q d\ln Q/dt-\sigma_P d\ln P/dt)/(\sigma_W-\delta_W).$$

Thus, just as in the classical analysis, the higher is the rate of growth of population the higher (lower) will be employment (wage) growth and the higher is output growth, the higher will be both employment and wage change. In contrast to the classical model, however, this is a state of steady disequilibrium rather than static equilibrium.

Once the rates of growth in employment and wages are determined by supply and demand, the vacancy and unemployment rates are determined so as to maintain this growth. From (1.12) and (1.15) we see that they are simultaneously determined by employment and wage dynamics:

(3.5) $$e\{U/E,V/E\} = d\ln E/dt$$

and

(3.6) $$w\{V/E,U/E\} = d\ln W/dt,$$

where $d\ln E/dt$ and $d\ln W/dt$ are given by (3.3) and (3.4). In contrast to Okun's Rule,[9] which relates the unemployment rate to the percentage gap between actual and potential output, (3.3)-(3.6) imply that U/E is constant as long as $d\ln Q/dt$ and $d\ln P/dt$ are constant. The rates of growth of actual and potential output need not be identical for the unemployment rate not to change.

As can be seen in Figure 2, the higher is employment growth the higher must be vacancy and unemployment rates to maintain this growth; the higher is the rate of change of wages, however, the higher will be the vacancy rate (V/E) relative to the unemployment rate. Thus, it follows unambiguously from (3.3)-(3.6) that an increase in output growth will increase the vacancy rate and that an increase in population growth will increase the unemployment rate. It is not immediately apparent, however, what will be the effect, in particular, of output on unemployment, since the increase in employment requires a greater unemployment rate, while the increase in wages is associated with a lower rate.

To explore further the effects of output and population growth on steady state unemployment and vacancy rates, we must make some assumptions regarding the particular forms of employment and wage dynamics. In particular it is assumed that $e\{U/E, V/E\}$ is of the form

$$(3.7) \qquad d\ln E/dt = 1/2\beta \ln\{(U/E)(V/E)\} + \varepsilon,$$

where $\beta > 0$ and $\varepsilon > 0$ measure the efficiency of search and the turnover rate. Correspondingly, it is assumed that $w\{(V/E)/(U/E)\}$ is of the form

$$(3.8) \qquad d\ln W/dt = 1/2\alpha \ln\{(V/E)/(U/E)\} + \gamma,$$

where $\alpha > 0$ and $\gamma > 0$ measure the relative willingness of firms and households to alter wage offers and reservation wages in response to a given level of tightness in the labor market.

It is now possible by equating (3.3) with (3.7) and (3.4) with (3.8) to obtain the steady-state vacancy and unemployment rates as a function of output and population growth:

$$(3.9) \qquad \ln\{V/E\} = [((\delta_Q/\alpha) + (\sigma_W/\beta))d\ln Q/dt$$

$$- ((\sigma_P/\alpha) - (-\delta_W/\beta))d\ln P/dt]/(\sigma_W-\delta_W)$$

$$+ [(-\varepsilon/\beta) - (\gamma/\alpha)]$$

and

$$(3.10) \qquad \ln\{U/E\} = [1-((\delta_Q/\alpha) - (\sigma_W/\beta))d\ln Q/dt$$

$$+ ((\sigma_P/\alpha) + (-\delta_W/\beta))d\ln P/dt]/(\sigma_W-\delta_W)$$

$$+ [(-\varepsilon/\beta) + (\gamma/\alpha)].$$

From the first term in (3.10) we can see that a change in output growth will not affect the unemployment rate if and only if $\delta_W/\alpha = \sigma_W/\beta$. Only if the increase in demand brought about by the increase in output (δ_Q) relative to the willingness of firms to increase wage offers and households to reduce reservation wages (α) is equal to the increase in supply brought about by the output-induced wage increase (σ_W) relative to the efficiency of job search (β), will there be a steady-state natural unemployment rate. Otherwise, an output increase will affect the steady-state U/E. In fact, if $\sigma_W/\beta > \delta_Q/\alpha$, an increase in the rate of growth of output will actually increase the unemployment rate, since more people are drawn into the labor force by the output increase than are employed by it. What will happen is an empirical matter that can differ from one labor market to another.

Since the rate of output growth affects the rate of wage change and the unemployment rate if $\delta_Q/\alpha \neq \sigma_W/\beta$, there is an underlying basis for a relation between dlnW/dt and U/E. This relation is derived by solving for

for dlnQ/dt in (3.10) and substituting the result into (3.4) to obtain:

$$(3.11)\qquad d\ln W/dt = [1/((\delta_Q/\alpha) - (\sigma_W/\beta))][-\ln\{U/E\}$$
$$+ ((1/\beta)(\sigma_W\sigma_P - \delta_W\delta_Q)/(\sigma_W - \delta_W))d\ln P/dt$$
$$+ ((-\varepsilon/\beta) + (\gamma/\alpha)(\sigma_W - \delta_W)].$$

Therefore, if $\delta_Q/\alpha > \sigma_W/\beta$, there is a trade off between wage change and the unemployment rate in the steady state. As is evident from the second term in (3.11), however, the higher is population growth (dlnP/dt) the worse will be the trade off. Moreover, if $\delta_Q/\alpha = \sigma_W/\beta$, there will be no trade off, only a natural unemployment rate given by (3.10). In fact if $\delta_Q/\alpha < \sigma_W/\beta$, then the labor market will exhibit both higher rates of wage increase and higher rates of unemployment as the rate of growth of output increases.

Now let us turn briefly to steady-state labor force participation behavior. Since employment and unemployment are both growing at the same rate labor force is also growing at this rate. Then by definition the proportionate rate of change in L/P is given by

$$(3.12)\qquad d\ln\{L/P\}/dt = d\ln E/dt - d\ln P/dt.$$

Therefore, changes in dlnE/dt caused by changes in output growth will be directly associated with changes in dln{L/P}/dt. The participation rate (L/P) can, however, be going down when employment is increasing. As can be seen from (3.12), whether L/P is increasing or not depends on whether output growth exceeds population growth or not. Finally, while there is no relation between labor force participation and the unemployment rate in static equilibrium, there is one between dln{L/P}/dt and U/E if $\delta_Q/\alpha \neq \sigma_W/\beta$. In particular, if there is a Phillips relation in the market, there is also an inverse relation between the unemployment rate and the rate of change in the labor force participation rate. Again, this relation does not depend on the discouraged worker hypothesis. Rather, it is the result of output increases yielding both higher employment growth and a lower unemployment rate.

IV. DISEQUILIBRIUM DYNAMICS

In this section, non-steady state labor market behavior is considered. In particular, the effects of alternative values of market parameters on the stability of the market is analyzed.

Thus far, we have concentrated attention on the behavior of the labor market in static equilibrium or in the steady state. For this analysis to be relevant, however, requires that the system be stable. Exogenous changes in output and population are always occurring so that the market is rarely exactly in a state of equilibrium or steady growth. But if the market is stable, so that vacancies, unemployment, employment, and wages tend to return to their equilibrium or steady state values, then the preceding analysis will be approximately correct even in a world of non-steady state behavior. Of course, the accuracy of the approximation will depend on the magnitude and frequency of the exogenous changes as well as the speed of response of the system, which is, again, an empirical matter.

To determine whether the labor market is stable requires that assumptions be made about the particular forms of the dynamic relations. Explicit assumptions have already been made regarding employment and wage dynamics in (3.7) and (3.8). Implicit assumptions of constant wage, output, and population elasticity demand and supply relations have also been made in (3.1) and

(3.2). Therefore, only vacancy and unemployment dynamics remain to be specified.

To maintain the log-linear character of the model, vacancy dynamics are assumed to be given by

$$(4.1) \qquad d\ln V/dt = \nu \ln\{D/E\},$$

where $\nu > 0$ measures the speed of vacancy response by firms to differences between their desired employment (D) and actual employment (E). Therefore, substituting the demand relation

$$(4.2) \qquad \ln D = \delta_0 + \delta_W \ln W + \delta_Q \ln Q,$$

into (4.1) we obtain

$$(4.3) \qquad d\ln V/dt = \nu\delta_0 + \nu\delta_W \ln W + \nu\delta_Q \ln Q - \nu \ln E.$$

Similarly, unemployment dynamics are described by

$$(4.4) \qquad d\ln U/dt = \mu \ln\{S/E\},$$

where $\mu > 0$ measures the speed of unemployment response by households to differences beween S and E. Again substituting the supply relation

$$(4.5) \qquad \ln S = \sigma_0 + \sigma_W \ln W + \sigma_P \ln P,$$

into (4.4) we get

$$(4.6) \qquad d\ln U/dt = \mu\sigma_0 + \mu\sigma_W \ln W + \mu\sigma_P \ln P - \mu \ln E.$$

Thus the model of the labor market is completely characterized by (3.7), (3.8), (4.3) and (4.6). The parameters of the model are δ_0, δ_W, δ_Q, σ_0, σ_W, σ_P, ν, μ,

β, ε, α, and γ; the time paths of Q and P are assumed to be given exogenously as are the initial values of vacancies, unemployment, employment and wages.

Since the model is linear in the logarithms of U, V, E, and W, the stability of the system can be readily analyzed by determining the conditions for which the eigenvalues of the system all have negative real roots. The eigenvalues are the roots to the fourth-order equation

$$(4.7) \qquad |A-\lambda I| = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda^4 + a_4 = 0,$$

where the system is described by

$$(4.8) \qquad dx/dt = Ax + Cz,$$

with $x = [\ln U, \ln V, \ln E, \ln W]$, $z = [1, \ln Q, \ln P]$,

$$(4.9) \qquad A = \begin{bmatrix} & & -\nu & \nu\delta_W \\ & & -\mu & \mu\sigma_W \\ 1/2\beta & 1/2\beta & -\beta & \\ 1/2\alpha & -1/2\alpha & & \end{bmatrix}$$

and

$$(4.10) \qquad C = \begin{bmatrix} \nu\delta_0 & \nu\delta_Q & \\ \mu\sigma_0 & & \mu\sigma_P \\ \varepsilon & & \\ \gamma & & \end{bmatrix} .$$

The Routh-Hurwitz criteria[9] for the stability of (4.8) are $a_1>0$, $a_4>0$, $a_1a_2-a_3>0$, and $a_3(a_1a_2-a_3)-a_1^2a_4>0$, where evaluating (4.7):

$$(4.11) \qquad a_1 = \beta,$$

$$a_1 = 1/2\beta(\nu+\mu) + 1/2\alpha(\mu\sigma_W+\nu(-\delta_W)), \tag{4.12}$$

$$a_3 = 1/2\alpha\beta(\mu\sigma_W+\nu(-\delta_W)), \tag{4.13}$$

and

$$a_4 = 1/2\alpha\beta\nu\mu(\sigma_W+(-\delta_W)). \tag{4.14}$$

The first three criteria are readily verified and the last one reduces to

$$\mu^2\sigma_W - \nu^2\delta_W - \nu\mu(\sigma_W-\delta_W) > 0. \tag{4.15}$$

Therefore, the stability depends only on the speeds of vacancy and unemployment response and the wage elasticities of demand and supply. By setting the left-hand side of (4.15) equal to zero, we can determine the ranges of stability for these parameters. The two solutions to this equation are $\nu=\mu$ and $\nu=(\sigma_W/(-\delta_W))\mu$.

The stability ranges are thus illustrated in Figures 3 and 4, corresponding to whether or not the supply elasticity is greater or less than the demand elasticity in absolute value. The ranges of instability are denoted by the hatched areas. From these figures we can see that if the system is not to generate explosive oscillations, the speeds of vacancy and unemployment response must be different. Either firms must be slow in their response to demand-employment differentials, and households fast, or vice-versa. In particular, if labor supply is wage inelastic ($\sigma_W=0$), then firms must adjust their level of search (V) more quickly than households adjust their level (U) to compensate for households' lack of response to wage changes in determining their desired level of employment (S). The less is the difference between the demand and supply elasticities, however, the closer can ν and μ be without the system

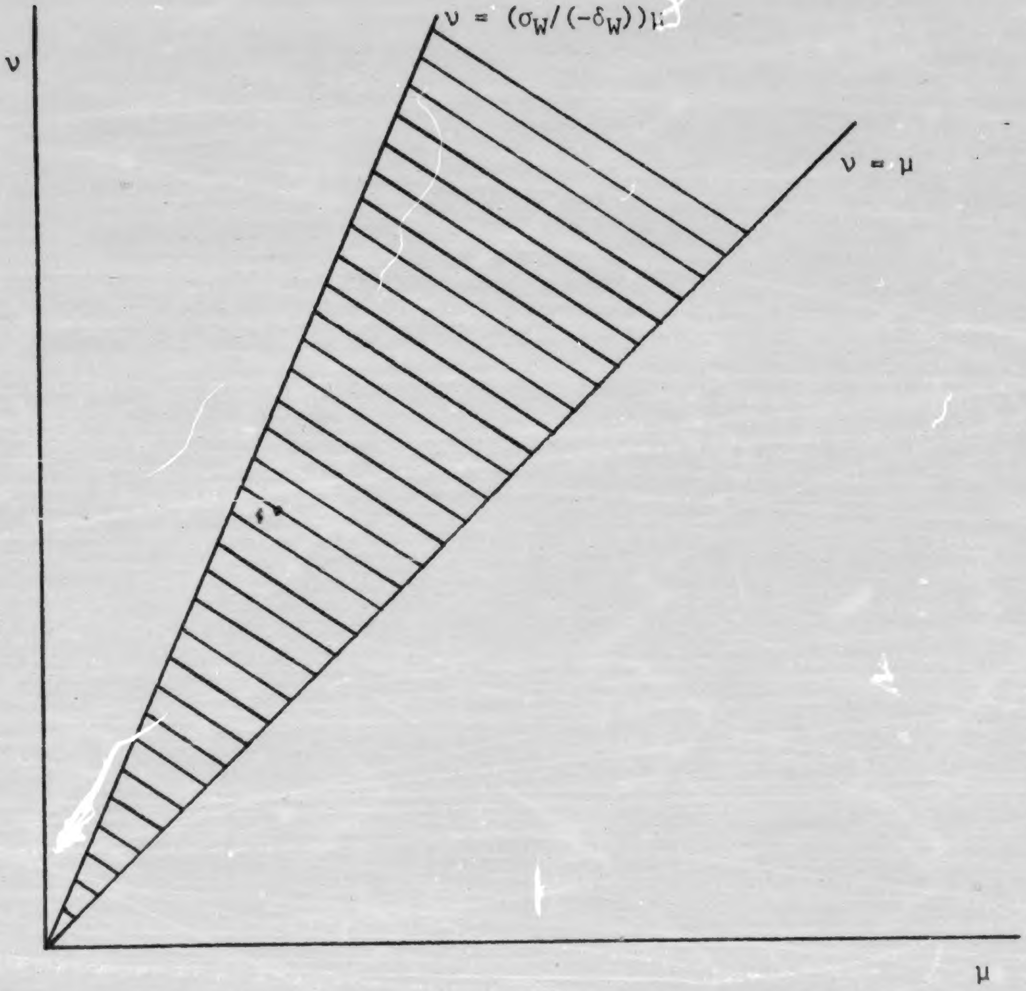
$\nu = (\sigma_W/(-\delta_W))\mu$
ν
$\nu = \mu$
μ

Figure 3

Stability Region if $\sigma_W > -\delta_W$

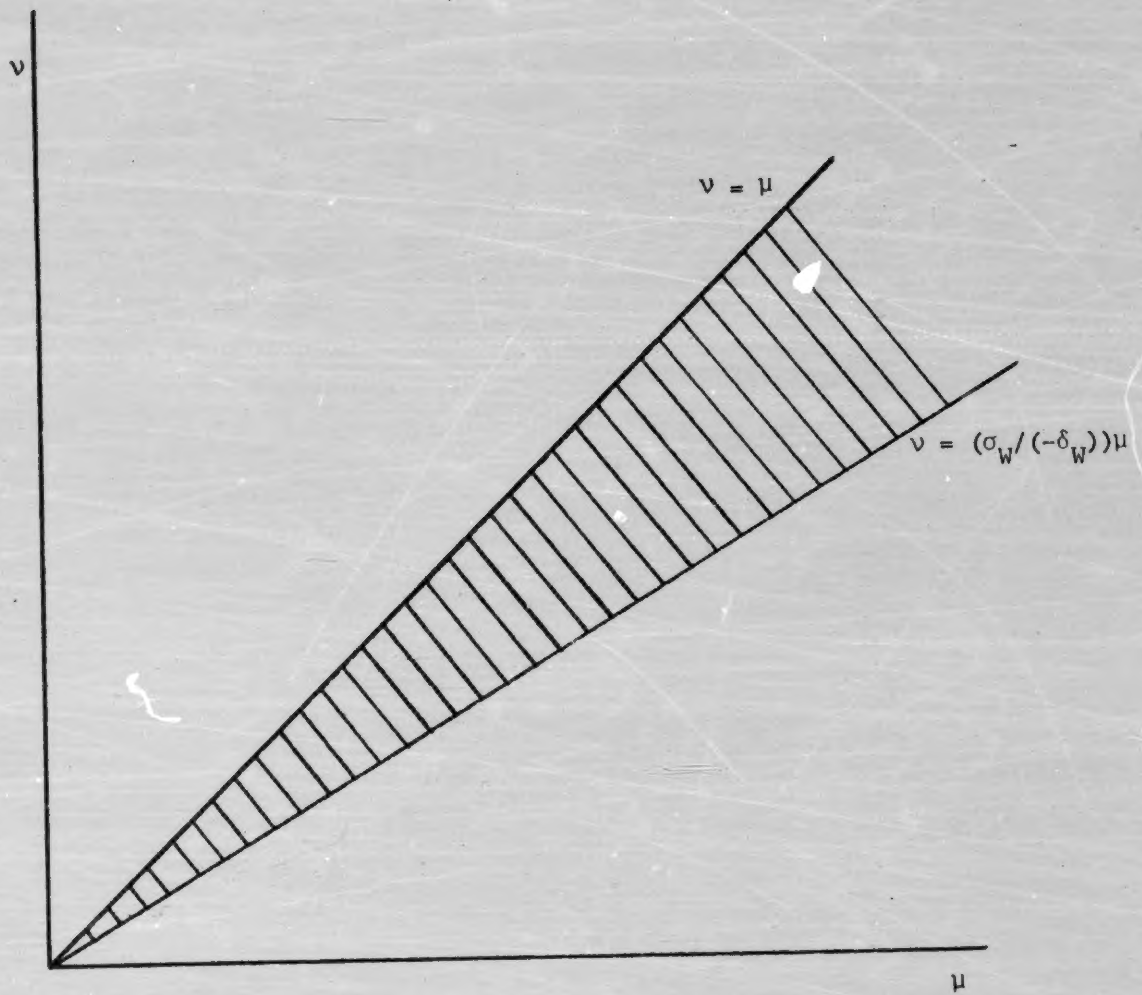
ν
$\nu = \mu$
$\nu = (\sigma_W/(-\delta_W))\mu$
μ

Figure 4

Stability Region if $\sigma_W < -\delta_W$

being unstable. In particular, if $\sigma_W = -\delta_W$, then the market is stable as long as $\nu \neq \mu$.

V. CONCLUSIONS

A dynamic disequilibrium model of the labor market has been presented and analyzed in this paper. This model integrates the classical theory of supply and demand with the modern theory of job search and labor turnover. It differs from traditional time-series labor market models in that firms are assumed to adjust vacancies rather than employment and households are assumed to adjust unemployment rather than labor force. It differs from cross-section models in that unemployment is not ignored or explained as the result solely of disequilibrium.

In equilibrium the model implies that there is a dichotomy between the determination of wages and employment and the determination of vacancy and unemployment rates. The levels of wages and employment on the one hand are determined by the equality of supply and demand so that the levels of employment desired by households and by firms are both equal to the actual level of employment. Therefore, cross-section models based on the assumption of labor market equilibrium can be viewed as not being inconsistent but only being incomplete. They need not imply that there is no unemployment, but they certainly do not explain it. Vacancy and unemployment rates on the other hand are determined by search-turnover behavior. Thus, there is a natural rate of unemployment, which is unaffected by changes in labor demand. This in turn implies that an increase in employment brought about by an increase in demand must be accompanied by an increase in the labor force participation rate. Therefore, the fact that labor demand and labor participation move together -

a fact which is commonly explained in terms of the discouraged worker phenomenon - is only a tautology given the existence of a natural rate of unemployment.

In a state of steady disequilibrium the growth in wages and in employment are still determined by the growth in supply and demand, but now vacancy and unemployment rates depend not only on search-turnover behavior, but also on labor demand and supply growth. The effect of output growth, in particular, on the unemployment rate depends on the balance between the increase in demand relative to the willingness of firms to increase wage offers and households to reduce reservation wages and the increase in supply brought about by output-induced wage increase relative to the efficiency of job search. If the former effect exceeds the latter, there is a trade off between wage change and the unemployment rate. Moreover, there is also an inverse relation between the unemployment rate and the rate of change in the labor force participation rate, which does not depend on the discouraged worker hypothesis. If the opposite is true, then both wage increases and the unemployment rate can be decreased by a reduction in the rate of output growth. Only if the two effects of a change in output growth balance out, will there be a natural rate of unemployment. Which effect dominates is an empirical matter, but we have seen that the relation between wages, output, and unemployment is stable as long as the rates at which firms and households ~~adjust~~ their rates of search are significantly different.

FOOTNOTES

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Opinions expressed are those of the author and do not necessarily represent the views of the Department of Labor, the Urban Institute, or its sponsors.

1. See Lipsey (1960, 1974).

2. While labor is treated as an aggregate in this analysis, demand can readily be disaggregated by industry and occupation.

3. Traditional time-series employment models assume that firms can directly control employment. For example, see Nadiri and Rosen (1973).

4. For an alternative job vacancy theory, in which firms make a desired level of vacancies, see Holt and David (1966).

5. Just as the demand for labor can readily be disaggregated, so can the supply of labor be disaggregated by demographic group and occupation. Moreover, the supply of labor can be made a function of not only the wage rate but other family income, with a distinction being made between primary and secondary labor supply.

6. Traditional time-series models of the labor market equate labor force with labor supply and then treat unemployment as a residual. For example, see Black and Kelejian (1970).

7. Price expectations are implicitly assumed to be justified but the analysis can readily be extended to include alternative forms of expectations behavior. See Phelps (1968) for a discussion of the role of expectations in wage dynamics.

8. Traditional time-series labor force models assume the discouraged worker hypothesis to explain the relation between labor force participation and labor demand. See, fo~~r ex~~ample Wachter (1974).

9. See Okun (1970), pp. 132-145.

10. See Bellman (1960), pp. 244-245.

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CATEGORICAL EARNINGS SUBSIDIES: MARKET EFFECTS AND PROGRAM COSTS

by Peter M. Greenston, C. Duncan MacRae and
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Abstract

The labor market effects and government budget costs of the Work Bonus Tax Credit proposed by the Senate Finance Committee are analyzed using an econometric model of state labor markets (pp. 1.1-1.75). The earnings subsidy is analyzed by translating the program into a shift in the market labor supply of demographically eligible families. The effects of this shift on both eligible and ineligible families are then simulated for 1976 on a state-by-state basis. The solutions indicate that the potential for an earnings subsidy to be market neutral, in the sense that on net the supply of labor by the subsidized workers is neither increased or decreased, would be approximately realized. The subsidy would neither dissipate the benefits of the subsidy through higher wages nor displace unsubsidized workers through lower wages. The implied costs of the program are in approximate accord with the projections based on static assumptions by the Senate Finance Committee.

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CATEGORICAL EARNINGS SUBSIDIES:
MARKET EFFECTS AND PROGRAM COSTS*

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A variety of labor subsidy programs have been proposed to deal with the problem of poverty. Foremost amongst them are negative income tax (NIT) plans. These combine an unearned income subsidy with an earnings tax so that a family's initial transfer is completely taxed back when its earnings exceed some maximum. While an NIT does indeed transfer income to those with low levels of income, by increasing income and taxing earnings it creates an incentive to decrease hours supplied to the labor market. This decrease in labor supply of the subsidized group tends to push the market wage up, thus dissipating some of the benefits of the program to nonparticipants.

Wage rate subsidy plans have been proposed as an alternative way of increasing incomes of the working poor. By rewarding work effort they provide an incentive to increase hours supplied. This increased supply of labor, however, causes the market wage to fall, resulting in the displacement of unsubsidized by subsidized workers. Thus, a wage subsidy tends to displace the unsubsidized workers, while an NIT tends to dissipate program benefits in terms of increased wage rates to the unsubsidized.

Recently a number of earnings subsidies, which combine features of both an NIT and a wage rate subsidy, have been proposed.[1] One of these is the Work Bonus Tax Credit proposed by the Senate Finance Committee. At low levels of earnings and income, an earnings subsidy is designed to operate like a wage subsidy, causing an increase in hours supplied. At higher levels of earnings and income it is designed to tax back the subsidy and thus resembles an NIT by causing a decrease in hours supplied. Since these changes in market supply act in opposite directions, there is the potential for an earnings subsidy to be market neutral, in the sense that on net the supply of labor by the subsidized workers is neither increased or decreased. This subsidy would then neither dissipate the benefits of the subsidy nor displace unsubsidized workers. To determine if this potential is realized requires an analysis of labor supply of those who are demographically eligible. However, if this potential is not fully realized, measuring the dissipation or displacement requires a market analysis that incorporates both supply and demand. An examination of the labor supply effects by themselves are not sufficient.

The purpose of this paper is to analyze the market effects and program costs of the Work Bonus Tax Credit using an econometric model of state labor markets. The earnings subsidy is analyzed by translating the program into a shift in the market labor supply of families with dependent children, hereafter referred to as demographically eligible families. The effects of this shift in labor supply on both demographically eligible and demographically ineligible families is then analyzed on a state-by-state basis by solving the econometric model for equilibrium wages, hours and earnings in each state, first in the absence and then in the presence of the subsidy.

We begin with a description of the earnings subsidy. By treating the program as a wage subsidy or tax combined with an unearned income subsidy we translate it into shifts in the labor supply of demographically eligible families. An econometric model of the state labor market, which has been designed to simulate the effects of a variety of human resource programs, is then briefly described. Using data from the 1970 Census Public Use Sample, we project the market effects and calculate the costs of the proposed program for 1976. Finally, conclusions regarding likely dissipation of benefits or displacement of unsubsidized workers are presented.

I. WORK BONUS TAX CREDIT

The subsidy proposed by the Senate Finance Committee is an earnings supplement for families with one or more dependent children.[2] It is conceived as a reform to the payroll tax. Those families with earnings which are less than or equal to $4000 receive a 10 percent subsidy on family earnings, while those participating families whose income has exceeded $4000 pay a 25 percent tax on the difference between their family income and $4000.

The way in which the subsidy depends on both earned and unearned income can be seen by first noting that by definition, gross family income, G, is the sum of family earnings, E, and unearned income, Y^n. When G is less than or equal to $4000, the subsidy is calculated according to

$$S = .10E, \tag{1}$$

where S is the amount of the subsidy. In this first range, the family receives the 10 percent subsidy and pays no tax. When G is greater than $4000, and E is less than or equal to $4000, the subsidy the family receives is calculated by

$$S = .10E - .25(G - 4000). \tag{2}$$

In this second range, the family receives a 10 percent subsidy on earnings, but also pays the 25 percent tax on income above $4000. Finally, in the third range when E is greater than $4000 and hence G is greater than $4000, the subsidy the family receives is determined by

$$S = 400 - .25(G - 4000). \tag{3}$$

The family no longer receives a 10 percent subsidy, although their earnings have generated a gross subsidy of $400. Since the family is subject to a 25 percent tax on income above $4000, the net subsidy received equals $400 less the tax paid.

Since participation in the program depends on both earnings and gross income, the level of unearned income is important. Families will participate only if they receive a non-negative subsidy. In each range, therefore, participation implies certain restrictions on the feasible combinations of E and Y^n. In the first range, corresponding to (1), the family must have some earnings in order to participate. Since gross income must be less than \$4000 in the first range, Y^n must be less than or equal to (4000-E). In the second range, corresponding to (2), we know that E is less than \$4000 and G is greater than \$4000. Moreover, a non-negative subsidy requires that Y^n must be less than or equal to (4000 - 0.6E) so that the maximum value for Y^n can vary between \$1600 and \$4000, depending on the level of earnings. If Y^n were greater than \$4000, the family would never receive a non-negative subsidy since they would be subject to the 25 percent tax before they received the 10 percent subsidy on their first dollar of earnings, and thus would not participate. In the third range, corresponding to (3), we know that both E and G are greater than \$4000. The non-negative subsidy requirement implies that Y^n must be less than or equal to (5600-E) so that the maximum value for Y^n can vary between zero and \$1600, depending on the level of earnings. As long as Y^n is less than or equal to \$1600, the family is ineligible for the program when income exceeds \$5600. If Y^n is greater than \$1600, the family is ineligible before gross income reaches \$5600. For these families, (3) is never used to calculate the subsidy because they are still receiving a 10 percent subsidy on their last dollar earned when they exit the program, and thus never face the 25 percent tax and also never achieve the maximum subsidy of \$400. Accordingly, the exact program exit level for these families is $(1000 - .10Y^n)/.15$, which can be derived from (2) by solving for G when the subsidy equals zero.[3] Therefore, when we analyze the labor supply effects of the program, we must distinguish between families with unearned income less than \$1600, and those with unearned income greater than \$1600, but less than \$4000.

The manner in which the subsidy (S) and hence net family income (Y) vary with earnings and unearned income is shown in Figure 1. Gross family income (G) is plotted on the horizontal axis and net family income which is the sum of G and S is plotted on the vertical axis. The line Y=G indicates the locus of zero subsidy points. The schedules illustrated correspond to different levels of unearned income (Y^n). In the first range G does not exceed \$4000 and the net income schedules are given by the equation

$$Y = 1.10G - .10Y^n. \tag{4}$$

As unearned income increases, the schedule shifts down in a parallel fashion. In the second range the family is receiving a 10 percent earnings subsidy while paying a tax on gross income exceeding \$4000. The net effect is to flatten out the net income schedules:

$$Y = .85G - .10Y^n + 1000 \tag{5}$$

As can be seen, families with Y^n exceeding \$1600 leave the program before gross income reaches \$5600, while those with Y^n less than or equal to \$1600 and earnings exceeding \$4000 enter the third range portrayed by the even flatter single schedule:

$$Y = .75G + 1400. \tag{6}$$

The crossover occurs at that level of G which equates (5) and (6) for a given level of Y^n. When Y^n is exactly equal to \$1600, the crossover and program exit point are one and the same.

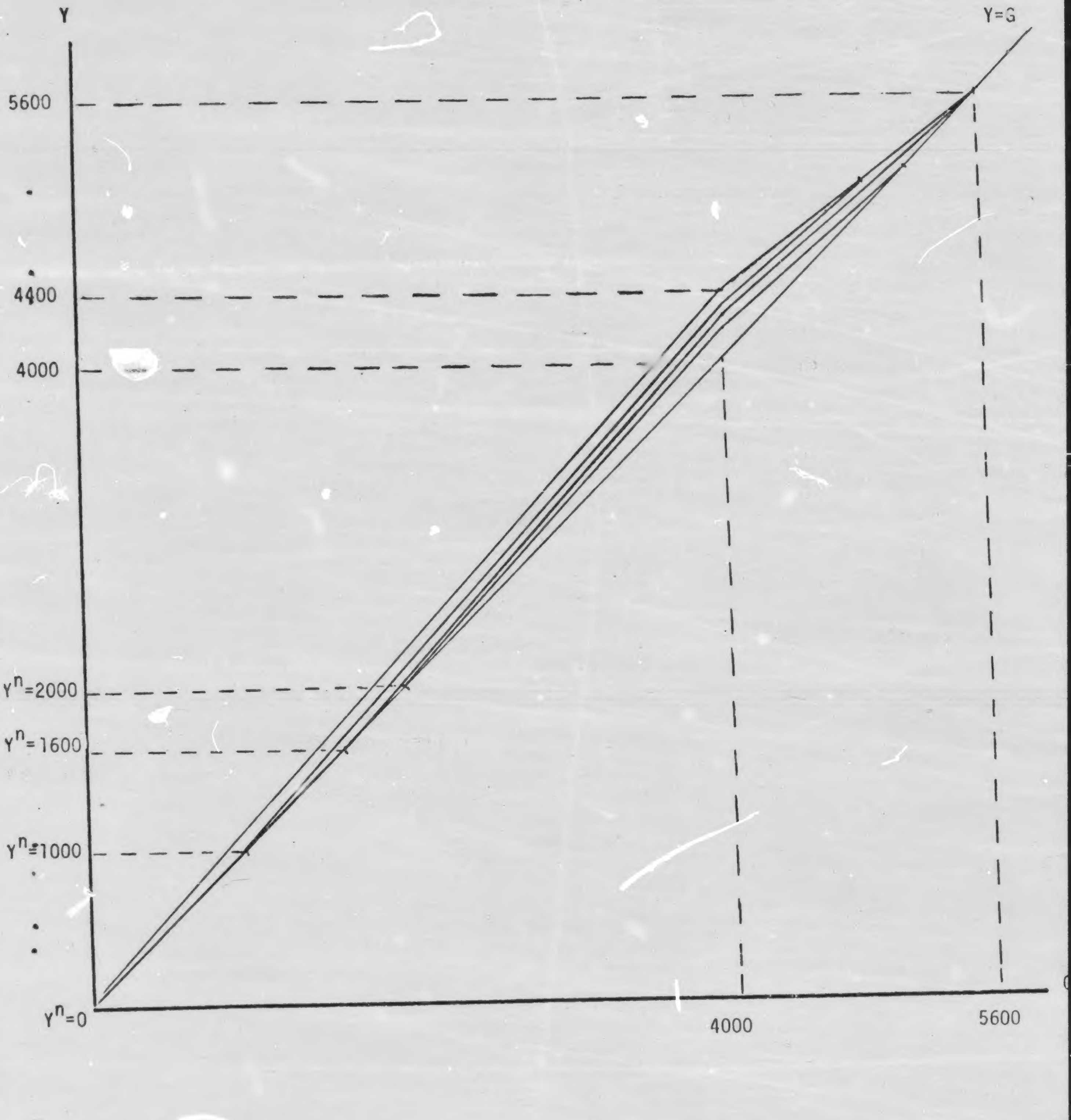

Figure 1

II. LABOR SUPPLY EFFECTS

Our method for simulating the Work Bonus Tax Credit is to translate the program into a shift in the market labor supply of eligible families. In the absence of the tax credit program, families determine their labor supply by maximizing a family utility function of the form

(7) $$U = u\{Y, T^p - H^p, T^s - H^s\},$$

where Y is equal to family income, T^p (T^s) is the number of available hours the primary (secondary) worker has for work and leisure, and H^p (H^s) is the number of hours worked by the primary (secondary) worker. Families maximize (7) subject to a budget constraint,

(8) $$Y = W^pH^p + W^sH^s + Y^n,$$

where W^p (W^s) is the primary (secondary) wage rate. First order conditions for utility maximization then yield the labor supply functions of primary and secondary workers:

(9a) $$H^p = h^p\{W^p, W^s, Y^n\};$$

(9b) $$H^s = h^s\{W^p, W^s, Y^n\}.$$

These are also the appropriate supply functions for those families who do not participate, either because they have no dependent children or because their earned-unearned income combination makes them ineligible.

Families participating in the tax credit receive a net subsidy which changes their budget constraint (8). When they are in the first range (1) so that $E \leq G < 4000$, their net income is given by

(10) $$Y = W^pH^p + W^sH^s + Y^n + .1(W^pH^p + W^sH^s),$$

which can be rewritten as

(11) $$Y = (1.1W^p)H^p + (1.1W^s)H^s + Y^n.$$

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This corresponds to the family receiving a 10 percent wage rate subsidy, and incurring no tax. The labor supply functions now become:

$$H^p = h^p\{1.1W^p,\ 1.1W^s,\ Y^n\}; \tag{12a}$$

$$H^s = h^s\{1.1W^p,\ 1.1W^s,\ Y^n\}. \tag{12b}$$

In the second range (2), $E \leq \$4000 < G$, and family income is given by

$$Y = W^pH^p + W^sH^s + Y^n + .10(W^pH^p + W^sH^s) \tag{13}$$
$$- .25(W^pH^p + W^sH^s + Y^n - 4000),$$

which can be rewritten as

$$Y = (.85W^p)H^p + (.85W^s)H^s + Y^n \tag{14}$$
$$+ .15(4000-Y^n) - (-.10(4000-Y^n)).$$

Here we observe that the family is simultaneously receiving the 10 percent earnings subsidy and incurring the 25 percent tax on income exceeding \$4000. On net, therefore, the family effectively incurs a 15 percent wage rate tax, while receiving an income transfer equal to the amount of tax saved on earnings by not paying at the current marginal rate up to that point $(.15(4000-Y^n))$, minus the tax actually paid up to that point $(-.10(4000-Y^n))$.[4] The family thus supplies labor as if it were subject to a 15 percent wage rate tax while receiving an income transfer of the amount $(.25(4000-Y^n))$.[5] In this range, therefore, the labor supply functions are:

$$H^p = h^p\{.85W^p,\ .85W^s,\ Y^n + .25(4000-Y^n)\}; \tag{15a}$$

$$H^s = h^s\{.85W^p,\ .85W^s,\ Y^n + .25(4000-Y^n)\}. \tag{15b}$$

In the third range (3), $E > 4000$ so that the family's net income is given by:

$$Y = W^pH^p + W^sH^s + Y^n + 400 \tag{16}$$
$$- .25(W^pH^p + W^sH^s + Y^n - 4000),$$

which can be rewritten as

$$Y = (.75W^p)H^p + (.75W^s)H^s + Y^n \tag{17}$$
$$+ .25(4000) - (.15Y^n + (-.10(4000-Y^n))).$$

The family has received a gross subsidy of \$400 and is paying tax at a 25 per cent rate on every dollar of income over \$4000. In effect, as indicated by (17), the family is incurring a 25 percent wage rate tax and receiving an income transfer of $.25(5600-Y^n)$, composed of the amount of tax saved by not paying at the current marginal rate of 25 percent, .25(4000), minus the amount of tax actually paid, $.15(4000 - (4000-Y^n)) + (-.10(4000-Y^n))$. Therefore, the labor supply functions are

$$H^p = h^p\{.75W^p,\ .75W^s,\ Y^n + .25(5600-Y^n)\}; \tag{18a}$$

$$H^s = h^s\{.75W^p,\ .75W^s,\ Y^n + .25(5600-Y^n)\}. \tag{18b}$$

The family supplies hours of work as if it were subject to a 25 percent wage tax and an income transfer of $(.25(5600-Y^n))$.

When family income is exactly \$4000, i.e. at the boundary between the first and second ranges, there is a discontinuous change in the marginal subsidy rate from 0.10 percent to -0.15 percent so that primary and secondary labor supply cannot be separately determined from knowledge of the wage rates alone. At this point, however, we know that family hours must satisfy

$$W^pH^p + W^sH^s = 4000 - Y^n, \tag{19}$$

so that family hours expressed in primary hour equivalents is in fact determinant as a function of the primary wage:

$$H^p + (W^s/W^p)H^s = (4000 - Y^n)/W^p. \tag{20}$$

Another discontinuity occurs when family earnings are exactly \$4000; that is, at the boundary between the second and third ranges, the marginal subsidy rate changes from -0.15 to -0.25 percent. Once again, we can solve for family hours expressed in primary hour equivalents, but cannot determine primary and secondary worker hours separately.

Finally, at the program exit point, the marginal subsidy rate jumps discontinuously from -0.25 to zero percent. Correspondingly, there will be a level of earnings where the family will "jump" discontinuously from participating in the program at "low" earnings to not participating at "high" earnings. We can make a reasonable assumption and approximate the jumping point, but we cannot do better without knowledge of the utility function.

For diagrammatic convenience, we illustrate the discussion with a single-worker family, such as a female-headed household with dependent children. In Figure 2, the pre-program budget constraint is ABC, where AB represents the level of unearned income, and the slope of BC is the negative of the wage rate. We also assume that this worker's supply elasticity with respect to her wage rate is positive and with respect to other family income is negative.

Suppose that Y^n is less than $1600. With the program in operation, the individual faces a new budget constraint ABDEFC determined by the three ranges spelled out in (1), (2), and (3). On BD the worker is receiving a 10 percent earnings subsidy, the absolute value of the slope of BD being 1.1 times the absolute value of the slope of BC. Thus, she supplies labor as if she were receiving a 10 percent wage subsidy. If the pre-program supply curve is SS' in Figure 3, receipt of the wage subsidy moves the individual up along her supply curve so that the supply curve seen by the market is shifted down as depicted by $\underline{AD}$.

The first program pivot point occurs at D in Figure 2. At this point, there are a variety of hours and wage rate combinations which produce income of exactly $4000, or earnings of exactly $(4000 - Y^n)$. These combinations are depicted by $\underline{\overline{DD}}$ in Figure 3, a constant earnings curve of $(4000 - Y^n)$ so that labor supply "jogs" back as the wage increases.

In Figure 2, segment DE portrays the range over which the worker simultaneously receives a 10 percent wage rate subsidy and incurs a 25

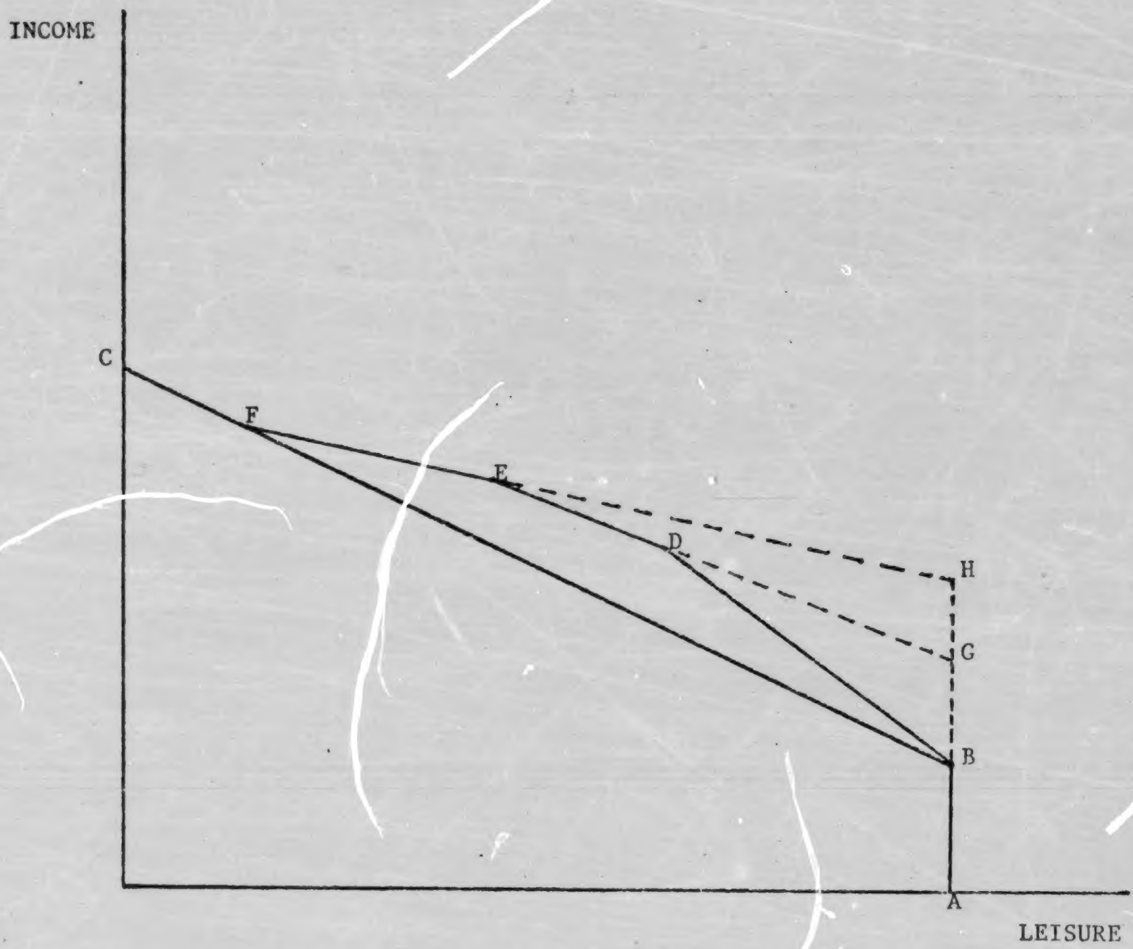
INCOME
C
F
E
D
H
G
B
A
LEISURE

FIGURE 2

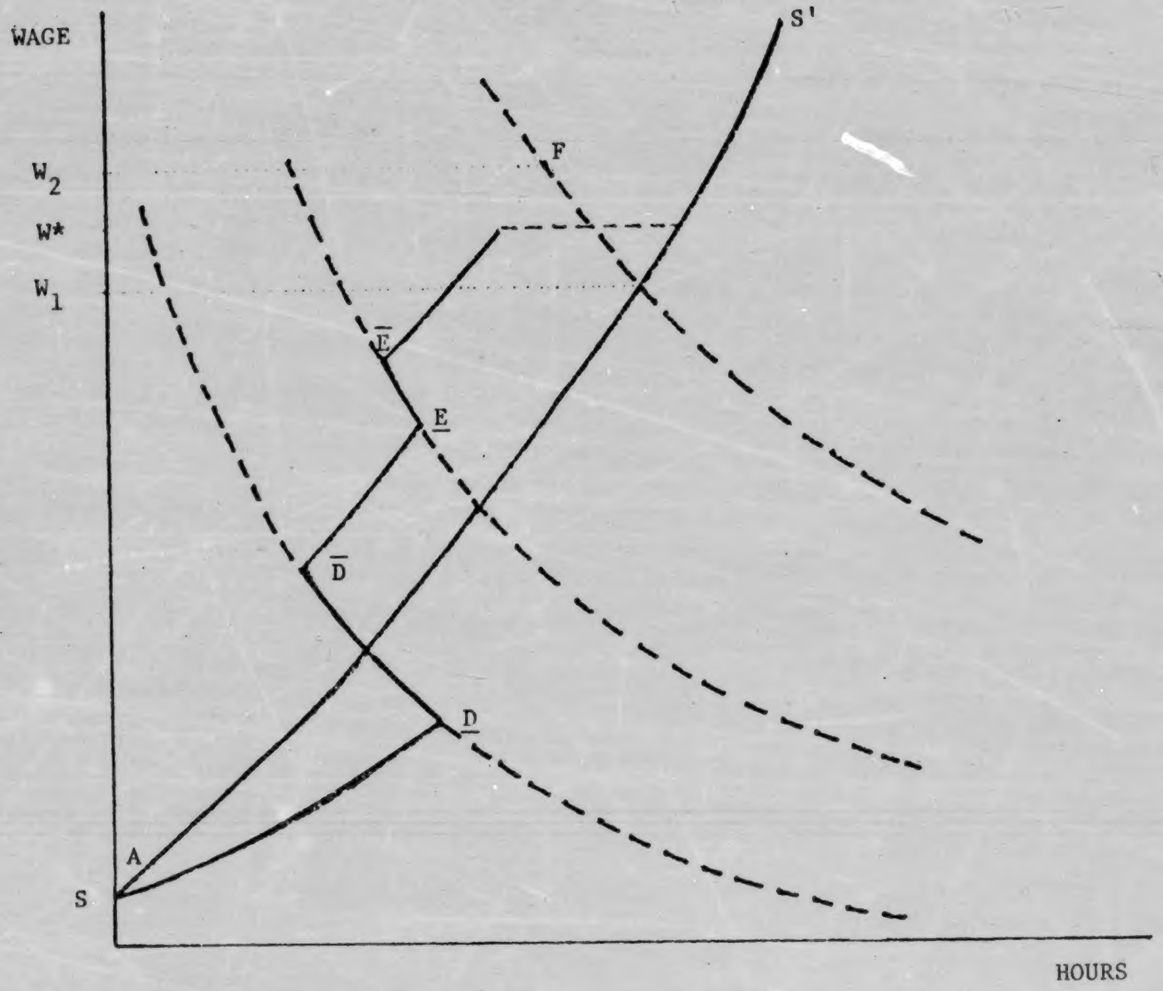
WAGE
W_2
W^*
W_1
S'
F
$\overline{E}$
E
$\overline{D}$
D
A
S
HOURS

FIGURE 3

percent tax on income exceeding \$4000. As we have shown, this is equivalent to a 15 percent wage rate tax, and an increase in unearned income of $(.25(4000-Y^n))$, which is given by BG. The corresponding market supply curve is shown as $\bar{D}\underline{E}$ in Figure 3, where the supply curve is shifted up and to the left of SS'. It shifts up because of a lower wage rate and shifts left because of an increase in unearned income.

The second jog occurs at E in Figure 2 where the marginal subsidy rate changes from -0.15 to -0.25 percent. Once again, there are various combinations of wage rates and hours producing earnings of exactly \$4000. These combinations are depicted by the constant earnings curve $\underline{E}\bar{E}$ in Figure 3.

On segment EF in Figure 2, the individual no longer receives the earnings subsidy, since her earnings exceed \$4000. In this range the maximum subsidy of \$400 is being taxed back at a 25 percent rate on income. As we have shown the individual behaves as if a 25 percent wage rate tax was in effect in conjunction with an income transfer of $.25(5600-Y^n)$ which is depicted by BH. The corresponding supply curve segment is $\bar{E}F$ in Figure 3.

In the neighborhood of F in Figure 2, the discontinuous jump occurs. At this point, the marginal tax rate jumps from +0.25 to zero percent and produces a non-convexity in the budget line which makes multiple optima of family utility possible. Thus it is difficult to determine the appropriate supply curve at a particular wage. This can be seen in Figure 3, where wage rates between W_1 and W_2 are compatible with two supply curves, one on which individuals participate and one on which they do not. We resolve the ambiguity by introducing a ratio test. We first calculate hours supplied by the individual, and thus her earnings, E_1, assuming she participates in the program. We do the same assuming she does not participate, denoting these earnings by E_2. We then compare to see whether

$$(21) \qquad E_1 \;/\; (5600-Y^n) \gtrless (5600-Y^n) \;/\; E_2.$$

If the left-hand side of (21) is greater than the right, we assume she reveals a preference not to participate in the program. This is because the wage rate she receives is closer to W_2 than W_1, and therefore it is more probable that she has already decided to jump out of the program. If the left-hand side is not greater, analagous reasoning holds for the individual to participate. Therefore, we have decided that the jump point, W^*, occurs at one-half the difference between W_1 and W_2.

III. STATE LABOR MARKET MODEL

In this section, we describe the salient features of the econometric labor market model with which we simulate the earnings subsidy.[6] The model describes the supply and demand for labor in a state, and has been estimated with data tabulated from the 1970 Census 1-in-1000 Public Use Sample across 30 states and aggregates of states using an instrumental variables method.

Families in each state labor market are partitioned into eight mutually exclusive and completely exhaustive types based on the demographic and education characteristics of the head. Taking the variance in hourly wage rates as a measure of the variance in utilized human capital, a one way, sequential analysis of variance algorithm was used to partition all primary workers in California according to their demographic and education characteristics so that the resulting groups display minimum intra-group and maximum inter-group wage rate variation.[7] The resulting structure of eight homogeneous groups was applied to each state to define the human capital groups in which family heads have these characteristics:

(i) Male and female, age 36 and over, college graduates or better (M/F-O-C)

(ii) Male and female, age 16-35, college graduates or better (M/F-Y-C)

(iii) Male, age 36 and over, schooling less than 12 years (M-O-NH)

(iv) Male, age 36 and over, high school graduate (M-O-H)

(v) Male, age 36 and over, schooling 13-15 years (M-O-SC)

(vi) Male, age 16-35, schooling 0-15 years (M-Y-NC)

(vii) Female, age 36 and over, schooling 0-15 years (F-O-NC)

(viii) Female, age 16-35, schooling 0-15 years, (F-Y-NC)

Within each family a distinction is made between family heads (primary workers)and other members (secondary workers) so that in each state there are eight groups of primary workers and eight groups of secondary workers (who have been grouped according to the characteristics of their head).

Representative primary and secondary workers are defined as the group average and used as observation units in each state labor market. The labor supply equations, which were derived from the traditional income-leisure model, explain the number of hours per family offered by the average primary and secondary workers in each group. In our notation, i indexes the group and j indexes the state, so that, for example, H^p_{ij} is the sum of primary worker hours offered by all males (or females) in group i and state j. If the number of families is given by F_{ij}, then the average number of hours per family is denoted by H^p_{ij}/F_{ij}. The model has eight primary and eight secondary worker supply equations. However, the coefficients in these equations are not all different. College graduate primary workers have the same set of coefficients, as do their secondary counterparts. Secondary workers from male-headed family groups (iii-vi) have the same set, as do those from the female-headed family groups (vii-viii). The estimated equations are reported in Table P and Table S with standard errors, where asterisks denote the significance of t tests at the 1 percent (***), 5 percent (**), and 10 percent (*) levels.

In the estimation, potential rather than actual wage rates were used, denoted by $\hat{W}^p_{ij}$ and $\hat{W}^s_{ij}$ and discussed below, while family unearned income is denoted by Y^n_{ij}/F_{ij}. Socio-demographic and family composition variables were used to adjust for any preference differences in tastes for market work, home work, and leisure. In the primary supply equation RACEP, ACEP,

TABLE P

PRIMARY SUPPLY EQUATIONS
Estimated Coefficients and Standard Errors

	GROUP	C	W^P	$(W^P)^2$	Y^n/F	RACEP	AGEP	SEXP	EDUP	URBAN	SPOUSE	DPENDP	R^2	S.E.E.	No. Obs.
1-2.	M/F-O-C	.487	.094	-.006	-.008	.008	.236	-.103	---	.199	.517	.573	.53	.112	60
	& M/F-Y-C	(.88)	(.23)	(.02)	(.03)	(.43)	(.26)	(.43)		(.26)	(.49)	(.42)			
3.	M-O-NH	- .613	.395	-.054	-.061	.094	2.78***	---	---	.350*	- .135	- .712	.82	.082	30
		(1.33)	(.59)	(.07)	(.10)	(.26)	(.83)			(.19)	(.91)	(.97)			
4.	M-O-H	-2.13	.783	-.091*	-.158***	-.010	.689*	---	---	.428	1.81***	.132	.76	.057	30
		(1.57)	(.51)	(.05)	(.04)	(.15)	(.38)			(.26)	(.52)	(.33)			
5.	M-O-SC	-2.21	.695	-.058	-.095**	.637	.779	---	---	-.202	1.23	.133	.57	.133	30
		(4.89)	(1.30)	(.11)	(.05)	(.41)	(.73)			(.33)	(1.69)	(.57)			
6.	M-Y-NC	- .092	.457	-.062	.232	-.214	-.011	---	-.152	.320*	.338	1.22*	.54	.073	30
		(1.39)	(.55)	(.07)	(.25)	(.29)	(.95)		(.37)	(.18)	(.63)	(.70)			
7.	F-O-NC	- .366	.403	-.058	.201	-.235	1.03**	---	-.165	.002	---	.111	.63	.085	30
		(1.24)	(.88)	(.15)	(.13)	(.28)	(.52)		(.36)	(.23)		(.85)			
8.	F-Y-NC	-1.37	1.25	-.268	.075	-.163	.797	---	.602	.149	---	- .112	.28	.153	30
		(3.35)	(2.67)	(.47)	(.30)	(.32)	(.58)		(.53)	(.53)		(.46)			

Note: Asterisks denote significance at the 1% (***), 5% (**), and 10% (*) levels for t tests on all parameters.

TABLE S

SECONDARY SUPPLY EQUATIONS
Estimated Coefficients and Standard Errors

GROUP	C	W^s	$(W^s)^2$	$(W^P H^P + Y^n)/F$	RACES	AGES	SEXS	EDUS	URBAN	SPOUSE	DPENDS	R^2	S.E.E.	No. Obs
1-2. M/F-O-C, & M/F-Y-C	.127 (.85)	- .012 (.51)	-.008 (.06)	-.031 (.02)	.094 (.23)	.227 (.45)	.272 (.39)	-.474* (.28)	.440** (.20)	.814*** (.29)	- .450 (.29)	.47	.076	60
3-6. M-O-NH, & M-O-H, & M-O-SC, & M-Y-NC	-2.46** (1.23)	1.64** (.79)	-.297** (.13)	.032* (.019)	- .312** (.13)	.389 (.33)	1.62*** (.36)	-.250 (.23)	.077 (.15)	.698* (.42)	- .063 (.22)	.57	.120	120
7-8. F-O-NC, & F-Y-NC	-2.07** (1.01)	1.29* (.72)	-.197 (.13)	-.029 (.08)	.012 (.09)	.309** (.15)	.098 (.09)	.307*** (.11)	-.093 (.21)	----	1.42** (.56)	.58	.084	57

SEXP, EDUP, SPOUSE, DPENDP, and URBAN denote, respectively, the proportion of primary workers in that group who are white, prime-age (22-54), male, high school graduates, domiciling with a spouse, responsible for dependents, and living in an urban area. In the secondary supply equations these variables are proportions of the total number of secondary workers of a given group.

Backward-bending supply curves were obtained for both primary and secondary workers at rates ranging from \$2.33 to \$7.83 per hour for the former and from \$2.76 to \$3.27 per hour for the latter. The income term coefficients are negative in most of the equations, but significantly different from zero in only two of them.

The market demand for labor is met by labor supplies of various qualities. Consequently, the sum of supplies forthcoming is expressed in terms of the hours of the numeraire group (M-O-H). If we let H^d_j denote this market sum of equivalent-quality hours, then

$$H^d_j = \sum_i (K^p_i/K^p_4)H^p_{ij} + \sum_i (K^p_i/K^p_4)(K^s_i/K^p_i)H^s_{ij}, \tag{22}$$

so that the market sum of supplies is comprised of hours offered by each group weighted by a measure of their relative (to the numeraire group) quality or productivity--denoted by K^p_i/K^p_4 and K^s_i/K^p_i--and then summed. Primary hours of the i-th group are converted into numeraire hours by K^p_i/K^p_4, while secondary hours are first converted into primary hours of the same family type by K^s_i/K^p_i and then into numeraire hours. Since labor quality reflects the underlying stock of utilized human capital, relative quality is then measured by relative human capital stocks.

According to human capital theory the wage rate of an individual or group of similar individuals is the product of the market rate of return to human capital and the stock of utilized human capital possessed by that individual or group. An important implication is that, in equilibrium, relative wage rates are equal to relative stocks of utilized human capital and are independent of the particular labor market:

$$K^p_i/K^p_4 = \widehat{(W^p_{ij}/W^p_{4j})} \tag{23}$$

and

$$K^s_i/K^p_i = \widehat{(W^s_{ij}/W^p_{ij})}. \tag{24}$$

The model assumes that there is a high correlation between demographic-education characteristics and utilized human capital so that relative human capital stocks can be estimated by regressing relative wages against variables representing the age-sex-education interactions. The primary and secondary relative wage estimates, and standard errors in parentheses, are given by

$$\begin{aligned} \log(W^p_{ij}/W^p_{4j}) = \underset{(.022)}{.399D_1} + \underset{(.022)}{.069D_2} - \underset{(.022)}{.185D_3} + \underset{(.022)}{.123D_5} - \underset{(.022)}{.254D_6} \\ - \underset{(.022)}{.553D_7} - \underset{(.022)}{.585D_8} + \underset{(.015)}{.054NS}, \end{aligned} \tag{25}$$

$$R^2 = .91,\ \text{S.E.E.} = .11,\ \text{NO.OBS.} = 210,$$

and

$$\begin{aligned} \log(W^s_{ij}/W^p_{ij}) = -\underset{(.035)}{.617D_1} - \underset{(.035)}{.145D_2} - \underset{(.035)}{.345D_3} - \underset{(.035)}{.458D_4} - \underset{(.035)}{.520D_5} \\ - \underset{(.035)}{.176D_6} + \underset{(.035)}{.074D_7} - \underset{(.037)}{.037D_8} - \underset{(.023)}{.046NS}, \end{aligned} \tag{26}$$

$$R^2 = .63,\ \text{S.E.E.} = .18,\ \text{NO.OBS.} = 237,$$

where the D's are variables representing the interactions and are listed in the same order as the family type codes defined above. The coefficients can be interpreted as the percentage deviation of the wage rate of the group in question from the wage rate of the numeraire group in the primary equation and from the corresponding primary wage rate in the secondary equation when the absolute difference is not large. A South/non-South dummy variable (NS) was included and found to be significant in both the primary and secondary equations, indicating that primary relative wage rates are higher in the non-south states but that secondary relative wage rates are higher in the southern states.

For primary workers relative wage rates are reasonably constant across labor markets—multiplicative standard errors are all two percent. Human capital stocks (or wage rates) of secondary relative to primary workers displayed slightly more variability (3 to 4 percent standard errors) across states. These fitted wage rates were used as estimates of relative quality in the aggregation of hours of the 16 human capital groups in each market and in the construction of potential or expected wage rates--to which we now briefly turn.

In the labor market model as specified supply and demand for labor determine the absolute level of the numeraire wage rate in each market, while relative wage rates are determined by relative stocks of human capital. Accordingly, in equilibrium the representative primary worker is viewed as having expectations of a wage rate proportional to that of the numeraire group, the constant of proportionality being a measure of his (her) relative productivity:

$$\hat{W}^P_{1j} = (K^P_1/K^P_4)W^P_{4j}. \qquad (27)$$

The representative secondary worker can expect to receive a wage rate proportional to that of the numeraire group, the constants of proportionality relating the productivity of the secondary worker to the numeraire group worker via his (her) primary worker:

$$\hat{W}^{s}_{ij} = (K^{s}_{i}/K^{p}_{i})(K^{p}_{i}/K^{p}_{4})W^{p}_{4j}. \tag{28}$$

As shown already, fitted relative wage rates are used to measure the human capital stock ratios.

Turning to the demand side, the demand function was derived from a constant elasticity of substitution production function assuming constant returns to scale. It is the market demand for equivalent-quality hours per dollar of output in the state:

$$\log(H^{d}_{j}/NOUT_{j}) = \underset{(.20)}{6.675^{***}} - \underset{(.13)}{1.049^{***}}\log W^{p}_{4} - \underset{(.13)}{.013}\ NOUTI, \tag{29}$$

and

$$R^{2} = .75, \quad S.E.E. = .064 \quad NO.OBS. = 30.$$

The demand for equivalent-quality labor displays an elasticity of unity and is insignificantly greater in states where labor intensive activities are higher - NOUTI being the proportion of state output in manufacturing and construction.

The simultaneous equation model of the labor market is composed of a demand equation for equivalent quality hours, primary and secondary worker relative wage and labor supply equations. Endogenous variables are primary and secondary hours and wage rates for 16 human capital groups defined by age, sex, and education characteristics. To predict with the model, this nonlinear system of equations is solved using an iterative solution technique for a market equilibrium, in which demand and supply are equal.

IV. MARKET EFFECTS AND PROGRAM COSTS

To simulate the effects and costs of the Work Bonus Tax Credit we assume that there are no labor supply response differences between families that are demographically eligible for the program and those that are not, so that in the absence of the program, primary and secondary worker hours are simply the sum of the offers of demographically eligible and demographically ineligible workers. The total number of hours offered by demographically eligible families of type i is the product of the proportion of type i families with dependent children and the number of families and the hours supplied per family. In the presence of the program, those demographically eligible workers who are also financially eligible and who, therefore, do participate, behave as if their wage rate is subsidized or taxed and as if they are receiving an income transfer.

The labor market model just described implicitly incorporates the existing federal/state/local tax and subsidy system. We also assume that the existing structure will not be altered by the tax credit so that the latter can be added onto the former. This is a reasonable assumption for low income people who pay little or no tax and who comprise the bulk of participants.

In the non-linear simultaneous equations model there are 32 supply equations: 8 family types and 2 classes of workers (primary and secondary) from demographically eligible and demographically ineligible families. Using an iterative technique to solve the model, equilibrium is achieved when the market demand for equivalent quality hours is just matched by the sum of supplies forthcoming from primary and secondary workers of demographically eligible and demographically ineligible families. The solution jointly determines the absolute level of wages or numeraire group wate rate, and by construction using relative human capital stocks, the other wage rates.

The parameters of the Work Bonus Tax Credit program are expressed in 1973 values. Accordingly, prior to simulation they are converted into 1969 values (using the appropriate GNP deflators) to correspond in real terms to the scale of the econometric model parameters. The effects of the program have been simulated for 1976 by updating and projecting the model's exogenous variables and then solving the model with and without the program. Predicted wage rates and earnings (but not hours) from the simulations are then inflated and reported in 1976 values.[8]

In projecting the exogenous variables, nonfarm output is assumed to grow in all states at the national rate of 24.7 percent from 1969-1976.[9] Corresponding to this real output growth, unearned income is assumed to grow at the same rate. The number of families in each state is projected to grow at the annual national rate of 1.48 percent (observed over the period 1969-1973). The demographic composition variables are assumed to remain at their 1969 values.

The effect of labor embodied technical progress is both to increase the number of equivalent-quality hours and to reduce the average cost of labor to firms. Labor productivity grew at an annual rate of 3.1 percent from 1969-1973 and is assumed to grow into 1976 at the same 3.0 percent rate that prevailed in the last two decades so that output per man hour is assumed in 1976 to be 22.7 percent above its 1969 level.[10] Hence, in the simulations for 1976 firms obtain 1.227 (H_j^d) equivalent-quality hours at a market wage of W_{4j}^p so that the average cost of labor to firms is only $W_{4j}^p/1.227$. Since demand is slightly elastic the reduction in average cost will just absorb the increase in effective supply. Therefore the effect of increased labor productivity alone is to increase real wages slightly.

The predicted values of the model are reported by states and groups. For each state we report an average value of primary (W^p) and secondary (W^s) wage rates, hours per family (H^p/F, H^s/F), earnings per family (E^p/F, E^s/F), and family unearned (Y^n/F) and total income (G/F) over the groups. Wage rates are weighted by hours, while hours and earnings per family are weighted by families. Similarly, for each group corresponding averages are calculated over the states.

In Tables 1 and 2, the pre-program predicted state average values for demographically eligible and ineligible families are reported. In the nation, an average of 41 percent of the families are demographically eligible. These families are composed of a greater proportion of older workers who have slightly higher wage rates, supply more hours, and hence have higher earnings, whereas demographically ineligible families consist primarily of younger workers. Nationally, primary workers from demographically eligible families earn $10,188 per year in contrast to $8,564

TABLE 1

PRE-PROGRAM PREDICTED VALUES FOR 1976, DEMOGRAPHICALLY ELIGIBLE FAMILIES

STATE	W^P	W^S	H^P/F	H^S/F	E^P/F	E^S/F	E/F	Y^n/F	C/F	ELIGIBLE FAMILIES (thousands)
ALA-MISS	4.77	3.42	1517.	587.	7236.	2011.	9247.	978.	10225.	654.
ALK-HAW-WASH	7.22	4.72	1541.	481.	11127.	2271.	13397.	1485.	14882.	654.
ARIZ-COLO-NMEX	5.70	3.86	1589.	544.	9062.	2099.	11161.	1642.	12803.	755.
ARK-OKLA	4.85	3.49	1542.	540.	7484.	1887.	9371.	1285.	10657.	639.
CALIFORNIA	7.22	4.78	1503.	435.	10851.	2077.	12928.	1703.	14632.	2991.
CONNECTICUT	8.28	5.21	1474.	333.	12203.	1732.	13935.	2039.	15974.	475.
DELA-VA	5.98	4.21	1592.	595.	9518.	2501.	12019.	1367.	13386.	758.
D.C.	5.89	4.51	1361.	487.	8014.	2195.	10209.	1572.	11781.	101.
FLORIDA	5.15	3.70	1420.	577.	7319.	2136.	9455.	2454.	11909.	941.
GEORGIA	5.78	4.09	1569.	610.	9068.	2495.	11563.	1110.	12673.	707.
ID-MT-NV-UT-WY	5.65	3.80	1661.	542.	9386.	2063.	11449.	1409.	12858.	493.
ILLINOIS	7.79	5.12	1477.	374.	11496.	1917.	13413.	1385.	14798.	1687.
INDIANA	7.25	4.89	1520.	402.	11019.	1963.	12982.	1302.	14284.	520.
IOWA-MINN	6.73	4.43	1540.	506.	10359.	2242.	12601.	1392.	13993.	919.
KAN-NEB-ND-SD	6.06	4.08	1566.	527.	9486.	2150.	11637.	1376.	13013.	613.
KENTUCKY	5.12	3.70	1520.	578.	7785.	2140.	9925.	1121.	11046.	483.
LOUISIANA	5.11	3.64	1588.	596.	8116.	2169.	10285.	881.	11166.	573.
ME-MA-NH-RI-VT	6.77	4.52	1573.	552.	10647.	2496.	13143.	1593.	14737.	1302.
MARYLAND	7.50	5.17	1551.	408.	11630.	2109.	13739.	1355.	15095.	607.
MICH-WISC	7.67	5.03	1524.	406.	11695.	2041.	13736.	1395.	15131.	2032.
MISSOURI	6.27	4.32	1555.	511.	9756.	2209.	11965.	1495.	13459.	683.
NEW JERSEY	8.02	5.20	1520.	414.	12184.	2153.	14337.	1631.	15968.	1092.
NEW YORK	7.89	5.13	1423.	371.	11220.	1903.	13122.	1667.	14789.	2700.
NO CAR-SO CAR	5.33	3.86	1593.	632.	8484.	2441.	10924.	1040.	11965.	1157.
OHIO	7.57	5.03	1504.	387.	11386.	1943.	13330.	1454.	14784.	1659.
OREGON	6.66	4.48	1500.	437.	9994.	1961.	11954.	1531.	13485.	313.
PENNSYLVANIA	7.16	4.74	1469.	478.	10523.	2265.	12789.	1399.	14187.	1722.
TENNESSEE	5.33	3.85	1520.	623.	8105.	2398.	10503.	1016.	11519.	626.
TEXAS	5.67	4.03	1616.	604.	9166.	2433.	11599.	1379.	12977.	1701.
WEST VIRGINIA	5.07	3.54	1442.	563.	7315.	2063.	9378.	1085.	10462.	281.
NATION	6.70	4.43	1521.	483.	10188.	2139.	12327.	1477.	13804.	30382.

TABLE 2

PRE-PROGRAM PREDICTED VALUES FOR 1976, DEMOGRAPHICALLY INELIGIBLE FAMILIES

STATE	W^p	W^s	H^p/F	H^s/F	E^p/F	E^s/F	E/F	Y^n/F	G/F	INELIGIBLE FAMILIES (thousands)
ALA-MISS	4.83	3.35	1254.	568.	6054.	1899.	7953.	973.	8932.	1013.
ALK-HAW-WASH	7.01	4.60	1319.	478.	~~9250.~~	2197.	11447.	1485.	12932.	953.
ARIZ-COLO-NMEX	5.66	3.76	1332.	503.	7539.	1891.	9430.	1642.	11072.	1004.
ARK-OKLA	4.84	3.35	1203.	501.	5815.	1681.	7496.	1285.	8761.	997.
CALIFORNIA	7.09	4.68	1352.	452.	9585.	2116.	11701.	1703.	13404.	4887.
CONNECTICUT	8.07	5.10	1280.	360.	10324.	1839.	12163.	2039.	14202.	665.
DELA-VA	5.93	4.12	1394.	587.	8268.	2421.	10689.	1367.	12056.	1001.
D.C.	6.45	4.65	1326.	439.	8547.	2041.	10588.	1572.	12160.	247.
FLORIDA	5.36	3.67	1206.	544.	6466.	2000.	8467.	2454.	10921.	1740.
GEORGIA	5.62	4.00	1373.	595.	7715.	2380.	10095.	1110.	11205.	838.
ID-MT-NV-UT-WY	5.59	3.65	1351.	521.	7552.	1904.	9456.	1409.	10865.	618.
ILLINOIS	7.65	5.03	1252.	416.	9580.	2091.	11671.	1385.	13056.	2465.
INDIANA	7.35	4.84	1272.	438.	9355.	2119.	11474.	1302.	12777.	1054.
IOWA-MINN	6.58	4.33	1232.	492.	8103.	2134.	10237.	1392.	11629.	1364.
KAN-NEB-ND-SD	6.04	3.99	1216.	500.	7337.	1993.	9330.	1376.	10706.	1016.
KENTUCKY	5.19	3.59	1221.	601.	6334.	2159.	8494.	1121.	9615.	627.
LOUISIANA	5.11	3.53	1335.	588.	6819.	2077.	8896.	881.	9778.	653.
ME-MA-NH-RI-VT	6.73	4.44	1283.	537.	8533.	2385.	11018.	1593.	12611.	1968.
MARYLAND	7.39	5.10	1407.	444.	10392.	2262.	12654.	1355.	14010.	777.
MICH-WISC	7.47	4.92	1282.	433.	9577.	2155.	11733.	1395.	13128.	2746.
MISSOURI	6.20	4.16	1219.	488.	7558.	2031.	9589.	1495.	11084.	1065.
NEW JERSEY	7.66	5.03	1314.	450.	10061.	2259.	12320.	1631.	13951.	1579.
NEW YORK	7.75	5.06	1220.	381.	9453.	1932.	11385.	1667.	13051.	4536.
NO CAR-SO CAR	5.41	3.80	1373.	621.	7424.	2360.	9784.	1040.	10824.	1302.
OHIO	7.52	4.95	1264.	412.	9507.	2038.	11545.	1454.	12999.	2221.
OREGON	6.36	4.31	1182.	430.	7513.	1854.	9367.	1531.	10898.	516.
PENNSYLVANIA	7.11	4.68	1218.	491.	8661.	2297.	10958.	1399.	12357.	2751.
TENNESSEE	5.35	3.77	1295.	624.	6926.	2351.	9277.	1016.	10293.	743.
TEXAS	5.66	3.94	1358.	574.	7683.	2260.	9943.	1379.	11322.	2149.
WEST VIRGINIA	5.06	3.48	1152.	543.	5831.	1892.	7723.	1085.	8808.	330.
NATION	6.68	4.39	1281.	482.	8564.	2116.	10679.	1477.	12156.	43876.

per year for workers from demographically ineligible families.

A comparison of pre-program predicted group averages for demographically eligible and ineligible families is made by referring to Tables 3 and 4. Here we observe that the wage rates, hours, and earnings of demographically eligible and ineligible families within a group are indeed very similar, with the national average wage rate being slightly higher for the demographically eligible families who again are represented more heavily by families with higher earnings.

The post-program predicted values of the number of participants, the average subsidy per participant, total family income with the subsidy (Y/F), and the total cost are reported along with equilibrium wage rates and hours in Table 5 and Table 7 for demographically eligible families by states and groups. The market effects of the program on the demographically ineligible are reported in Table 6 and Table 8. Of those families that are demographically eligible, 8 percent are also financially eligible and therefore are participants in the program. State participation rates vary widely, however, from a low of 3.7 percent of the demographically eligible families in the New England states, to a high in Florida of 14.6 percent participation.

The family composition of participation, however, does not vary. Only families headed by females who are not college graduates are projected to participate in the program. The participation rate for group F-O-NC is 45 percent, and for group F-Y-NC it is over 99 percent. No other group is projected to participate in any state. Of those families participating, we found that for the nation as a whole 48 percent have income below $4000 and accordingly had an incentive to increase hours

TABLE 3

PRE-PROGRAM PREDICTED VALUES FOR 1976, DEMOGRAPHICALLY ELIGIBLE FAMILIES

	GROUP TYPE SEX	AGE	ED	W^P	W^S	H^P/F	H^S/F	E^P/F	E^S/F	E/F	Y^n	G/F	ELIGIBLE FAMILIES (thousands)
1.	M/F	O	C	10.97	5.61	1623.21	392.10	17302.30	2198.13	20000.43	3222.70	23223.13	2907.
2.	M/F	Y	C	7.82	6.34	1755.24	299.51	13724.48	1597.45	15621.93	577.75	16199.68	1794.
3.	M	O	NH	6.01	4.08	1245.65	658.05	7487.41	2804.10	10291.51	1599.14	11890.65	6452.
4.	M	O	H	7.09	4.31	1688.35	725.03	11962.04	3125.65	15087.70	1470.25	16557.95	4400.
5.	M	O	SC	8.29	4.61	1753.41	619.80	14527.29	2856.02	17383.31	2424.47	19807.78	1751.
6.	M	Y	NC	5.60	4.32	1796.46	371.04	10063.34	1602.35	11665.69	254.11	11919.60	9504.
7.	F	O	NC	4.16	4.27	707.71	255.16	2943.35	1090.42	4033.77	1966.80	6000.57	1916.
8.	F	Y	NC	4.00	3.94	330.27	157.33	3320.59	619.28	3939.87	436.32	4376.19	1658.
9.	ALL TYPES			6.70	4.43	1520.94	462.77	10188.02	2136.68	12326.90	1476.98	13603.88	30382.

TABLE 4

PRE-PROGRAM PREDICTED VALUES FOR 1976, DEMOGRAPHICALLY INELIGIBLE FAMILIES

	GROUP TYPE SEX	AGE	ED	W^P	W^S	H^P/F	H^S/F	E^P/F	E^S/F	E/F	Y^n/F	G/F	INELIGIBLE FAMILIES (thousands)
1.	M/F	O	C	10.94	5.58	1614.65	394.92	17656.54	2205.54	19862.08	3222.70	23084.78	3931.
2.	M/F	Y	C	8.00	6.49	1727.38	285.63	13817.54	1853.60	15671.14	577.75	16248.89	1861.
3.	M	O	NH	6.07	4.11	1229.90	676.41	7464.07	2777.92	10241.98	1599.14	11841.12	13346.
4.	M	O	H	7.07	4.30	1682.16	723.96	11897.50	3114.64	15012.14	1470.25	16482.39	5270.
5.	M	O	SC	8.25	4.59	1742.17	623.56	14374.56	2865.21	17239.77	2424.47	19664.24	2178.
6.	M	Y	NC	5.66	4.37	1785.40	360.46	10111.31	1576.67	11687.98	254.11	11942.10	4349.
7.	F	O	NC	4.24	4.35	704.70	249.52	2989.16	1085.72	4074.88	1966.80	6041.68	10405.
8.	F	Y	NC	4.06	3.97	827.53	154.40	3358.18	612.58	3970.75	436.32	4407.07	1837.
9.	ALL TYPES			6.68	4.39	1281.42	481.87	8563.61	2115.75	10678.36	1476.98	1476.98	43376.

TABLE 5

POST-PROGRAM PREDICTED VALUES FOR 1976, DEMOGRAPHICALLY ELIGIBLE FAMILIES

STATE	W^P	W^s	H^P/F	H^s/F	E^P/F	E^S/F	E/F	Y^n/F	G/F	SUBSIDY/ PARTICIPANT	PARTICIP. FAMILIES (thousands)	Y/F	TOTAL COST (millions)
ALA-MISS	4.72	3.41	1544.	569.	7287.	1941.	9228.	978.	10206.	311.29	117.	10249	36.48
ALK-HAW-WASH	7.25	4.73	1533.	479.	11114.	2265.	13379.	1485.	14864.	164.33	67.	14881	10.98
ARIZ-COLO-NMEX	5.70	3.87	1593.	535.	9082.	2070.	11152.	1642.	12793.	295.11	81.	12825	23.67
ARK-OKLA	4.85	3.48	1544.	548.	7481.	1909.	9390.	1285.	10676.	283.11	74.	10709	20.90
CALIFORNIA	7.15	4.81	1529.	417.	10937.	2003.	12941.	1703.	14644.	163.69	243.	14657	39.71
CONNECTICUT	8.29	5.19	1463.	337.	12126.	1752.	13873.	2039.	15917.	144.41	23.	15924	3.38
DELA-VA	5.97	4.22	1603.	579.	9563.	2444.	12007.	1367.	13374.	200.49	94.	13399	15.60
D.C.	5.86	4.56	1385.	456.	8112.	2078.	10190.	1572.	11761.	375.36	12.	11806	4.40
FLORIDA	5.16	3.70	1421.	569.	7329.	2107.	9436.	2454.	11891.	206.58	139.	11922	28.51
GEORGIA	5.78	4.09	1569.	606.	9077.	2477.	11553.	1110.	12663.	256.36	96.	12698	24.64
ID-MT-NV-UT-WY	5.64	3.80	1665.	538.	9396.	2045.	11440.	1409.	12849.	327.94	52.	12884	16.91
ILLINOIS	7.79	5.10	1466.	382.	11410.	1948.	13358.	1365.	14743.	274.54	93.	14758	25.42
INDIANA	7.26	4.88	1508.	406.	10949.	1978.	12927.	1302.	14229.	303.75	46.	14246	13.83
IOWA-MINN	6.74	4.44	1538.	503.	10359.	2232.	12591.	1392.	13983.	169.50	76.	13997	12.91
KAN-NEB-ND-SD	6.07	4.08	1563.	525.	9484.	2141.	11625.	1376.	13001.	201.65	61.	13021	12.29
KENTUCKY	5.12	3.70	1522.	573.	7797.	2119.	9916.	1121.	11038.	291.07	61.	11074	17.74
LOUISIANA	5.07	3.64	1612.	572.	8172.	2080.	10253.	881.	11134.	321.55	75.	11176	24.12
ME-MA-NH-RI-VT	6.76	4.52	1567.	553.	10631.	2501.	13132.	1593.	14726.	348.27	48.	14739	16.73
MARYLAND	7.58	5.21	1532.	395.	11610.	2058.	13668.	1355.	15024.	345.94	36.	15045	12.57
MICH-WISC	7.68	5.03	1515.	408.	11642.	2051.	13693.	1395.	15088.	266.64	81.	15029	21.56
MISSOURI	6.29	4.32	1550.	510.	9746.	2204.	11951.	1495.	13446.	202.51	70.	13467	14.24
NEW JERSEY	8.03	5.19	1507.	418.	12102.	2170.	14272.	1631.	15903.	296.50	61.	15920	18.37
NEW YORK	7.92	5.13	1406.	373.	11134.	1911.	13045.	1667.	14712.	269.22	177.	14730	47.64
NO CAR-SO CAR	5.31	3.88	1606.	611.	8533.	2370.	10904.	1040.	11944.	294.95	149.	11982	43.90
OHIO	7.57	5.01	1495.	394.	11308.	1973.	13281.	1454.	14735.	296.30	86.	14750	25.35
OREGON	6.68	4.48	1492.	439.	9973.	1970.	11943.	1531.	13473.	290.71	18.	13490	5.11
PENNSYLVANIA	7.23	4.77	1454.	471.	10514.	2245.	12759.	1399.	14158.	333.56	77.	14173	25.80
TENNESSEE	5.33	3.85	1524.	615.	8125.	2367.	10492.	1016.	11508.	325.53	88.	11554	28.61
TEXAS	5.67	4.03	1616.	601.	9172.	2419.	11591.	1379.	12969.	291.84	191.	13002	55.75
WEST VIRGINIA	5.06	3.53	1444.	589.	7313.	2082.	9395.	1085.	10480.	304.50	27.	10509	8.21
NATION	6.70	4.44	1520.	479.	10176.	2124.	12299.	1477.	13776.	261.70	2517.	13798	658.51

TABLE 6

POST-PROGRAM PREDICTED VALUES FOR 1976, DEMOGRAPHICALLY INELIGIBLE FAMILIES

STATE	W^P	W^S	H^P/F	H^S/F	E^P/F	E^S/F	E/F	Y^n/F	G/F	INELIGIBLE FAMILIES (thousands)
ALA-MISS	4.81	3.33	1253.	565.	6027.	1885.	7912.	978.	8891.	1013.
ALK-HAW-WASH	7.03	4.61	1318.	476.	9263.	2194.	11457.	1485.	12942.	953.
ARIZ-COLO-NMEX	5.66	3.76	1332.	503.	7544.	1892.	9436.	1642.	11078.	1004.
ARK-OKLA	4.83	3.35	1202.	501.	5603.	1677.	7486.	1285.	8771.	997.
CALIFORNIA	7.07	4.67	1353.	454.	9569.	2121.	11690.	1703.	13394.	4887.
CONNECTICUT	8.05	5.09	1282.	364.	10314.	1854.	12165.	2039.	14207.	665.
DELA-VA	5.94	4.13	1394.	587.	8274.	2422.	10695.	1367.	12063.	1001.
D.C.	6.45	4.65	1325.	439.	8550.	2041.	10591.	1572.	12163.	247.
FLORIDA	5.37	3.68	1206.	545.	6473.	2003.	8475.	2454.	10930.	1740.
GEORGIA	5.62	4.00	1373.	595.	7720.	2382.	10101.	1110.	11211.	838.
ID-MT-NV-UT-WY	5.59	3.65	1351.	521.	7548.	1903.	9452.	1409.	10861.	618.
ILLINOIS	7.62	5.01	1255.	422.	9566.	2112.	11678.	1365.	13063.	2465.
INDIANA	7.34	4.83	1273.	440.	9348.	2127.	11475.	1302.	12777.	1054.
IOWA-MINN	6.58	4.34	1232.	492.	8108.	2134.	10242.	1392.	11634.	1364.
KAN-NEB-ND-SD	6.04	3.99	1216.	500.	7342.	1994.	9336.	1376.	10712.	1018.
KENTUCKY	5.19	3.60	1221.	601.	6338.	2161.	8499.	1121.	9621.	627.
LOUISIANA	5.10	3.52	1335.	587.	6802.	2069.	8871.	861.	9753.	653.
ME-MA-NH-RI-VT	6.74	4.44	1283.	537.	8640.	2385.	11024.	1593.	12618.	1968.
MARYLAND	7.44	5.13	1403.	433.	10433.	2223.	12656.	1355.	14011.	777.
MICH-WISC	7.46	4.91	1263.	440.	9572.	2161.	11733.	1395.	13129.	2746.
MISSOURI	6.20	4.17	1219.	486.	7565.	2032.	9597.	1495.	11092.	1065.
NEW JERSEY	7.64	5.02	1316.	453.	10052.	2271.	12323.	1631.	13954.	1579.
NEW YORK	7.74	5.06	1220.	383.	9449.	1936.	11386.	1667.	13052.	4535.
NO CAR-SO CAR	5.41	3.60	1373.	621.	7428.	2361.	9789.	1040.	10830.	1302.
OHIO	7.49	4.93	1267.	417.	9491.	2058.	11548.	1454.	13002.	2221.
OREGON	6.37	4.32	1181.	430.	7520.	1854.	9374.	1531.	10905.	516.
PENNSYLVANIA	7.16	4.71	1214.	485.	8690.	2282.	10971.	1399.	12370.	2751.
TENNESSEE	5.35	3.77	1295.	625.	6931.	2353.	9284.	1016.	10300.	743.
TEXAS	5.66	3.94	1358.	574.	7687.	2261.	9949.	1379.	11327.	2149.
WEST VIRGINIA	5.06	3.48	1151.	543.	5824.	1888.	7712.	1085.	8797.	380.
NATION	6.68	4.39	1282.	482.	8563.	2118.	10681.	1477.	12158.	43876.

TABLE 7

POST-PROGRAM PREDICTED VALUES FOR 1976, DEMOGRAPHICALLY ELIGIBLE FAMILIES

	GROUP TYPE															
	SEX	AGE	ED	W^p	W^s	H^p/F	H^s/F	E^p/F	E^s/F	E/F	Y^n/F	G/F	SUBSIDY/ PARTICIPANT	PARTICIP. FAMILIES (thousands)	Y/F	TOTAL COST (millions)
1.	M/F	O	C	10.96	5.60	1623.	392.	17795.	2199.	19994.	3223.	23216.	.00	0.	23216.	.00
2.	M/F	Y	C	7.82	6.33	1755.	300.	13718.	1899.	15617.	578.	16195.	.00	0.	16195.	.00
3.	M	O	NH	6.01	4.08	1246.	689.	7487.	2806.	10293.	1599.	11893.	.00	0.	11893.	.00
4.	M	O	H	7.08	4.31	1689.	726.	11964.	3129.	15093.	1470.	16563.	.00	0.	16563.	.00
5.	M	O	SC	8.28	4.61	1754.	622.	14523.	2864.	17387.	2424.	19811.	.00	0.	19811.	.00
6.	M	Y	NC	5.60	4.32	1797.	372.	10061.	1608.	11669.	254.	11923.	.00	0.	11923.	.00
7.	F	O	NC	4.11	4.45	749.	198.	3080.	882.	3962.	1967.	5929.	188.85	865.	6014.	163.34
8.	F	Y	NC	3.93	3.87	756.	134.	2968.	521.	3489.	436.	3925.	299.82	1653.	4224.	495.46
9.	ALL TYPES			6.70	4.44	1520.	479.	10176.	2124.	12299.	1477.	13776.	261.70	2518.	13798.	658.81

TABLE 8

POST-PROGRAM PREDICTED VALUES FOR 1976, DEMOGRAPHICALLY INELIGIBLE FAMILIES

	GROUP TYPE			W^p	W^s	H^p/F	H^s/F	E^p/F	E^s/F	E/F	Y^n/F	G/F	INELIGIBLE FAMILIES (thousands)
	SEX	AGE	ED										
1.	M/F	O	C	10.93	5.58	1615.	395.	17650.	2206.	19857.	3223.	23079.	3931.
2.	M/F	Y	C	8.00	6.49	1727.	286.	13811.	1855.	15666.	578.	16244.	1851.
3.	M	O	NH	6.07	4.11	1230.	677.	7464.	2780.	10244.	1599.	11843.	13846.
4.	M	O	H	7.07	4.30	1683.	725.	11899.	3118.	15017.	1470.	16487.	5270.
5.	M	O	SC	8.25	4.60	1743.	625.	14371.	2873.	17244.	2424.	19668.	2178.
6.	M	Y	NC	5.66	4.38	1786.	362.	10109.	1583.	11691.	254.	11945.	4549.
7.	F	O	NC	4.24	4.35	705.	250.	2988.	1087.	4075.	1967.	6042.	10405.
8.	F	Y	NC	4.06	3.96	829.	154.	3363.	611.	3974.	436.	4411.	1837.
9.	ALL TYPES			6.68	4.39	1282.	482.	8563.	2118.	10681.	1477.	12158.	43876.

supplies, while 40 percent have income above $4000 and on average decreased hours supplied. Twelve percent of the families are located at one of the two jogs or program pivot points. Finally, while we do believe that participation in the program would be dominated by female-headed families, we also believe that we have overestimated female-headed participation and underestimated male-headed participation. The reason for this is that we employ group averages, which do not take into account the variation within the group. Only a procedure which incorporates this variation, such as a microsimulation model, could produce a finer distribution of participants. Nevertheless, we have no a priori reason to believe that our estimates of total participation are biased by the use of group averages.

The effects of the program expressed as percentage differences (post-program less pre-program values as a percent of the latter) are reported for demographically eligible (Table 9 and Table 11) and demographically ineligible (Table 10 and Table 12) families. Looking at the state averages in Table 9, hours per family fall in half the states by small amounts for demographically eligible families (i.e., in these states the program segments which incorporate an incentive to decrease hours supplied outweight the program segment where there is an incentive to increase hours supplied). Nationally, primary and secondary hours have fallen by only 0.1 percent and 1.0 percent, and wage rates have increased slightly for secondary workers (0.14 percent) and decreased slightly for primary workers (0.05 percent). Although the net effects of the program by states are small, Table 11 shows that for demographically eligible families there are some rather sizeable changes in hours for the participating groups.

TABLE 9

PRE- AND POST-PROGRAM STATE DIFFERENCES, DEMOGRAPHICALLY ELIGIBLE FAMILIES (PERCENTAGE)

STATE	W^P	W^S	H^P/F	H^S/F	E^P/F	E^S/F	E/F	C/F	Y/F
ALA-MISS	-1.02	-.35	1.75	-3.17	.71	-3.50	-.20	-.18	.23
ALK-HAW-WASH	.42	.14	-.53	-.36	-.12	-.24	-.14	-.13	.00
ARIZ-COLO-NMEX	-.04	.16	.25	-1.54	.21	-1.38	-.09	-.06	.17
ARK-OKLA	-.15	-.27	.11	1.45	-.04	1.18	.20	.18	.48
CALIFORNIA	-.90	.62	1.71	-4.15	.79	-3.55	.10	.08	.17
CONNECTICUT	.14	-.23	-.77	1.38	-.63	1.15	-.41	-.36	.31
DELA-VA	-.18	.38	.65	-2.67	.47	-2.30	-.10	-.09	.09
D.C.	-.52	1.06	1.75	-6.34	1.22	-5.35	-.19	-.17	.21
FLORIDA	.06	.05	.07	-1.39	.14	-1.34	-.20	-.16	.10
GEORGIA	.06	.01	.03	-.76	.09	-.75	-.09	-.08	.19
ID-MT-NV-UT-WY	-.13	-.10	.24	-.61	.11	-.90	-.07	-.06	.20
ILLINOIS	-.01	-.37	-.74	2.01	-.75	1.63	-.41	-.37	-.27
INDIANA	.15	-.19	-.78	.94	-.64	.76	-.42	-.39	-.26
IOWA-MINN	.14	.05	-.14	-.50	-.00	-.45	-.08	-.07	.02
KAN-NEB-ND-SD	.14	.00	-.16	-.43	-.03	-.43	-.10	-.09	.06
KENTUCKY	.01	.01	.15	-.96	.16	-.97	-.09	-.06	.25
LOUISIANA	-.77	-.11	1.48	-3.97	.69	-4.07	-.31	-.29	.08
ME-MA-NH-RI-VT	.25	.05	-.40	.14	-.15	.20	-.08	-.07	.01
MARYLAND	1.08	.66	-1.24	-3.04	-.17	-2.41	-.52	-.47	-.33
MICH-WISC	.14	-.13	-.59	.63	-.45	.50	-.31	-.28	-.21
MISSOURI	.23	.02	-.32	-.22	-.10	-.20	-.11	-.10	.05
NEW JERSEY	.19	-.22	.86	.98	-.67	.76	-.46	-.41	-.03
NEW YORK	.40	-.11	1.16	.58	-.77	.46	-.59	-.52	-.39
NO CAR-SO CAR	-.23	.35	.82	-3.22	.59	-2.88	-.19	-.17	.14
OHIO	-.09	-.40	-.60	1.97	-.69	1.56	-.36	-.33	-.22
OREGON	.33	-.01	-.54	.49	-.21	.48	-.10	-.09	.03
PENNSYLVANIA	.89	.55	-.97	-1.46	-.08	-.92	-.23	-.21	-.09
TENNESSEE	-.01	.04	.26	-1.33	.24	-1.29	-.11	-.10	.30
TEXAS	.05	.00	.02	-.59	.07	-.58	-.07	-.06	.19
WEST VIRGINIA	-.16	-.17	.14	1.10	-.02	.92	.18	.17	.44
NATION	-.05	.14	-.08	-.86	-.12	-.72	-.22	-.20	-.04

TABLE 10

PRE- AND POST-PROGRAM STATE DIFFERENCES, DEMOGRAPHICALLY INELIGIBLE FAMILIES (PERCENTAGE)

STATE	W^P	W^S	H^P/F	H^S/F	E^P/F	E^S/F	E/F	G/F
ALA-MISS	-.38	-.36	-.07	-.37	-.45	-.74	-.52	-.46
ALK-HAW-WASH	.22	.20	-.07	-.33	.14	-.13	.09	.08
ARIZ-COLO-NMEX	.06	.05	.00	.03	.06	.08	.06	.05
ARK-OKLA	-.10	-.10	-.02	-.11	-.12	-.21	-.14	-.12
CALIFORNIA	-.26	-.24	.10	.46	-.16	.22	-.09	-.08
CONNECTICUT	-.25	-.23	.16	1.05	-.09	.82	.04	.04
DELA-VA	.07	.07	-.00	-.02	.07	.05	.06	.06
D.C.	.04	.04	-.01	-.05	.04	-.02	.03	.02
FLORIDA	.09	.08	.01	.05	.10	.13	.10	.08
GEORGIA	.07	.07	-.00	-.01	.07	.05	.07	.06
ID-MT-NV-UT-WY	-.04	-.04	-.00	-.02	-.04	-.06	-.05	-.04
ILLINOIS	-.40	-.37	.25	1.39	-.15	1.02	.06	.05
INDIANA	-.18	-.17	.10	.53	-.08	.36	.00	.00
IOWA-MINN	.09	.08	-.02	-.08	.06	.00	.05	.05
KAN-NEB-ND-SD	.07	.07	-.00	.00	.07	.07	.07	.06
KENTUCKY	.06	.05	.01	.03	.06	.08	.07	.06
LOUISIANA	-.23	-.21	-.02	-.16	-.25	-.37	-.28	-.25
ME-MA-NH-RI-VT	.11	.10	-.03	-.11	.08	-.00	.06	.05
MARYLAND	.69	.66	-.30	-2.36	.39	-1.74	.01	.01
MICH-WISC	-.13	-.12	.07	.38	-.05	.26	.01	.00
MISSOURI	.11	.10	.02	-.05	.09	.05	.08	.07
NEW JERSEY	-.21	-.21	.13	.72	-.08	.52	.03	.02
NEW YORK	-.10	-.09	.06	.35	-.04	.26	.01	.01
NO CAR-SO CAR	.05	.05	.00	.01	.06	.06	.06	.05
OHIO	-.41	-.39	.24	1.36	-.17	.97	.03	.03
OREGON	.12	.11	-.03	-.11	.09	.00	.07	.06
PENNSYLVANIA	.63	.60	-.29	-1.27	.33	-.68	.12	.11
TENNESSEE	.07	.07	.00	.02	.08	.09	.08	.07
TEXAS	.05	.05	.00	.01	.05	.06	.05	.05
WEST VIRGINIA	-.11	-.10	-.02	-.09	-.12	-.19	-.14	-.12
NATION	-.04	-.00	.03	.12	-.01	.12	.01	.01

TABLE 11

PRE- AND POST PROGRAM FAMILY TYPE DIFFERENCES, DEMOGRAPHICALLY ELIGIBLE FAMILIES
(PERCENTAGE)

	GROUP TYPE SEX	AGE	ED	W^P	W^S	H^P/F	H^S/F	E^P/F	E^S/F	E/F	G/F	Y/F
1.	M/F	O	C	-.05	-.03	.01	.07	-.04	.04	-.03	-.03	- .03
2.	M/F	Y	C	-.04	-.01	-.00	.09	-.04	.08	-.03	-.03	- .02
3.	M	O	NH	-.02	-.00	.02	.07	-.00	.07	.02	.02	.01
4.	M	O	H	-.03	-.01	.05	.11	.02	.10	.03	.03	.03
5.	M	O	SC	-.06	.00	.03	.28	-.03	.28	.02	.02	.01
6.	M	Y	NC	-.04	.06	.02	.29	-.02	.35	.03	.03	.02
7.	F	O	NC	-1.08	4.09	5.80	-22.32	4.66	-19.15	-1.78	-1.19	.22
8.	F	Y	NC	-1.83	-1.60	-8.95	-14.57	-10.62	-15.94	-11.46	-10.31	-11.05
9.	ALL TYPES			-.05	.14	-.08	-.86	-.12	-.72	-.22	-.20	- .04

TABLE 12

PRE- AND POST PROGRAM FAMILY TYPE DIFFERENCES, DEMOGRAPHICALLY INELIGIBLE FAMILIES
(PERCENTAGE)

	GROUP TYPE										
	SEX	AGE	ED	W^P	W^S	H^P/F	H^S/F	E^P/F	E^S/F	E/F	C/F
1.	M/F	O	C	-.04	-.02	.01	.06	-.03	.04	-.03	-.02
2.	M/F	Y	C	-.05	-.01	-.00	.11	-.05	.10	-.03	-.03
3.	M	O	NH	-.03	-.00	.03	.09	-.00	.08	.02	.02
4.	M	O	H	-.04	-.02	.05	.12	.02	.10	.03	.03
5.	M	O	SC	-.05	.01	.03	.25	-.02	.26	.02	.02
6.	M	Y	NC	-.05	.05	.02	.32	-.03	.37	.03	.03
7.	F	O	NC	-.04	.01	.01	.12	-.03	.13	.01	.01
8.	F	Y	NC	-.03	-.11	.17	-.07	.14	-.18	.09	.08
9.	ALL TYPES			-.04	-.00	.03	.12	-.01	.12	.01	.01

Among older female-headed families, (F-O-NC) primary workers increase hours on average about 6 percent while secondary workers decrease hours approximately 22 percent. Moreover, younger female primary workers (F-Y-NC) decrease hours substantially (9 percent). Total family income for the entire F-O-NC group has increased from $5929 to $6014 on average, composed of a fall in earnings per eligible of almost two percent and a subsidy per participant of $189. For the F-Y-NC group, however, total family income has decreased from $4376 to $4224, the decrease in earnings per eligible of 11 percent outweighing the subsidy per participant of $300. For this group income seems to be an inferior good.

Turning to Table 10 and Table 12, for demographically ineligible families both the average primary and secondary wage rates decrease very slightly so that we project little dissipation of benefits upon implementation of this program. Moreover, primary and secondary hours of demographically ineligible families even increase slightly so there is no substitution, let alone large scale substitution, of subsidized labor for unsubsidized labor. Thus, the potential of the Work Bonus Tax Credit to be market neutral is approximately realized.

V. CONCLUSIONS

In this paper, we have analyzed the market effects and program costs of the Work Bonus Tax Credit. We first described the program as outlined by the Senate Finance Committee and then interpreted the proposal, showing that there are three distinct income ranges in which a family could participate. In the first range, family income and hence earnings are less than $4,000 and the family receives a ten percent subsidy on earnings. If family income exceeds $4,000 while earnings remain below that amount, the family is in the second range. It continues to receive the earnings subsidy but also pays a 25 percent tax on income which exceeds $4,000. In the third range earnings also exceed $4,000 and the family no longer receives the earnings subsidy. It pays the tax on income exceeding $4,000 until the initial transfer has been completely taxed away. The program exit level depends on the family's level of unearned income, occurring at $5,600 for families without unearned income and decreasing as unearned income increases.

We next used the theory of family labor supply to analyze the labor supply effects of each of three program ranges, emphasizing that the program combines a wage tax with an income transfer. We determined the income transfer using a concept of tax saved minus tax paid previously developed for the analysis of the payroll tax and the income tax. We then illustrated this theory with the case of a single-worker family deriving the appropriate program budget constraints, and emphasizing in the analysis of labor supply the importance of "jogs", where marginal rates increase, and "jumps", where marginal tax rates increase discontinuously. A brief overview of the state labor market model with which we simulated the Work Bonus Tax Credit was then given.

Projecting the proposed program into 1976, we forecast that while 42 percent of the families will be demographically eligible, only 8 percent of these eligible families will, in fact, participate. State participation rates will vary widely, however, ranging from 3.7 percent of the eligible families in the New England states to 14.6 percent of the eligible families in Florida. Participation in the program, however, will be dominated by families headed by females who are not college graduates. We also project that while the Work Bonus Tax Credit will cause sizeable changes in hours supplied to the market by individual families, the labor supply effects of the different segments will offset each other in each state so that the program is almost market neutral. In other words, the dissipation of benefits through wage increases to the demographically ineligible will be minimal as will be the substitution of subsidized for unsubsidized labor. Thus, we project that the program could be introduced without causing large readjustments of the non-target population. Using a labor market model which allows for labor supply effects, we estimate that the program will cost $658 million in 1976, while the Senate Finance Committee, using a static analysis not allowing for changes in labor supply, estimates the program will cost $700 million in 1974.

FOOTNOTES

*This research was supported by funds from the Office of Research and Development, Manpower Administration, U.S. Department of Labor, under Grant No. 21-11-74-09 to the Urban Institute.

Opinions expressed are those of the authors and do not necessarily represent the views of the Department of Labor, the Urban Institute, or its sponsors.

We wish to thank Linda Royster for programming assistance.

1. Haveman [3] and Haveman and Lampman [4] have proposed several different earnings subsidies.

2. For a capsule summary of the Work Bonus Tax Credit, see [7, pp. 6-7].

3. The Senate Finance Committee [7] states that the exit level occurs when family income equals $5600. However, this is only true when unearned income does not exceed $1600.

4. For a discussion of the tax paid, tax saved concept in terms of the personal income tax, see MacRae and Yezer [6, pp. 6-7].

5. For an analysis of the effect of the Social Security tax on labor supply which distinguishes between effective decreases in the wage rate and decreases in unearned income, see MacRae and MacRae [5].

6. A complete discussion of the specification, estimation, and evaluation of the model is given by Greenston and MacRae [2]. This model has evolved from the low-skill labor market model developed by Crandall, MacRae and Yap [1]. In contrast to the earlier model, which partitions the labor market by occupation and focuses only on the low-skill sector, this model describes the supply and demand for all workers and partitions them by demographic group.

7. See Greenston and Riordan [3] for a description of the algorithm and its application in forming human capital groups.

8. Forecasts of the GNP Implicit Price Deflator are contained in [10]. The 1976 to 1969 ratio of Deflators is applied to the simulation output.

9. The 1976 estimate of $905 billion (in 1958 dollars) is a Chase Econometric Model forecast of early 1974.

10. Productivity trends are discussed in [9, pp. 11-12].

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JOBS AND INCOME (JOIN): A LABOR MARKET ANALYSIS

by Robert I. Lerman, C. Duncan MacRae and
Anthony M.J. Yezer

Abstract

The major labor market effects of a work subsidy program developed by the staff of the Subcommittee on Fiscal Policy of the Joint Economic Committee are analyzed in this paper. The Jobs and Income proposal (JOIN) has been designed to incorporate some features which increase labor supply to the private sector and others which should have the opposite effect. The income guarantee and earnings tax tend to decrease labor supply, while wage subsidies, including that implicit in its public employment component, work in the opposite direction. The simulation of a high benefit version with an econometric model of state labor markets (pp. 1.1-1.75) in 1976 indicates that hours supplied by participating families tend to increase as a result of the implementation of JOIN. However, wages do not fall significantly and non-participating families appear to be little affected by the introduction of JOIN. Therefore, we conclude that the increase in labor supply would be absorbed by the public sector leaving the private sector relatively unaffected.

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JOBS AND INCOME (JOIN): A LABOR MARKET ANALYSIS

by R.I. Lerman, C.D. MacRae and A.M.J. Yezer*

I. INTRODUCTION

A major controversy in the welfare reform debate is whether to adopt an employment subsidy program or a purely income-conditioned program. An important limitation of such income-conditioned programs as the negative income tax (NIT) is their effect on the financial reward for working. In order to assure poor families a moderate income at reasonable costs to the taxpayer under an NIT, the implicit tax rate on earned income of recipients would have to reach near 50 percent or higher. The substantial reduction in the financial return to work and the provision of an income guarantee could trigger a reduction in work hours. And, if not, the NIT might still be considered unfair because it narrows greatly the income differences among persons working at the same wage but for considerably different numbers of hours.

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Many work subsidy programs have been proposed as alternatives to the NIT, but these programs also may have some undesirable economic effects. Wage subsidies themselves could lower work hours, as much or more than an NIT.[1] To some extent they may reduce the wages employers pay, thereby offsetting government attempts to raise the incomes of low wage workers. And wage subsidies may channel too large a share of government dollars toward secondary workers in middle and upper income families. Guaranteed employment plans may draw laborers from the private sector and may cause employers to reduce their employment of low-wage workers, again raising the government costs per dollar of improvement in the incomes of poor and near-poor families.

Theoretically, work subsidy plans could induce a wide range of effects on labor supply, wage rates, and poverty.[2] The actual outcome depends on the precise nature of the subsidy program and on the reactions of workers and employers. Although the few empirical studies of work subsidy plans provide some useful results, their value is limited by the failure to take account of the demand side of the labor market.[3]

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[1] Irwin Garfinkel, "A Skeptical Note on 'The Optimality' of Wage Subsidy Programs," American Economic Review, June 1973, pp. 447-453; and Jonathan Kesselman, "Incentive Effects of Transfer Systems Once Again, Journal of Human Resources, Winter 1973, pp. 119-129.

[2] See, for example, Peter Mieszkowski, "The Indirect Market Effects of Wage-Subsidies and Public Employment Programs," Studies in Public Welfare, Paper No. 19, Public Employment and Wage Subsidies, Joint Economic Committee, (Washington, D.C.: U.S. Government Printing Office, 1975); and Jonathan Kesselman, "A Comprehensive Approach to Income Maintenance: SWIFT," Journal of Public Economics, February 1973, pp. 59-88.

The purpose of this paper is to simulate the wage, employment and, hence, earnings effects of a proposed work subsidy program. The Jobs and Income proposal (JOIN) is a comprehensive one that would replace several existing welfare programs. The JOIN design attempts to overcome some important criticisms of work subsidy programs. To focus program benefits on the poorest families, JOIN utilizes a surtax on total family income. This surtax varies somewhat with the presumed needs of different kinds of families. To avoid substantial reductions in the wages employers pay or in the number of low-wage workers they hire, JOIN has both wage subsidy and guaranteed jobs components.

Using either program alone, however, could result in low cost-effectiveness, depending on how demand for low-wage workers changes with wage rate changes. The substantial uncertainty about employer reactions makes the combination wage-subsidy, guaranteed-job approach advisable. A third component of JOIN is an income guarantee available only to one-parent families with at least one child under age 14. Although this feature is not efficient because of its categorical nature, the only alternative may be that the Federal government provide child care to all such families to free the parents for full-time work.

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[3] Samuel Rea, Jr., "Trade-Offs Between Alternative Income Maintenance Programs," in Studies in Public Welfare, Paper No. 13, A Volume of Studies Prepared for the use of the Subcommittee on Fiscal Policy, Joint Economic Committee, U.S. Congress, U.S. Government Printing Office, Washington 1974. See, however, Michael Barth, "Universal-Wage-Rate Subsidy: Benefits and Effects," in U.S. Congress, Joint Economic Committee, The Economics of Federal Subsidy Programs: Special Study Papers, 1974.

Estimating the impact of JOIN on the wages employers pay, on the levels of employment in conventional jobs and in special public jobs, and on the incomes of low-income families is an extraordinarily difficult task. Perhaps the largest problem is the uncertainty about how workers and employers would react to the JOIN program. Even the direction of JOIN effects on the labor supplied by workers and the number of workers demanded by employers is ambiguous. A higher wage through the wage subsidy or special public job may cause workers to increase or decrease their time at work. The surtax on other family earnings might cause the second earner to reduce his work time but the surtax on family unearned income could raise the work time of all family members. Demand for workers by conventional employers could rise if the JOIN wage subsidy allows employers to pay lower wages. But employment demand could also fall since the public employment component of JOIN might increase the wages employers must pay.

These worker and employer responses could exert a considerable impact on the government costs and the income gains that result from the JOIN program. In order to take such responses into account, the paper utilizes an econometric model of the market for labor. The model includes equations representing how workers react to wage and income changes and how employers react to wage changes. The model also uses the notion that adjustments in the market yield a wage rate at which the amount of labor workers are willing to provide is equal to the amount of labor firms are willing to employ. The model offers a systematic approach to estimating the impact of JOIN on changes in labor supply and demand, which in turn determine changes in wage rates, conventional employment, and special public employment.

A combined program of wage subsidies, public employment and income guarantees has the potential for transferring a work subsidy to participants without dissipating the benefits of the subsidy to non-participants through higher market wage rates or displacing them with lower wage rates. The reason for this potential is that the wage subsidy would tend to increase manhours supplied to the private sector while the income guarantee and public employment would tend to work in the opposite direction. If this potential is realized then the program would be labor market neutral. By this we mean that it would transfer benefits to participants through the fiscal process without also transferring benefits or costs to non-participants through the labor market. If the program is truly market neutral, then the costs are borne solely through the fiscal mechanism.

The primary objective of this paper is to determine the labor market effects of JOIN, in particular the degree to which the program would be market neutral. A detailed description and analysis of JOIN appears elsewhere.[4] In this paper we simulate a JOIN program with the same structure but with a higher level of family benefits. We simulate an alternative set of program parameters both because there is room for reasonable debate regarding the appropriate parameters and because we believe that within limits the scale of benefits is less important than the structure of the program in determining market neutrality. Moreover, since we are concerned with the market effects of a combined wage subsidy, public employment, and income guarantee program, we believe that these effects will stand out more clearly with higher benefit levels. Lower benefits would in general only diminish labor market effects.

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[4] Robert I. Lerman, "JOIN: A Jobs and Income Program for American Families," Studies in Public Welfare, Paper No. 19, Public Employment and Wage Subsidies, Joint Economic Committee, (Washington, D.C.: U.S. Government Printing Office, 1975).

The next section describes the JOIN program. Following this description is a translation of the program's benefit structure into effects on wages and non-employment income. How JOIN influences the wage rates and non-employment income of each family type largely determines JOIN's impact on labor supply. The fourth section explains the State labor market model used to simulate JOIN. The simulations provide predictions of how JOIN would affect wages, employment, incomes, and program payments to participants if introduced in 1976. The next section reports and interprets these simulation results and points out their limitations. The final section presents a summary and conclusions.

II. A DESCRIPTION OF JOIN

The Jobs and Income (JOIN) Program is a comprehensive one embodying wage subsidies, guaranteed public jobs, and income guarantees. JOIN is universal in the sense that all families and individuals are potentially eligible for benefits, and categorical in the sense that benefit generosity depends partly on family type. This section outlines the basic financial structure of the program.

All families or individuals fall into one and only one filing unit group. The five filing unit categories are: 1) two -parent families with at least one child under age 18; 2) one-parent families with at least one child under age 18 and no children under age 14; 3) one-parent families with at least one child under age 14; 4) childless married couples; and 5) single individuals over age 18. Filing units 1, 2, and 3 exclude all family members other than a parent, spouse, or child under age 18. Children age 18 and over are not in the same unit with their parents, whether or not they live together.

Each filing unit is eligible for one wage subsidy or one public job. Filing units in category 3 are also eligible for an income guarantee. Finally all filing units are subject to a surtax based on total family earnings and total family non-employment income.[5] All filing units could designate one and only one person 18 or over to receive the wage subsidy or public job. In the version of JOIN with higher family benefits considered in this paper, if the person worked in a private job or in a conventional public job paying between $2.10 and $4.00 per hour, he would be eligible for a wage subsidy equalling one-half the difference between \$4.00 and the person's wage. For example, a worker earning \$2.50 per hour would receive a \$.75 per hour subsidy, thereby raising his total wage to \$3.25 per hour. As the worker's net wage varied from \$2.10 to \$4.00, his wage subsidy would decline but his gross wage would rise. If the applicant's job paid less than \$2.10 per hour, he would be ineligible for the wage subsidy, but he could work at a special public job paying \$3.00 per hour.

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[5] The JOIN plan outlined in the paper by Lerman (see footnote 4) includes a provision for transforming the current $750 personal income tax exemption into refundable tax credits. The value of the tax credits could equal about $200 per person with no less in federal tax revenues. Because the complexities of integrating the income tax changes into the model would have required substantial time and money, the simulations included in this paper do not take account of this provision.

Group 3 filing units would be eligible for an income guarantee in addition to the wage subsidy or job guarantee. The income guarantee would depend on family size as follows:

Family Size	Income Guarantee
2	\$ 2800
3	3100
4	3400
5	3700
6 or more	3900

All filing units receiving JOIN benefits would be subject to a surtax. The following surtax formula would apply:

$$(1) \qquad T = .25(E_1 + E_2 + S - D_i) + .5Y^n, \qquad \text{for } i=1,2,3,4,$$

$$T = .35(E_1 + E_2 + S) + .5Y^n \qquad \text{for } i = 5$$

where $0 < T < E_2 + S + G$

and T = the surtax payment;

E_1 = total annual family earnings other than earnings from the special public job;

E_2 = annual dollar earnings from the special public job;

S = the wage subsidy payment, in dollars per year;

Y^n = annual family non-employment income other than JOIN benefits;

G = the annual income guarantee available to group 3, and;

D_i = the annual earnings disregard that applies to filing unit group i.

The disregard parameters for 1976 are:

$$D_1 = D_2 = D_4 = \$6600, \quad \text{and } D_3 = D_5 = 0.$$

The surtax formulas are identical for groups except primarily for differences in the amount of earnings disregarded. For example, two-parent families with children under 18 would pay no surtax on family earnings below $6600 while single parent families with children under 14 would be subject to a 25 percent surtax on all family earnings. The effect of the zero disregard on single individuals (group 5) is to render their wage subsidy alternative unprofitable. Single individuals would not choose to apply for a wage subsidy, because at wage rates between $2.10 and $4.00, their surtax would always equal or exceed their wage subsidy. Group 3 filing units would find the wage subsidy profitable in spite of the zero earnings disregard. For example, at a private wage of $2.50, a working mother heading a family and receiving an income guarantee would face a $1.88 effective wage without the wage subsidy and a$2.44 effective wage with the subsidy (.75 x 3.25).

One may derive two expressions for the net income of JOIN recipients, after benefits and surtaxes. These formulas are:

$$\text{(2)} \qquad Y_1 = E_1 + E_2 + S + .5Y^n + G \qquad \text{for } i = 1,2,3,4 \text{ where } E_1 + E_2 + S \leq D_1,$$

$$Y_1 = .75(E_1 + E_2 + S) + .25D_i + .5Y^n + G \qquad \text{for } i = 1,2,3,4 \text{ where } E_1 + E_2 + S \geq D_i.$$

and

$$(3) \qquad Y_1 = .65(E_1 + E_2 + S) + .5Y^n \qquad \text{for } i = 5,$$

with Y_1 = total annual family income of JOIN recipients after benefits and surtax payments. Equation (2) applies to those filing units whose earnings are below the earnings disregard levels and therefore not subject to any surtax. Such units would still have to pay a surtax equal to one-half of non-employment income. Equation (3) applies to units receiving JOIN benefits whose earnings are above the unit's disregard level. Since the disregard level for groups 3 and 5 is zero, each dollar of earnings is subject to the surtax and therefore equation (3) always applies. Equations (2) and (3) cover only those units whose net income after JOIN benefits and surtaxes exceeds their net income from earned and non-employment income.

This description is sufficient for analyzing how JOIN influences the individual's total supply of labor, but not the distribution of labor between the public and private sectors. The distribution depends partly on the nature of the public jobs. Although many of the jobs would fill public needs not met currently, some of their work would undoubtedly be similar to work that conventional public employees might perform.[6] In other words, governments may use JOIN funds to substitute special public workers for conventional workers on the public payroll. Thus, the next expansion in public jobs would be less than the number of jobs funded by JOIN. In this situation, however, we assume for the sake of simplicity that there is no substitution of special public employees for conventional ones, so that all public jobs funded by JOIN add to the total demand for labor.

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[6] The substitution of special public workers for conventional public workers may occur in a variety of ways. One is simply to replace low-skill conventional public workers will low-skill special public workers. A second is to alter the mix of labor toward low-skill and away from medium and high skill workers to perform a given task. A third is to replace projects of conventional public agencies with projects performed by outside project sponsors using special public workers that accomplish the same goal.

III. THE IMPACT OF JOIN ON LABOR SUPPLY

The first step in simulating the effects of the JOIN program is to analyze how recipients would change the amount they work. A family member or individual might have a larger or smaller probability of working and might work longer or shorter hours as a result of the JOIN program. The wage subsidy and special public jobs would improve the wage opportunities for many JOIN recipients. But the higher wage both increases the return to added hours of work, inducing an increase in labor supply, and allows workers to afford added leisure, inducing a decrease in labor supply. The JOIN surtax provisions might also raise or lower work effort. The surtax on non-employment income reduces the family's ability to afford leisure while the surtax on earnings causes a decline in the effective wage rate for some workers in JOIN filing units.

These considerations alone suggest that the analysis of labor supply changes is a complex and difficult one. This section describes in as simple a fashion as possible the techniques used in this paper to estimate such labor supply changes. Unfortunately, the presentation is of necessity somewhat technical. Thus, those readers who are not interested in the analytical approach may want to skim this section and move to the following sections.

The labor supply analysis draws on economic theory and on estimates of how workers currently respond to wage and income changes. Economic theory suggests that the amount individuals choose to work depends primarily on their wage rate, on their non-employment income, and on their preferences for leisure and income. Faced with a wage rate and a level of non-employment income that is beyond the worker's control in the short run, it is assumed that the worker chooses that amount of work time which maximizes his satisfaction level. JOIN has a potentially important effect on this choice since it influences the worker's wage and non-employment income opportunities. Estimating JOIN's impact first requires specifying exactly how the program would alter each person's wage and income opportunities. Then, using statistically derived relationships between the amounts different people work and their wage rates, non-employment income, and some other factors, one can estimate how wage and non-employment income changes induced by JOIN would influence the amount recipients work.

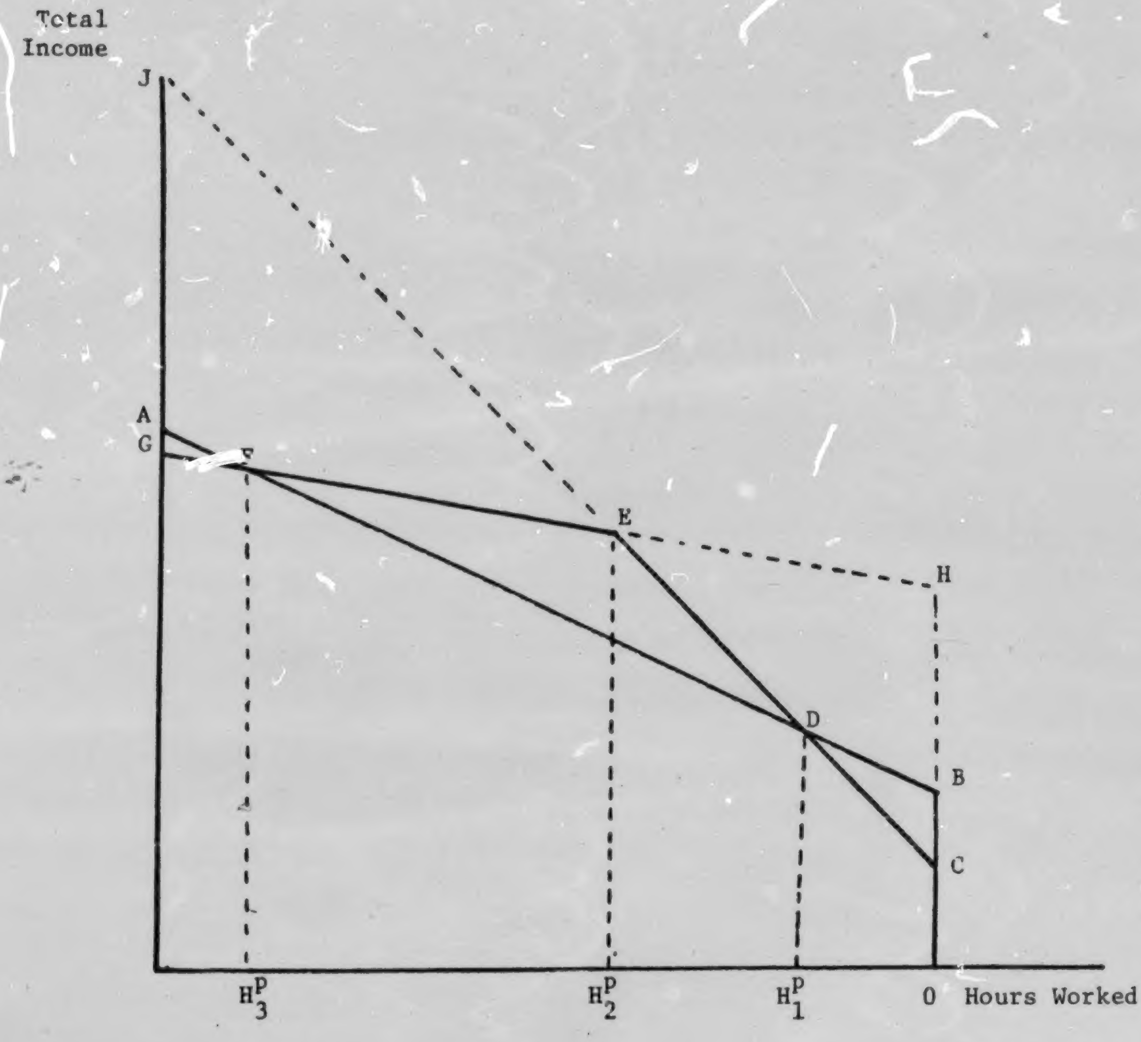
This way in which JOIN alters a particular filing unit's wage and income opportunities depends on the wage rates of the filing unit members, the category of the filing unit, and the filing unit's non-employment income. For exposition purposes it is worthwhile to begin with the case of a multi-person filing unit with a single worker.

The object is to consider how JOIN alters the worker's wage rate and non-employment income at every level of hours worked. As noted above, JOIN offers benefits in the form of a wage subsidy, a public job at a fixed wage rate, and/or an income guarantee to category 3 filing units. But JOIN applies a surtax on earnings and non-employment income that may be charged against benefit payments. Except for category 3 units, workers are eligible for JOIN benefits only if their wage rate is below the target wage under the wage subsidy.

Consider first those units whose wage rates are in the wage subsidy range. Figure 1 illustrates such a worker's alternatives for income and hours worked with and without the JOIN program. Corresponding to the lines in Figure 1 indicating the worker's options are equations that relate total income to the level of hours worked. Without the JOIN program, the worker's income is based

Figure 1

Budget Constraint Of A Single Worker With And Without The JOIN Wage Subsidy *



* This figure applies to JOIN family types 1, 2, and 4. See Figure 2 for the budget constraint for type 3. Type 5 families are not eligible for the wage subsidy.

on the following equation:

$$(4) \qquad Y = W^{p}H^{p} + Y^{n},$$

where Y = total income in dollars per year;

W^{p} = the hourly wage rate of the primary worker;

H^{p} = annual hours worked by the primary worker, and

Y^{n} = annual non-employment income of the filing unit.

Line AB illustrates this equation. Eligibility for the JOIN program may increase the worker's wage and non-employment income. At earnings levels below the filing unit's earnings disregard under the JOIN surtax, the worker's income is based on the following equation:

$$(5) \qquad Y = (2.00 + .5W^{p})H^{p} + .5Y^{n}.$$

Segment CE illustrates this equation. Note that at work hours below point D the worker attains a higher income by remaining out of JOIN. This is because the surtax applied to his non-employment income is larger than the benefit from the wage subsidy payments. As work hours increase beyond H_1^{p}, the worker gains by receiving the wage subsidy even after paying the JOIN surtax. The hours level at which participating in JOIN increases family income is determined by setting income in equations (4) and (5) equal and solving for hours to obtain the following equation:

$$(6) \qquad H_1^{p} = \mathrm{Max}|0,\ (.5Y^{n})/(2.00 - .5W^{p})|.$$

If non-employment income is zero, then JOIN becomes immediately profitable.

A third segment applies to those JOIN filing units whose earnings exceed the JOIN disregard. Such families face a 25 percent surtax on earnings and a tax on non-employment income, except type 5 families for whom the surtax is 35 percent. At hours yielding earnings above the filing unit's disregard, D_i, family income equals:

$$(7) \qquad Y = (1 - 0.25)(2.00 + .50W^p)H^p + .5Y^n + .25D_i .$$

Line GFEH illustrates this equation. The line is applicable to the individual at hours worked levels between points E and F. In this range of hours worked, the JOIN subsidy is financially profitable but the surtax on added earnings applies. The hours level at point E equals:

$$(8) \qquad H_2^p = \frac{D_i}{2.00 + .5W^p} .$$

At hours levels beyond point F, the JOIN program becomes no longer profitable for the worker. The fact that segment AF lies above segment GF illustrates that income is higher without the JOIN benefits and JOIN surtax than with them. Segment AF lies along line AB, which represents equation (4). The hours level at point F, at which income is equal whether or not the unit participates in JOIN, is equal to:

$$(9) \qquad H_3^p = (.25D - .5Y^n)/[W^p - .75(2.00 - .5W^p)].$$

In summary, one may see how the JOIN program can raise the income attainable over some ranges of hours worked by comparing line BDEFA with line BA. The problem of the labor supply analysis is to determine how the new options illustrated by segment DEF would influence the worker's choice of hours worked. According to economic theory, the worker will choose the point which maximizes his satisfaction level. Given the worker's preferences for leisure and income, it is possible to derive a general expression for how the hours level he chooses depends on his wage rate and his level of non-employment income. In the absence of JOIN, this expression is:

$$(10)\qquad H^P = h^P(W^P, Y^P),$$

where h^P is a function the form of which will be estimated later in this paper. In principle, one could easily determine h^P by simply plugging into (10) the worker's new wage rate and level of non-employment income under JOIN. The problem is that the worker's effective wage, (his net increase in income for an added hour worked) and his net non-employment income level depend on the hours range the worker chooses. That is, there is no single wage rate and non-employment income level appropriate for all hours levels.

A general solution to this problem is to assume that workers make only small changes in hours worked in response to JOIN. Under this assumption, workers whose original level of hours worked was observed in a particular region would choose a new hours level based on effective wages and non-employment incomes relevant to that range. For example, workers supplying hours in the AF and DB ranges would face the same wage rates and non-employment income levels after JOIN and would be expected not to change their hours choice. But effective wage rates and non-employment incomes would differ over the hours range between H_1^P and H_3^P. Those whose original hours levels were between H_1^P and H_2^P would choose hours levels under the JOIN program based on the following equation:

$$(11)\qquad H^P = h^P((2.00 + .5W^P), .5Y^n).$$

The workers in this range would face a new effective wage, $2.00 + .5W^P$, and a new effective non-employment income level, $.5Y^n$. Those whose original hours levels were between H_2^P and H_3^P would choose hours levels under JOIN based on the following equation:

(12) $$H^P = h^P(.75(2.00 + .5W^P), .5Y^n + .25D_i).$$

For this group, the effective wage is $.75(2.00 + .5W^P)$ or three-quarters of the effective wage for the group in range DE. The reduction results from the application of the surtax on earnings. The group would also act as if its non-employment income were equal to $.5Y^n + .25D_i$, or OH in Figure 1. To the worker choosing over range, DE, JOIN acts as if it places a surtax on all his earnings and as if it provides an income transfer equal to the surtax not paid on the first D_i dollars of income.

One aspect of the JOIN program is that at certain levels of income workers are confronted with a discontinuous increase in marginal tax rates. Labor supply theory says that under these circumstances workers at a variety of wage rates will work just the number of hours that maintains their earnings at the level where the tax rate increases. This result occurs in the transition between receipt of the wage subsidy with no surtax on earnings, segment DE, and a wage subsidy in which earnings are subject to the surtax, segment EF. Point E is a peak which represents the best choice for workers at many wage rates. This point represents the hours level at which earnings are just equal to the earnings disregard, D_i.

Although the general approach of using equations (7), (8), and (9) is appropriate in many cases, this procedure can yield incorrect results. One problem occurs for those whose predicted hours are near points D and F. For this group, it is clearly inappropriate to base predictions as if they were faced with only a single set of wage rate, non-employment income opportunities. It can be demonstrated that workers who would appear to choose points very near D and F if their choice were based on only one option would actually choose other points if confronted with the entire range of JOIN options.

Unfortunately, it is impossible without substantial information about individual preferences to determine which option or which range of hours worked would yield the best choice for any individual. However, there is a presumption that points near D and F are inferior to points further away from D and F. Thus, the choice predicted in this simulation is the one further away from D or F. To determine which point is closer requires using a ratio test that compares, for example, the hours levels at point D with the hours levels chosen using equation (10), segment BD, and equation (11), segment DE.

Although the analysis discussed above covers only single worker filing units who are eligible for a wage subsidy, the same techniques are applicable to other situations. Consider workers who would be ineligible for the wage subsidy because their highest wage was less than $2.10 per hour. This group would be eligible for a special public job paying $3.00 per hour. Figure 1 again illustrates the options facing such workers in filing units with only one worker. But the equations would have to be adjusted. Equations (4) and (10) would remain the same since they represent the individual's wage and non-employment income in the absence of JOIN. In equations (5) and (11), one would replace the effective hourly wage term, $2.00 + .5W^P$, with the figure $3,00. This would cover the range in which none of the workers' earnings would be subject to the surtax. In equations (7) and (12), one would again replace the effective wage term, $.75(2.00 + .50W^P)$, with the figure $2.25. Over the hours range covered by these equations, earnings are subject to a 25 percent surtax, thereby reducing the public employment wage from $3.00 to an effective wage of $2.25.

JOIN has a slightly different effect on the budget constraint of category 3 filing units. Since this group may receive an income guarantee, they receive an immediate benefit of $G - .5Y^n$ at zero hours of work (G denotes the dollar guarantee level). Those in this category would find their options altered by JOIN to CEFA in Figure 2. BC represents the guarantee minus one-half other non-employment income and we assume that this number is positive. Notice that the worker in Figure 2 will participate in JOIN unless hours exceed H_3^P.

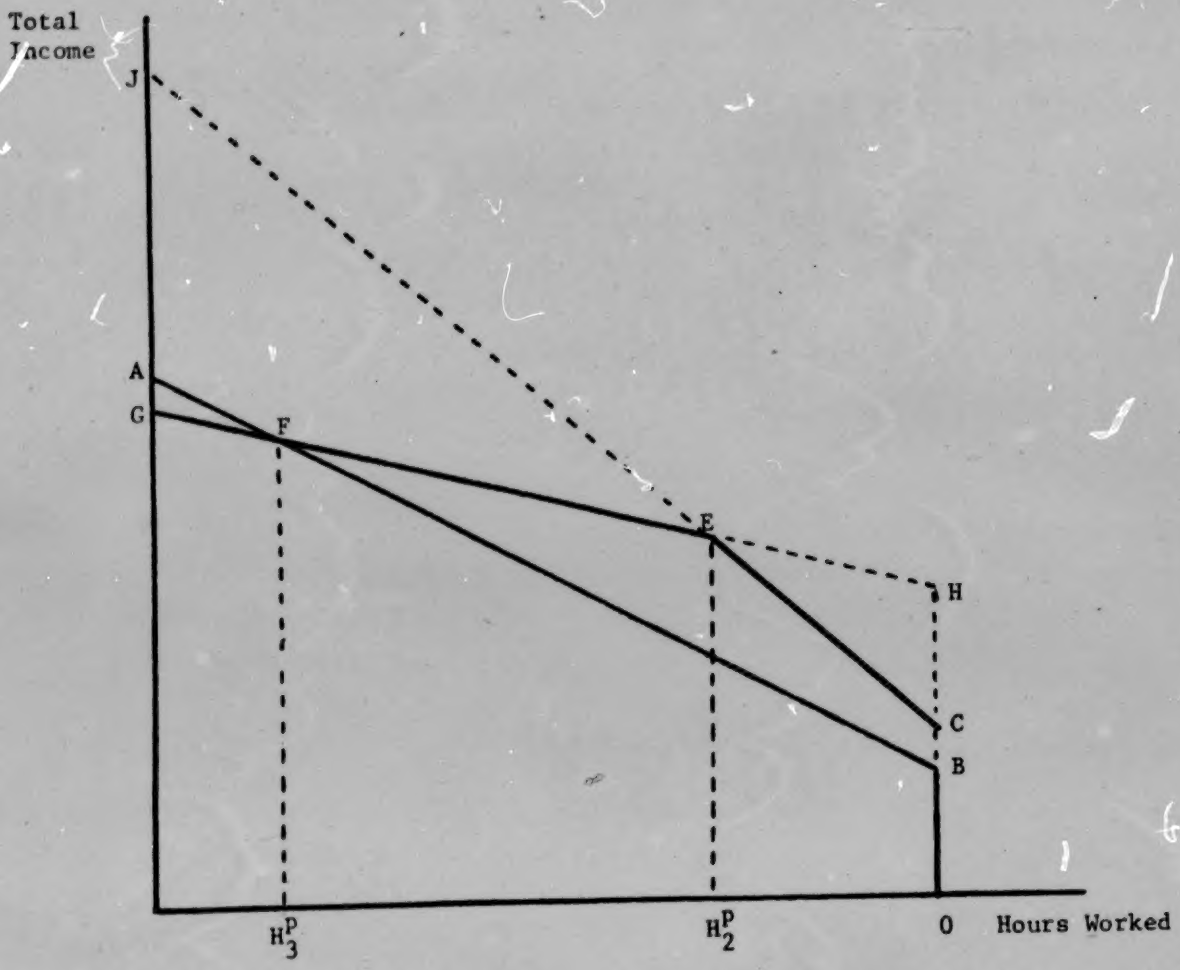
Single individuals (category 5) would find that part of Figure 1 describes their budget constraint. As noted above, only the special public job is potentially advantageous for this group. If the category 5 worker's best wage were less than \$2.25, then JOIN might prove profitable. The worker's options under JOIN would resemble BDEJ in Figure 1. At hours levels under H_1^P, the surtax on the worker's non-employment income would exceed the effective earnings gain from the \$3.00 public job less the 25 percent surtax on earnings. Beyond H_1^P, the worker's effective wage would be \$2.25 and he would participate in public employment.

The important groups not covered by the preceding discussion are those filing units with more than one worker. These cases are highly complex to analyze, but the same general techniques are applicable. Although the analysis of this case is left for the appendix, the reader should be aware of the general approach. One basic assumption is that the filing unit acts so as to maximize its satisfaction, where its satisfaction level depends on total income of the unit and the leisure of each of its members. It follows that one worker's choice concerning his work time depends partly on the time spent at work and the earnings of the other worker. The filing unit has a broad range of options since either worker may accept the work subsidy. Presumably, the unit will choose the hours of work for each member, and choose the person who is to receive the subsidy that maximizes satisfaction for the filing unit as a whole. The analytical tech-

Figure 2

Budget Constraint Of A Single Worker In A JOIN Category

Three Family With And Without A JOIN Wage Subsidy


Total Income
J
A
G
F
E
H
C
B
H_3^P
H_2^P
0
Hours Worked

niques described in the appendix attempt to simulate those decisions.

IV. USING THE STATE LABOR MARKET MODEL TO SIMULATE JOIN

A comprehensive analysis of JOIN requires an assessment of its impact on the hours people work and on the wages employers pay. If workers eligible to receive the JOIN wage subsidy increase their time at work, JOIN's cost to the government may rise as a result of the additional hours subsidized. In addition, the subsidy cost per hour will rise if the increase in the labor force causes a fall in the wages employers pay. Alternatively, employers may have to increase their wage offers in order to retain workers who otherwise might accept JOIN's public job guarantee. Increased wage rates, in turn, will reduce JOIN costs as more people stay in private employment and fewer people go into special public jobs.

Use of a State labor market model allows one to simulate how JOIN would influence wage rates paid in private employment, the total employment levels in public and private employment, the extent to which JOIN draws new workers into the labor force, and JOIN's benefits to recipients and nonrecipients. The model represents the wage determination process in each State of the United States with a system of equations.[7] One set of equations relates the way work hours supplied by workers depend on their wage rates, their non-employment income, their dependents, and their other personal characteristics. Another equation expresses the relationship between the amount of labor demanded by firms, and the wages they pay, and the overall level of output. The model determines the market wage by finding that wage rate which equates labor supplied and labor demanded.

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[7] See Peter M. Greenston and C. Duncan MacRae, "Labor Markets, Human Capital, and the Structure of Earnings," pp. 1.1-1.75.

In common sense terms, the model assumes that the hours which persons of each demographic type choose to work are a function of the wage rate they can obtain and their non-employment income. In other words, it assumes that Americans in every State have the same relative preferences between income and leisure, and that they behave differently in different States only because of variations in the wage rate and the availability of non-employment income. Similarly, the model assumes that the number of labor hours which employers in each State wish to purchase depends on the level and composition of State output and the wage rate prevailing in the State. In other words, the model assumes that given similar output composition, employers in each State will vary their demand for labor hours per dollar of output solely on the basis of the prevailing wage rates in the State. Finally, the model assumes that employers enjoy perfect flexibility in substituting workers of various skill levels for one another. As a consequence, the model assumes that the effect of a rise in the wage rate is that employers will choose to substitute capital for labor. Given these assumptions, the State labor market model tells us how American workers and employers can be expected to respond to variations in wage rates.

To simulate the effects of JOIN, one first specifies how JOIN would affect the wage and income opportunities of all families and individuals. The previous section performed this task. Next, one plugs these new effective wage and income figures into the labor supply equations derived from State-by-State analysis to determine their effect of JOIN on the amount people work and on the numbers of worker remaining in private jobs and the numbers taking the special public jobs offered under JOIN. This next step does not take account of the feedback effect of changes in labor supply on the wages employers pay. If JOIN changes the amount of labor supplied to private firms, the pre-JOIN wage will no longer result in equality between labor supplied and labor demanded. Thus, the model must find the new wage that will bring supply and demand into equality. As the model solves new for the wage level, it simultaneously determines the amount people work in private and in public employment, their total earnings, and their total subsidies from JOIN.

To understand the logic of the model requires an examination of its equations and how they describe the behavior of workers and firms. Consider first the labor supply equations. Since different population groups are expected to respond differently to changes in wage rates and non-employment income, separate equations are necessary for each group. For example, family heads vary their work patterns in different ways than wives do; young persons in different ways than old persons; and well educated workers in different ways than less educated workers. Of primary importance is the distinction between labor supplied by the primary worker and labor supplied by secondary workers in each family. The other distinctions of significance are between different types of families. To determine which types of families show similar labor supply behavior, a special statistical technique was employed to select the groupings on the basis of minimum within-group wage variation and maximum between-group variation. This technique classified all families into eight groups based on the characteristics of the family head. The most relevant groupings were found to be families headed by those with the following characteristics:

1) male and female, age 36 and over, 16 or more years of schooling completed;
2) male and female, age 16-35, 16 or more years of schooling completed;
3) male, age 36 and over, 0-11 years of schooling completed;
4) male, age 36 and over, 12 years of schooling completed;
5) male, age 36 and over, 13-15 years of schooling completed;
6) male, age 16-35, 0-15 years of schooling completed;
7) female, age 36 and over, 0-15 years of schooling completed; and
8) female, age 16-35, 0-15 years of schooling completed.

The model of labor supply behavior consists of separate equations for primary and secondary workers in each of the eight family types. The equations explain the variation across 30 States and groups of States in the average number of hours worked per family per year on the basis of economic and demographic variables. For example, State variations in average hours worked by secondary workers of a particular family type depend on variations in their average wage, in the average wage of the primary worker in that family type, in the family's non-employment income plus the earnings of the primary workers, and variations in the percentage of secondary workers who are white, between age 22 and 54, male, living with a spouse, responsible for dependents, and residing in an urban area. Econometric estimates of the relationship between hours worked and the explanatory variables use data on each variable derived from the 1970 Census Public Use Sample of 1 of every 1000 households. The estimated relationships determine the quantitative significance of each of the variables. One finding is that State variations in wages of secondary workers in some family types stimulate significant increases in hours worked while for secondary workers in other family types wages have no discernible impact on hours worked.

Although separate estimates of labor supply relationships were performed for sixteen groups (primary and secondary workers in eight family types), the results were similar for some groups. In fact, only three separate equations were necessary to explain variations in hours worked of secondary workers. Behavior of primary workers differed sufficiently by family type to require seven separate equations. In most of the equations, it was found that higher wages and lower non-employment income induce longer work hours. In some cases, work hours increase as wages increase only to a certain level after which further wage increases appear to reduce or to leave average work hours constant.

Equations representing the demand for workers also take account of worker differences. But for employers, the relevant differences are those affecting worker productivity rather than those affecting family status. The demand for college graduates will clearly differ from the demand for workers without a high school education. In order to simplify the analysis, the model translates hours worked by different types of workers into equivalent productivity units as measured by relative wages. The hours worked by one group of workers is used as the basis of comparison. Hours worked by all other groups become translated into hours of the base (or numeraire) group on the basis of wage differences. The base group is male, primary workers, high school graduates, over age 36. One hour of work by male college graduates over 36 might become 2 hours in terms of the base group's hours. Alternatively, the value in numeraire hours of an hour worked by males with less than a high school education might be only one-half an hour. The precise ratio of one group's hours to the numeraire's hours is equal to the ratio of the wage rates. In theory, relative wages are equal to relative productivities when markets are in equilibrium. To determine which relative wage weights to assign to workers of different productivity in all State areas, an equation relating relative wage rates to age-sex-education characteristics was estimated.[8] Although some differences in relative wage rates appeared between southern and non-southern States, relative wage rates are reasonably constant over most areas. Using the wages derived in the relative wage equations, hours worked by those with different productivities were aggregated into a single hours measure.

The demand for labor part of the model is an equation intended to represent the impact of wage rates on employers' demand for workers. The actual equation relates State variations in the number of equivalent-quality hours worked (per dollar of State output) to State variations in wage rates of the base group and to State variations in the proportion of state output in manufacturing and construction. The estimated equation is based on 1970 data. The results indicate that a one percent higher wage of the base group is associated with about a one percent lower demand for labor hours. The output proportion variable is not statistically significant.

The demand for labor equation along with the condition that hours demanded equal hours supplied completes the model. An iterative solution technique solves for the wage for the base group that equates supply and demand. The wages of other groups of workers follow directly from the ratio of their wage rates to the wage rate of the base group. One important assumption embodied in the model's demand equation is that workers of different skill levels are perfectly substitutable at some fixed ratio. That is, if the wage level of a high skill is three times that of a low skill worker, firms may substitute three low skill workers for one high skill worker at no loss or gain in output. This assumption implies that relative wage rates of different classes of workers are fixed. Thus, JOIN's impact on the wages of one class of workers, say the base group, will have an equal percentage effect on wages of all other classes of workers. By assumption, JOIN cannot improve or worsen the relative position of low wage workers as measured by their market wage rate relative to that of other workers.

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[8] See pp. 1.26-1.46 for the exact form of this equation and for the estimated regression results.

One problem in using the State labor market model to simulate the effects of JOIN is the difference in units of analysis. JOIN eligibility criteria for various filing unit categories are based on the nuclear family and on legal responsibility. One filing unit group consists of both parents and all children under eighteen. Children 18 and over and other household members not married to the family head are in different filing units. In contrast, the family unit used in the model and in most labor supply analysis includes all household members related to the head, regardless of their ages. The solution adopted here is to treat all secondary family workers not married to the primary worker as eligible for JOIN. This assumes that secondary workers in families with no spouse present are not under age 18 and that the secondary worker in families with a spouse present is the spouse.

A second problem arises because JOIN is universal but the State labor market model includes only the civilian non-agricultural labor force. Since wage rates in the agricultural sector are low, a significant number of workers excluded from this analysis may in fact be eligible for JOIN benefits.

V SIMULATION RESULTS AND IMPLICATIONS

We have discussed the inputs into the simulation process: labor supply analysis of JOIN and the structure of the State labor market model. More detailed discussions of both appear in the Appendices. This section is divided into a discussion of assumptions made to update the model to make forecasts for 1976 and an analysis of macro and micro effects of the JOIN program.

The program parameters of JOIN are designed to apply to a program implemented in 1976. Basic exogenous inputs into the State labor market model were updated to 1976 by projecting the growth of output, unearned income, population, and labor productivity from the 1970 Census data base used to estimate the model. Furthermore, since all monetary values (wage rates, earnings, income and output) in the econometric model are expressed in 1969 dollars, the parameters of the 1976 JOIN were first converted to 1969 dollar values. This is accomplished by multiplying them by the ratio of the 1969 GNP deflator to an estimate of the 1976 GNP deflator (obtained from the National Planning Association).[9] Then the above mentioned variables are projected into 1976 and the model is simulated with and without the program. Finally, the predicted values are translated into 1976 dollar values by multiplying them by the ratio of the 1976 GNP deflator to the 1969 GNP deflator. The projections are based on the following trends and assumptions. Nonfarm output is assumed to grow in all states at the same rate as national output, which has been projected to increase from $837.3 billion in 1973 to $905.0 billion in 1976 (in constant 1958 dollars), or a real growth of 24.7 percent from 1969 to 1976.[10] Corresponding to this growth in real output, unearned income is projected to grow at the same rate. The number of families in each state is assumed to continue to grow at the same rate of 1.48 percent a year which occurred over the period 1960-73. The demographic composition variables are assumed to remain at their 1969 values.

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[9] Forecasts of the GNP Implicit Price Deflator are contained in National Planning Association, The U.S. Economy: 1973-1983, Washington, D.C., 1974.

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Labor productivity increased at a 3.1 percent annual rate over the period 1969-73 and is projected to grow in 1974-76 at the same 3.0 percent rate that prevailed in the last two decades.[11] The effect of the labor embodied technical progress is both to increase the number of equivalent-quality hours and to reduce the average cost of labor to firms. Since quality per man hour is assumed to increase by 22.7 percent over 1969-76, firms obtain 1.227 (H_j^d) equivalent-quality hours at a market wage of W_{4j}^p so that the average cost of labor to firms is only $W_{4j}^p/1.227$. The reduction in average cost will just absorb the increase in effective supply since demand is slightly elastic. Therefore, on net, the effect of increased labor productivity is to increase wage rates slightly.

The predicted values of the model are reported by JOIN category. But some individuals who are classified by JOIN as eligible category 5 families, are actually members of extended family units living together. Such individuals appear in our tables as secondary workers and are associated with the JOIN category appropriate for their family head.

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[10] The 1976 estimate is a Chase Econometric forecast.

[11] Productivity trends are discussed in Manpower Report of the President, Washington, D.C., March 1973, pp. 11-12.

Insofar as such extended families pool income from all sources, it would be misleading to list family members under two JOIN categories, and give the impression that they were separate units each with rather low income.[12]

For each category we report an average value of primary (W^p) and secondary (W^s) wage rates, hours per family (H^p/F, H^s/F), earnings per family (E^p/F, E^s/F), and family unearned income (Y^n) over all states. Wage rates are weighted by hours, while hours and earnings per family are weighted by families.

In Tables 1 and 2 the pre-program predicted average values by JOIN category for participants and non-participants are reported. In the nation, an average of 4.0 percent of families have primary workers participating while .7 percent of families have secondary workers participating in JOIN. In the absence of the program workers in these families on the average have significantly lower wage rates, levels of employment and unearned income than workers in non-participating families. Thus both their earned and their total income is lower. Nationally, workers from participating families are predicted to earn $2500 per year in contrast to $8100 per year for workers from non-participating families.

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[12] It should be noted that pre-program values for categories 2 and 3 are identical because they differ only in the age of dependents. Since we did not have state-by-state observations on these categories we allocated one-parent families with dependents, which we do observe, between the categories on the basis of the proportions observed for the entire American population.

TABLE 1

PRE-PROGRAM PREDICATED VALUES FOR 1976 BY JOIN CATEGORY, NON-PARTICIPANTS

JOIN CATEGORY	W^p #	W^s	H^p	H^s	E^p	E^s	Y^n	NON-PARTP	NON-PARTS
1	7.80	4.13	1068	266	8337	1098	1287	20712000	20673000
2	8.12	4.32	816	178	6617	770	1421	2445000	2540000
3	8.69	4.32	1149	178	9982	770	1577	3133000	5158000
4	8.12	4.07	960	289	7801	1175	1679	24269000	24416000
5	6.96	4.24	650	152	4526	643	1824	15964000	16964000
6 *	7.84	4.13	919	237	7201	978	1584	57524000	69753000

* Weighted average of 1 through 5.

KEY

W^p - private market wage of primary worker in dollars per man-hour

W^s - private market wage of secondary worker in dollars per man-hour

H^p - annual man-hours of primary worker

H^s - annual man-hours of secondary worker

E^p - annual earnings of primary worker

E^s - annual earnings of secondary worker

Y^n - annual family non-labor income in dollars

NON-PARTP - number of families with no participating primary worker

NON-PARTS - number of families with no participating secondary worker

TABLE 2

PRE-PROGRAM PREDICTED VALUES FOR 1976 BY JOIN CATEGORY, PROGRAM PARTICIPANTS

JOIN CATEGORY	W^p#	W^s	H^p	H^s	E^p	E^s	Y^n	PARTP	PARTS
1	3.21	2.48	1521	222	4887	550	871	268000	307000
2	3.00	2.16	1271	273	3804	593	412	111000	16000
3	3.31	2.16	359	273	1191	593	1076	2059000	33000
4	3.12	2.48	1331	219	4150	543	785	258000	110000
5	--	--	0	0	0	0	0	0	0
6 *	3.22	2.44	607	226	1954	553	983	2697000	468000

* Weighted average of 1 through 5.

KEY

All notation is as described in Table 1 except:

PARTP - number of families with a primary worker participating

PARTS - number of families with a secondary worker participating

The post-program predicted values of the annual government cost per family (COSTP, COSTS) and annual subsidy per family (SUBP, SUBS) are reported along with equilibrium wage rates and annual hours worked in Tables 3 and 4. Government costs include the wage subsidy net of the JOIN tax, the income guarantee cost for category 3 families, and the wage bill in public employment net of JOIN tax on participants. Figures for the annual subsidy per worker for primary and secondary workers indicate the increase in earnings above those determined by private market wage rates. Thus the entire amount of wage subsidy payments and any income guarantee is included in this figure. But, for individuals participating in public employment, the subsidy is an implicit wage subsidy equal to wage differences in public and private employment. Thus our estimate of subsidy is the earnings differential associated with this wage subsidy.

Perhaps the most notable feature of the results on participants in Tables 2 and 4 is the concentration of participation in category 3 families and the lack of category 5 participants. JOIN is very attractive for category 3 families because of the income guarantee which these families receive. Since non-labor income for this group is about $1000, the average value of the income guarantee net of the 50 percent non-labor income JOIN tax will be about $2800 per year. Primary and secondary worker hours tend to be small for category three families which means that the earnings tax does not push these families near the break even level of income even if they have high wages. JOIN is unattractive to category 5 families because of the higher earnings tax which they face, 35 percent as opposed to 25 percent for other categories, and the lack of an earnings disregard. Given a public employment wage of $3.00 the

TABLE 3

POST-PROGRAM PREDICTED VALUES FOR 1976 BY JOIN CATEGORY, NON-PARTICIPANTS

JOIN CATEGORY	W^{P}#	W^{s}	H^{P}	H^{s}	E^{P}	E^{s}	Y^{n}	NON-PARTP	NON-PARTS
1	7.76	4.11	1073	268	8333	1100	1287	20712000	20673000
2	8.07	4.30	819	179	6609	771	1422	2445000	2540000
3	8.63	4.30	1155	176	9973	755	1577	3133000	5158000
4	8.08	4.05	966	290	7806	1175	1679	24269000	24416000
5	6.93	4.21	652	152	4516	640	1824	16964000	16964000
6 *	7.80	4.12	923	237	7198	977	1584	67524000	69753000

* Weighted average of 1 through 5.

# KEY

All notation as described in Table 2.

TABLE 4

POST-PROGRAM PREDICTED VALUES FOR 1976 BY JOIN CATEGORY, PARTICIPANTS

JOIN CATEGORY	W^P#	W^s	H^P	H^s	E^P	E^s	Y^n	COSTP	COSTS	SUBP	SUBS	PARTP	PARTS
1	3.55	3.15	1660	367	5889	1155	870	272	210	272	210	268000	307000
2	3.46	3.04	1335	311	4609	944	411	661	708	470	213	111000	16000
3	3.59	3.04	474	398	1700	1209	1076	2998	907	2932	269	2059000	33000
4	3.50	3.15	1436	370	5033	1164	783	166	211	166	211	258000	110000
5	--	--	0	0	0	0	0	0	0	0	0	0	0
6 *	3.55	3.14	719	368	2557	1154	983	2359	277	2301	214	2697000	468000

* Weighted average of 1 through 5.

# KEY

All notation is as described in Table 2 except:

COSTP - annual government cost per primary worker participating in dollars

COSTS - annual government cost per secondary worker participating in dollars

SUBP - annual subsidy per primary worker participating in dollars

SUBS - annual subsidy per secondary worker participating in dollars after-tax wage available to category five workers choosing to participate in JOIN is only $1.95 per hour. This is generally below private market wages projected to be available to workers in 1976. Also some category 5 families consist of individuals who are members of extended families that are eligible for JOIN under categories 1 through 4. Hours and earnings for such workers appear in descriptions of secondary worker behavior under the category of the head of the extended family.

Changes in the circumstances of families that participate in JOIN and those that do not participate are given in Tables 5 and 6. As can be seen, JOIN is nearly market-neutral in that it has little net effect on market-clearing wage rates. Because of this feature there is little net effect on the behavior or income of non-participant families. Table 6 indicates that, for both primary and secondary workers, there is a slight fall in market wages of non-participating families. But because workers in these families are on the backward-bending portion of their uncompensated labor supply curves, the decline in wages evokes an increase in hours which leaves earnings virtually unchanged.

Families that participate in JOIN show sharply higher earnings and hours of work after the program is implemented. Table 6 indicates that these increases in hours and earnings extend to both primary and secondary workers of family types 1 through 4. But the percentage gains in hours are particularly large for category 3. Given the shape of the underlying labor supply functions, and the design of JOIN, one would anticipate that increases in hours would be most pronounced for the lowest wage workers. These individuals tend to be on the portion of the labor supply curve which has the smallest positive slope and they are unlikely to earn enough to exhaust the value of their disregard from the JOIN earnings

TABLE 5

POST- LESS PRE-PROGRAM DIFFERENCES IN PREDICTED VALUES FOR 1976 BY JOIN CATEGORY, NON-PARTICIPANTS

JOIN CATEGORY	ΔW^P#	ΔW^S	ΔH^P	ΔH^S	ΔE^P	ΔE^S	NON-PARTP	NON-PARTS
1	-0.04	-0.02	+5	+2	- 4	+ 2	20712000	20673000
2	-0.05	-0.02	+3	+1	- 8	+ 1	2445000	2540000
3	-0.06	-0.02	+6	-2	- 9	-15	3133000	5158000
4	-0.04	-0.02	+6	+1	+ 5	0	24269000	24416000
5	-0.03	-0.03	+2	0	-10	- 3	16964000	16964000
6 *	-0.04	-0.01	+4	0	- 3	- 1	67524000	69753000

* Weighted average of 1 through 5.

# KEY

ΔW^P - post-program private market wages of primary workers less pre-program wages

ΔW^S - post-program private market wages of secondary workers less pre-program wages

ΔH^P - post-program annual hours of primary workers less pre-program hours

ΔH^S - post-program annual hours of secondary workers less pre-program hours

ΔE^P - post-program annual earnings of primary workers less pre-program earnings

ΔE^S - post-program annual earnings of secondary workers less pre-program earnings

NON-PARTP - number of families without a primary worker participating

NON-PARTS - number of families without a secondary worker participating

TABLE 6

POST- LESS PRE-PROGRAM DIFFERENCES IN PREDICTED VALUES FOR 1976 BY JOIN CATEGORY, PROGRAM PARTICIPANTS

JOIN CATEGORY	ΔW^P#	ΔW^s	ΔH^P	ΔH^s	ΔE^P	ΔE^s	PARTP	PARTS
1	+.34	+.67	+139	+145	+1002	+605	268000	307000
2	+.46	+.88	+ 64	+ 38	+ 805	+351	111000	16000
3	+.28	+.99	+115	+125	+ 509	+616	2059000	33000
4	+.38	+.67	+105	+151	+ 583	+621	258000	110000
5	--	--	--	--	--	--	0	0
6 *	+.33	+.70	+112	+142	+ 603	+601	2697000	468000

* Weighted sum 1 through 5.

# KEY

All notation is identical to Table 6 except:

PARTP - number of primary workers participating

PARTS - number of secondary workers participating tax. The tendency of the income guarantee to discourage work effort on the part of category 3 families apparently did not outweigh the positive effects of the wage subsidy and public employment on work effort.

In interpreting the results presented above, limitations of the analysis should be recognized. We have assumed a minimum wage in private employment of $2.10 per hour would be permitted in spite of legislation which mandates higher minimum wages by 1976. Of course workers receiving such a wage in private employment would actually get $3.05 per hour including the wage subsidy. Secondly, in analyzing the choice of private vs. public employment, we have ignored the element of job satisfaction and amenity. It may be that workers would accept a lower wage in public employment rather than work in an undesirable job in the private sector. Also we have ignored any role of the employment services associated with JOIN in encouraging workers to accept positions in the private sector. Instead workers consistently choose the position which yields the highest earnings for given hours of work. Finally, we have not dealt with the existence of other taxes and subsidies which would persist, be modified, or be eliminated by the introduction of JOIN.

VI. SUMMARY AND CONCLUSIONS

In this paper we have analyzed the market effects of the high benefit version of the JOIN proposal. We first reviewed the basic financial structure of the program. Each family or individual falls into one out of five possible filing unit categories depending on family composition. Each filing unit is eligible for one wage subsidy or one public job. A worker receiving between $2.10 and $4.00 per hour is eligible for a wage subsidy equalling one-half the difference between $4.00 and his or her wage rate. A worker earning less than $2.10 per hour is ineligible for the wage subsidy but eligible for a special public job paying $3.00 per hour. Single parent families with a child less than 14 years old are also eligible for an income guarantee. All filing units, however, are subject to a surtax based on total family earnings and total family non-employment income.

Since the JOIN program affects both the wage rate and non-employment income opportunities of participants, we next used the theory of family labor supply to analyze the consequent labor supply effects of the program on the different filing unit categories. The analysis focused on specifying how the several program components alter the budget opportunities confronting the family and reflecting these changes in the parameters which determine labor supply behavior. JOIN has been designed to incorporate some features which increase labor supply to the private sector and others which should have the opposite effect. While the wage subsidy and special public jobs would improve the wage opportunities for many JOIN recipients, the higher wage not only increases the return to added hours of work, inducing an increase in labor supply, but also allows workers to afford added leisure, inducing a decrease in labor supply. Moreover, the JOIN surtax provisions might also raise or lower work effort. The surtax on non-employment income reduces the family's ability to afford leisure while the surtax on earnings causes a decline in the effective wage rate for some workers in JOIN filing units.

Given the labor supply effects of the JOIN program on those who are eligible to participate, a model of State labor markets was employed to determine the changes in wage rates and work effort on both participants and nonparticipants brought about by the program through its effect on the market supply and demand for labor. Projecting the proposed program into 1976, we forecast that an average of four percent of families would have primary workers participating, and almost one percent of families would have secondary workers participating. We found, however, that participation would be dominated by single parent families eligible for the income guarantee, and that single individuals would be only occasional participants. The simulation indicated that hours supplied by participating families would tend to increase as a result of the implementation of JOIN. These increases would extend across both primary and secondary workers in all participating family types. However, wages would not fall significantly and non-participating families appear to be little changed by the introduction of JOIN. Therefore, we conclude that the increase in labor supply would be absorbed by the public sector leaving the private sector relatively unaffected. Thus, it seems that the potential for JOIN to be market neutral would be realized and the program could be introduced without causing large readjustments of the non-target population. In other words, the dissipation of benefits through wage increases to the demographically ineligible will be minimal as will be the substitution of subsidized for unsubsidized labor.

APPENDIX - LABOR SUPPLY ANALYSIS OF THE MULTI-WORKER FAMILY

The multi-worker family has two or more potential participants in the labor force. We will consider the case in which there are two workers per family, the primary worker and a single secondary worker. Such a family supplies primary hours, H^p, and secondary hours, H^s, to employment in a manner which maximizes utility of the family as a whole from income, and leisure of family members. The budget constraint of a multi-worker family not participating in JOIN is given by equation (1):

$$Y = W^p H^p + W^s H^s + Y^n, \tag{1}$$

where Y = total family income in dollars per year;

Y^n = total annual non-labor income of the family;

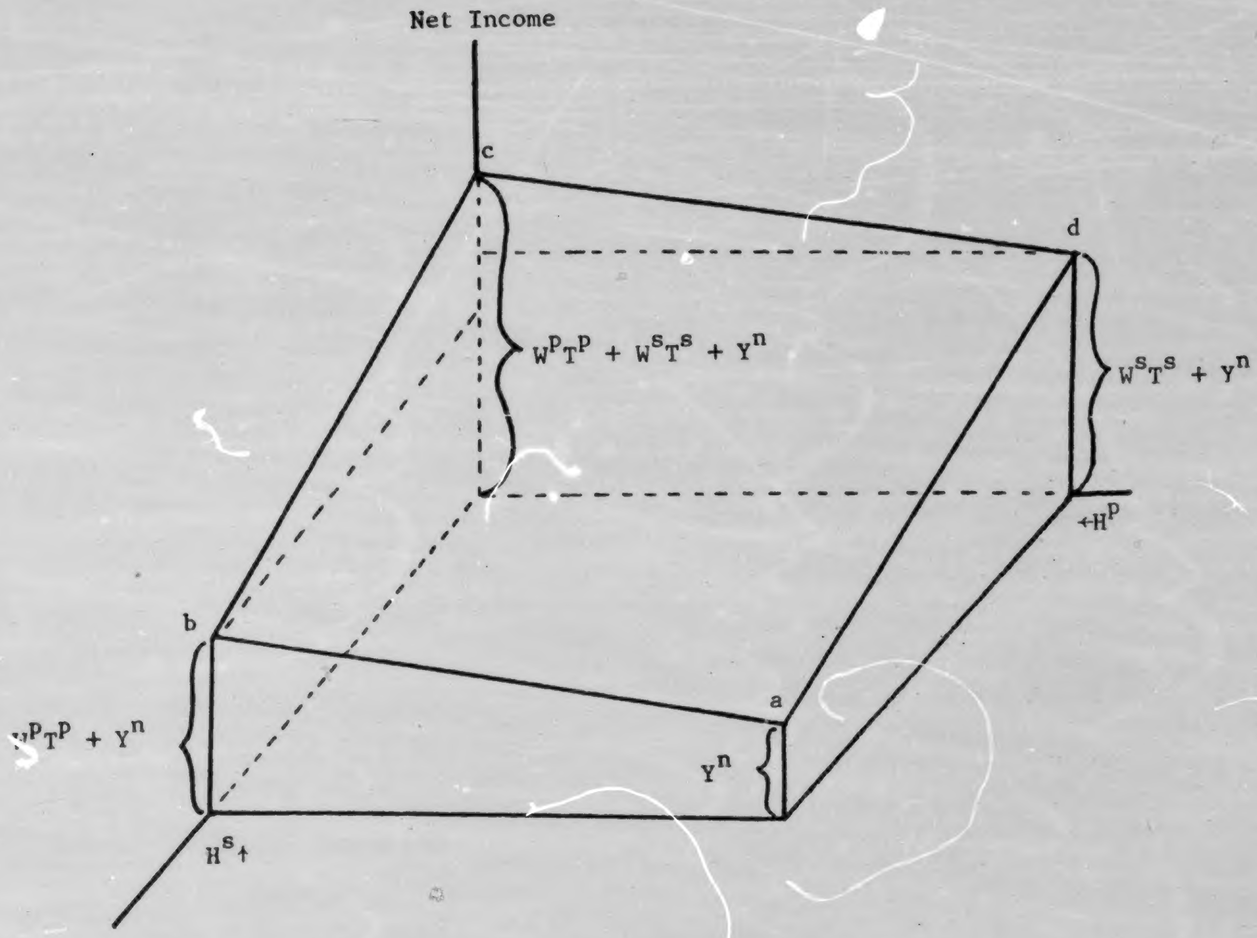
W^p = market wage rate of the primary worker in dollars per man-hour;

W^s = market wage rate of the secondary worker in dollars per man-hour.

We can represent this budget constraint by a plane in (H^p, H^s, Y) space, specifically plane abcd in Figure 1. Note that the budget plane slopes up from the point where $Y = Y^n$ and $H^p = H^s = 0$. The steepness of the slope as H^p increases varies directly with W^p and similarly steepness increases in the H^s direction as W^s increases. This will be called the initial budget plane and, in the absence of JOIN, families would maximize utility by supplying labor at a point of tangency between an indifference surface in (H^p, H^s, Y) space and the initial budget plane, determining the H^p and H^s supplied annually. Labor supply of each family member will, in general, depend on family non-labor income, his own wage, and that of the other family member as shown below:

Figure 1

Budget Constraint of a Multi-Worker Family in the Absence of JOIN


Net Income
c
d
$W^pT^p + W^sT^s + Y^n$
$W^sT^s + Y^n$
←H^p
b
a
$W^pT^p + Y^n$
Y^n
H^s↑

Equation of the Budget Constraint

$$Y = W^p(T^p - L^p) + W^s(T^s - L^s) + Y^n = W^pH^p + W^sH^s + Y^n$$

$$(2) \qquad H^p = h^p(W^p, W^s, Y^n)$$

$$H^s = h^s(W^s, W^p, Y^n).$$

The JOIN wage subsidy modifies the family's budget constraint by introducing other budget planes into the diagram. Consider first the case in which the primary worker is eligible for a wage subsidy and the secondary worker is not eligible for any of the JOIN subsidies. Figure 2 shows the initial budget plane, abcd, with a slope of $-W^p$ in the (Y, H^p) plane and $-W^s$ in the (Y, H^s) plane. If the primary worker receives a JOIN wage subsidy, but total family earnings are below the exemption level so that the earnings surtax does not apply, net family income is given by:

$$(3) \qquad Y = (2.00 + .5W^p)H^p + W^sH^s + .5Y^n + G .$$

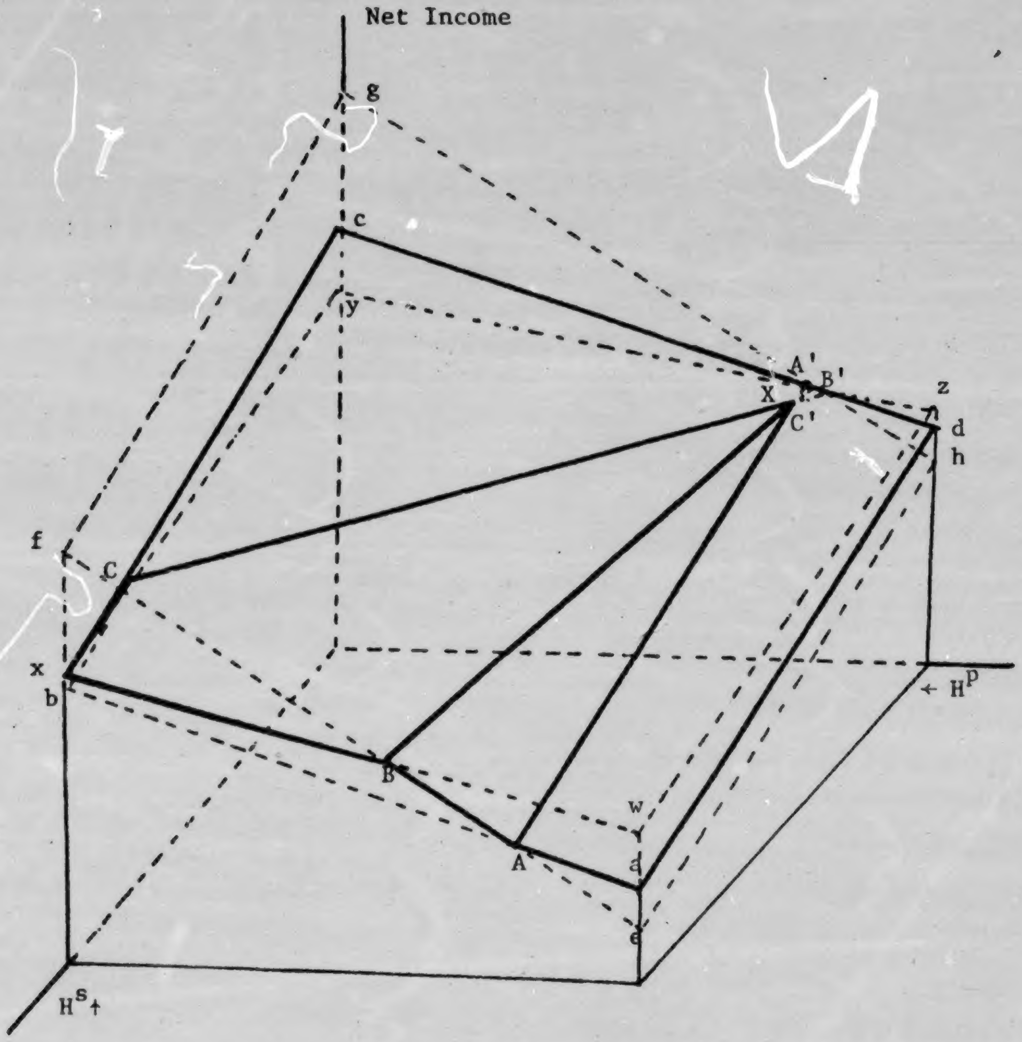
Equation (3) generates the plane efgh shown in Figure 2. As noted for single worker families the existence of the income guarantee for category 3 filing units would shift plane efgh vertically and modify the appearance of the diagram slightly. Plane efgh has a slope of $-(2.00 + .5W^p)$ in the (Y, H^p) plane and $-W^s$ in the (Y, H^s) plane and its height when $H^p = H^s = 0$ is $.5Y^n$. If the family finds a utility maximizing point on plane efgh, it will supply labor as if the initial supply equations in (2) above included a wage subsidy for the primary worker and a change in non-labor income.

$$(4) \qquad H^p = h^p(2.00 + .5W^p, W^s, .5Y^n + G)$$

$$H^s = h^s(W , 2.00 + .5W^p, .5Y^n + G).$$

Figure 2

Budget Constraint of a Multi-Worker Family with Primary Worker Eligible for the JOIN Wage Subsidy


Net Income
g
c
y
A'
B'
X
z
C'
d
h
f
C
x
b
$\leftarrow H^P$
B
w
a
A
e
$H^S\uparrow$

Budget Surface--

Equation of:

Plane abcd, $Y = W^PH^P + W^SH^S + Y^n$;

Plane efgh, $Y = (2.00 + 0.5W^P)H^P + W^SH^S + 0.5Y^n + G$

Plane wxyz, $Y = 0.25(2.00 + 0.5W^P)H^P + W^SH^S) + 0.5Y^n + G + 0.25E$

The change in non-labor income may be thought of as an income transfer equal to $-.5Y^n$ for groups 1, 2, and 4 and equal to $G-.5Y^n$ for group 3 which receives an income guarantee of G dollars for participating in JOIN. Note that in a multi-worker family policies which influence primary supply also change secondary worker behavior because the labor supply decisions are linked.

The intersection of planes abcd and efgh is the line AA' which is the locus of points formed by combinations of primary and secondary hours such that net incomes with and without the wage subsidy are equal. For any value of secondary hours, the level of primary hours along AA' is fixed and equal to:

$$(5) \qquad H_1^P = \text{Max}|0, (.5Y^n - G)/(2.00 - 0.5W^P)|$$

Equation (5) is precisely the same as the equation used to determine break even hours in the case of the single worker family. The region of the budget surface around AA' formed by planes abcd and efgh is concave downward introducing the likelihood of multiple equilibria, or points at which there is more than one point at which an indifference surface is tangent to the budget surface. At such points where equations (2) and equations (4) indicate that primary hours are below and above H_1^P respectively we simulate labor supply by assuming that the solution in which primary hours are furthest removed from H_1^P is the one which maximizes family utility. Given the concave downward shape of indifference surfaces, it is likely that the tangency with the highest indifference surface occurs at points relatively further removed from H_1^P. For a further discussion of this point in the context of a single worker see Kesselman.[13]

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[13] Jonathan Kesselman, "Conditional Subsidies in Income Maintenace," *Western Economic Journal*, March 1971 , 11, pp. 1-20.

Families receiving a wage subsidy and having earnings above the level of exemptions are subject to a surtax of 25 percent on earnings net of exemptions. For such families, net income is given by:

$$\text{(6)} \qquad Y = (1 - .25)((2.00 + .5W^p)H^p + W^sH^s) + .5Y^n + .25D_i + G$$

$$= .75\ ((2.00 + .5W^p)H^p + W^sH^s) + .25D_i + .5Y^n + G$$

Plane wxyz in Figure 2 is generated by equation (6). This plane has a slope of $-.75(2.00 + .5W^p)$ in the (Y, H^p) plane and $-.75W^s$ in the (Y, H^s) plane, and its height when $H^p = H^s = 0$ is $.5Y^n + .25D_i + G$. Plane wxzy is displaced from the initial budget plane by an income transfer equal to the difference of $G + .5Y^n + .25D_i$ and Y^n; a wage tax on primary workers equal to the difference of $.75(2.00 + 0.5W^p)$ and W^p; and a 25 percent wage tax on secondary workers. Thus the labor supply equations for family members can be written as:

$$\text{(7)} \qquad H^p = h^p(.75(2.00 + .5W^p),\ .75W^s,\ .5Y^n + .25D_i + G)$$

$$H^s = h^s(.75W^s,\ .75(2.00 + .5W^p),\ .5Y^n + .25D_i + G)$$

Planes efgh and wxyz intersect in line BB' whose equation may be written as:

$$\text{(8)} \qquad H_2^p = (D_i - W^sH^s)/(2.00 + .5W^p).$$

This is a generalization of the result for H_2^p calculated for single worker families above. Note that the line BB' runs diagonally across the budget constraint surface in Figure 2, intersecting line AA' at point X, and that lines BB' and AA' together determine the portion of plane efgh that lies on the budget surface. Only the triangular wedge, AXB, of plane efgh lies within the budget surface. This means that only equilibrium tangencies of indifference surfaces on this wedge can produce labor supply equilibria in that portion of the JOIN program where there is a wage subsidy without an earnings surtax. The size of the wedge AXB varies directly with the level of exemptions and inversely with wages of both primary and secondary workers and family non-labor income. This can be seen by solving for the level of secondary worker hours at which the point X occurs:

$$(9) \quad H^s = [D_i - (0.5Y^n - G)[(2.00 + .5W^p)/(2.00 - .5W^p)]]/W^s.$$

As H^s decreases, the point X moves down the budget plane toward the line where H^s equals zero and the size of the triangle AXB decreases. Equation (9) shows that H^s varies directly with D_i, and inversely with Y^n, W^p, and W^s.

The existence of kinks in the budget surface, such as that along line BB' in Figure 2, creates special problems for labor supply analysis. Line BB is part of the budget surface but it is possible for an indifference surface to be tangent to line BB' and hence to the budget surface without being tangent to either plane efgh or plane wxyz. Since labor supply equations (4) and (7) both assume that supply is determined by tangency of an indifference surface with planes efgh and wxyz respectively, they cannot be used to determine family labor supply produced by the tangencies with BB' described above. Equation (8), the equation of line BB', gives us the relationship between primary and secondary hours that must hold on BB', but the exact mix of primary and secondary hours is indeterminate. Our approach to handling this indeterminacy in the labor supply simulation involves setting secondary hours equal to the mean of secondary hours computed using equations (4) and (7) and then solving for primary hours, given these secondary hours, using equation (8). This approach to the indeterminate mix of primary and secondary hours has the virtue of yielding hours which are consistent with equation (8).

The line CC' on Figure 2 is the intersection of plane wxyz with the initial plane abcd and it indicates combinations of primary and secondary hours at which the net value of the wage subsidy is completely taxed away by the earnings surtax:

$$(10) \qquad H^P_3 = (G + .25D_i - .5Y^n - .25W^sH^s)/(W^P - .75(2.00 + .5W^P))$$

This is analogous to the equation for H^P_3 found for the single worker family. The quadraliteral CXBx in Figure 2 represents the portion of plane wxzy that lies within the budget surface and hence indicates possible combinations of hours for which supply equations (7) apply and the family is in the tax back section of the JOIN program. The size of CXBx decreases as point X moves down in Figure 2 to the point where $H^s = 0$, and the location of point X is determined according to equation (9). Thus the size of the budget surface falling in plane wxyz varies directly with the exemption level, D_i, and inversely with Y^n, W^P, and W^s.

Above the break even point along line CC', labor supply returns to that described by the initial supply equations (2) as the budget surface returns to plane abcd. However, the region of the budget surface around CC', formed by planes wxyz and abcd, is concave downward introducing the possibility of multiple equilibria analogous to the situation found around line AA'. There may be more than one point at which an indifference surface is tangent to the budget surface. The choice is between the labor supply equilibrium given by equations (2) on abce and that given by equations (7) indicating tangency on plane wxzy.

Diagrammatic analysis of the case in which the secondary worker is eligible for the wage subsidy follows simply by reversing the labels on the H^P and H^S

axes in Figure 2. Analysis of cases in which both family workers have market wages which render them eligible for the wage subsidy is very complex. The budget constraint facing a family under such circumstances would consist of portions of five budget planes. Unfortunately, such a surface would introduce the possibility of positions in which there were three equilibrium points. Design of tests to differentiate between such multiple equilibria would be most difficult. The case in which both workers were eligible for the wage subsidy was simulated by first attributing potential participation in the subsidy to the worker for whom the subsidy per hour would be largest given the prevailing market wage. Then the final equilibrium of the family is analyzed as described above.

The analysis of multi-worker family participation in the public employment program proceeds in a manner analogous to the simulation of the wage subsidy. Figure 2 can be reused to analyze family labor supply when only the primary worker is eligible for public employment. The initial plane could be represented by abcd. Net family income for public employment participants not subject to the earnings surtax would be given by:

$$Y = 3.00H^P + W^sH^s + .5Y^n + G. \tag{11}$$

This is the equation of a plane displaced from the initial plane much as efgh is displaced from abcd in Figure 2. Labor supply responses associated with the budget constraint in equation 11 will be equivalent to those associated with a primary worker wage subsidy equal to $(3.00 - W^P)$ and an income transfer of $G-.5Y^n$;

$$\begin{aligned} H^P &= h^P(3.00,\ W^s,\ .5Y^n + G) \\ H^s &= h^s(W^s,\ 3.00,\ .5Y^n + G). \end{aligned} \tag{12}$$

Finally if family earnings were above the earnings disregard, D_i, the appropriate budget constraint[14] would be given by:

$$(13) \qquad Y = 0.75(3.00H^P + W^sH^s) + .25D_i + .5Y^n + G.$$

The plane generated by equation (13) relates to the other two planes as plane wxyz relates to the other planes generating the budget surface in Figure 1. Equation (13) implies that the family supplies labor as if confronted with a wage tax equal to $W^P - 0.75 \times 3.00$ dollars per hour and receives an income transfer of $.25D_i + G - .5Y^n$ dollars per year. The resulting supply equations for this public employment tax-back segment of JOIN are:

$$(14) \qquad H^P = h^P(0.75 \times 3.00,\ W^s,\ .5Y^n + .25D_i + G).$$

$$H^s = h^s(W^s,\ 0.75 \times 3.00,\ .5Y^n + .25D_i + G)$$

To complete the analysis of supply responses of families eligible for public-employment, we use techniques for dealing with supply along or around lines where planes intersect analogous to those developed for the wage subsidy simulation. The case of secondary eligibility is handled symmetrically to primary supply analysis. Finally, cases of multiple eligibility for public employment or mixed wage subsidy-public employment eligibility were simulated by testing first to determine which worker could receive the largest absolute subsidy per man-hour. Family labor supply responses were then simulated assuming that only the worker getting the largest subsidy was eligible for participation in JOIN.

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[14] For type 5 families $Y = .065(3.00H^P + W H) + .5Y^n$. Also, since $D_i = 0$, $i = 3.5$, equation (11) does not apply. Individuals participate in public employment only if their market wage is below the public employment wage net of the earnings surtax.

A DIFFUSION ANALYSIS OF PARTICIPATION IN THE AID TO FAMILIES WITH DEPENDENT CHILDREN (AFDC) PROGRAM BY STATES

by Peter M. Greenston and C. Duncan MacRae

Abstract

The significant growth in the AFDC program during the 1960's has been attributed to growth in the number of families eligible for the program and the rate of participation by those who are eligible. In this paper a model of AFDC participation in twenty states over the period 1960-1973 is developed which allows for both change in the number of eligibles and in the participation rate over time. The number of families eligible for AFDC is a function of the number of female-headed families and the level of unemployment. Participation is the result of the diffusion of information from those participating to those who are not but who are eligible for the program. For almost every state, the model does an excellent job, tracking not only upturns but also downturns. It not only predicts participation, but also estimates eligibility so that participation rates can be derived. Since changes in these rates have influenced the level of participation in most of the states, it is concluded that the diffusion of information along with the growth in eligibility has played an important role in explaining the growth of AFDC.

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5.1

A DIFFUSION ANALYSIS OF PARTICIPATION IN THE AID TO FAMILIES WITH DEPENDENT CHILDREN (AFDC) PROGRAM BY STATES*

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The number of families participating in the AFDC program surged upward [illegible] the decade of the sixties. In 1960, there were 0.8 million participating families, while by 1970 there were almost 2.6 million families. The rate of growth from 1962 through 1966 fluctuated but was relatively unexceptional, averaging 4.1 percent. Subsequently, however, it increased each year through 1970, climbing from a rate of 15.1 percent to 36.1 percent. Finally, it slowed down in 1971 to 14 percent, to 7 percent in 1972, and to less than 1 percent in 1973.

A large proportion of the national increase has occurred in a handful of states -- on average, twelve states have accounted for 74 percent of the 1960-1970 increase, and twenty states have accounted for 85 percent.[1] Rates of growth, however, have not been uniform across these states. Some exhibit the national pattern, some are still accelerating, while others have already registered negative rates of increase. It is this diversity which makes an analysis of the growth of AFDC, at least at the state level, mandatory.

A multiplicity of factors has been advanced to explain the increase in the number of AFDC recipients. In general, participation levels may grow through an increase in the (demographically and financially) eligible population or through an increase in the rate of participation by those already eligible. Closely associated with growth in the eligible population is an increasing number of female-headed families, whether growing independently or as the result of increasing benefit levels; an increase in the unemployment rate and consequent difficulty in finding low-skill employment by both male workers and female heads; increasing state standards of need which expand eligibility up the income scale; and changes in program eligibility rules, including eligibility for new groups and more liberal acceptance procedures by local agencies. Increased rates of participation are brought about by increases in the amount of information about a program circulating within a community as well as changes in community norms.

Surprisingly, there have been relatively few studies of the growth of AFDC. Rather, most of the research has focused on explaining the level of participation in a cross-sectional framework and then inferring from this static framework the factors most important in explaining growth. These studies fall into two groups. The earlier work of Collins [4] and Lurie [10] attempts to explain the level of participation by social, economic and program factors. In explaining the ADC recipient rate (the number of child recipients per 1000 persons in the eligible age group) in 1960, using states as observations, Collins attributes her "relatively weaker than expected" results to state control over eligibility and standards coupled with diverse social structures and attitudes, implying that non-quantifiable factors may loom large in determining the dependent variables. Collins' analysis does indicate, however, the importance of understanding the role of eligibility rules as prerequisite to explaining recipient rates, and the corresponding necessity of distinguishing between eligibility and participation phenomena.

In a comparison of 1960 with 1966 levels of participation, Lurie attributed approximately 40 percent of the increase (in the number of AFDC families) to eligibility factors and implied that the remainder was due to an increased demand for AFDC by existing eligibles. During the subsequent period, it would seem that an increased demand was as much of a factor as before. This was a period of agitation by civil and welfare rights groups to get people onto the rolls, and a time when welfare lost its stigma and became a "right." Boland [2, pp. 15-24] estimates that the participation rate for all types was 56 percent in 1967, and 78 percent in 1970. In fact, the rate for female-headed families increased from 63 to 91 percent. This notion of an increasing level of awareness within the community is further supported by the fact that AFDC participation grew relatively rapidly in metropolitan areas where information is easily exchanged, even though poor female-headed families (albeit a suspect proxy for eligibles) grew slowly in metropolitan areas relative to non-metropolitan areas.

Recent research by Honig [8] and Durbin [5] explore the relationship between the AFDC program, eligibility, and participation. From their work we can infer that both growth in the eligible population and an increase in the rate of participation have been in evidence during the last decade. Using a cross-section of SMSA's, Honig measures the extent to which the program has increased the number of eligibles -- by inducing the formation of female-headed families -- and then estimates the participation response to higher payment levels that is due to larger numbers of welfare-induced female heads. She found that increases in the AFDC stipend increased participation directly and by inducing an increase in the formation of female-headed families.

Durbin's model of New York City health districts is the most comprehensive in that participation is dependent upon economic and demographic factors and the number of female-headed families which itself is a function of welfare, labor market, and social parameters. She suggests that increases in the number of women of child bearing age, especially among non-whites, and the automatic expansion of the financially eligible population as standards of need have increased, have contributed to the growth of AFDC. Relaxation of administrative stringency as reflected in falling rejection rates and the increased size of welfare benefits relative to market earnings (which also affects the rate of formation of female-headed families) have also acted to increase participation.

The purpose of this paper is to develop a time series analysis of AFDC growth by focusing on the implicit diffusion of information about the program from participants to nonparticipant eligibles throughout the community. Although the number of eligibles is not an observable variable (i.e., it cannot be measured easily), we hypothesize it to be a function of economic and demographic variables and do derive an estimate of it from the model. Saks [11] and Rydell [12] have also developed time series models of AFDC participation in New York City. Their models rely on social, economic, and programmatic variables just as the cross-section work does, but they do not deal explicitly with what we believe is central -- the program-eligibility-participation relationship.

I. ELIGIBILITY AND PARTICIPATION

Eligibility and participation are different phenomena and ought to be explicitly distinguished, especially in an analysis of the growth of AFDC. Beginning in 1961 demographic eligibility was extended in several states from female-headed families with dependent children to male-headed families with dependent children in which the head is unemployed. This set of families is known as the unemployed parent (UP) segment.[2] In 1968, the Supreme Court abolished the "man-in-the-house" rule so that an additional set of male-headed families (in the Census definition) became demographically eligible. The Court struck down a one-year state residency requirement in 1969, and thus increased the number of male- and female-headed eligible families. A family is financially or economically eligible if its total income is less than the appropriate break-even level determined by its earnings, unearned income, and a host of state established parameters. In the subsequent discussion, eligibility connotes both demographic and financial eligibility.

We hypothesize that at a particular time not all eligible families are participating because not all of them know about the program, and that it is the diffusion of information which has increased the rate of participation over time. Moreover, we implicitly assume that once sufficient information is available, the eligible family does participate in the program. Accordingly, the task is first to specify a relationship between eligibility and the demographic, social, and economic factors which influence it, and then to simulate the flow of information from participants to eligible nonparticipants.

Demographic eligibility is satisfied by a family with dependent children if (i) the head is a female; or if (ii) the head is an unemployed father and the family lives in a state with a UP program, or the head is an incapacitated father; or if (iii) the head is a male and no spouse is present. Children living with relatives other than parents are also demographically eligible. As mentioned already, families in which the male is not the father of the children and has no legal requirement to support them are also demographically eligible.

Financial eligibility is determined by the same variables which underlie the traditional microeconomic income-leisure analysis. This framework, in fact, has been applied to the General Assistance Program by treating it as a negative income tax.[3] The decision of how much labor to supply to the market depends upon market wage rates (and earnings) relative to AFDC subsidies and tax rates, and individual preferences for work vis-a-vis welfare (leisure). As a first approximation to the determination of eligibility, we assume there is a reduced form equation which relates the number of eligible families to certain demographic and economic variables as follows:

$$(1) \qquad E_t = a_o + a_1 FHF_t + a_2 D_t FHF_t + a_3 U_t + a_4 B_t / W_t + a_5 S_t ,$$

where t refers to the time period in quarters (I, II, III, IV), E denotes the number of eligible families, and the other variables are defined below. Since a complete enumeration of the demographically eligible families in any period t is not feasible, and since the bulk of participating families are composed of female-headed and UP segments, we assume that the number of demographically eligible families for the period 1960-73 is a linear combination of the number of female-headed families (FHF), the number of unemployed people (U) reflecting the size of the UP segment, and an additional proportion of the FHF population from 1968III - 1973I, which is assumed to be highly correlated with the number of "man-in-the-house" families, where $D_t = 0$ for 1960I - 1968II, and $D_t = 1$ for 1968III - 1973I. The number of financially eligible families is assumed to be related to the level of unemployment as a proxy for the likelihood that a demographically eligible family is a poor family, to the average expenditure per AFDC family relative to the average manufacturing wage rate which is used to measure the attractiveness of welfare relative to market work and is denoted by B/W, and to the state standard of need, S, which is presumably correlated with variation in the income exit level of the program.

Consider now the diffusion of information from participants to eligible nonparticipants (i.e., potential participants). In particular, we assume that all participants are linked with all potential participants in a social structure where a constant proportion β of possible contacts per period lead to new participants.[4] Since the number of possible contacts in any period is the product of the number of participants and the number of potential participants, we can express the change in the number of participants during period t as:

$$(2) \qquad P_t - P_{t-1} = \beta P_{t-1}(E_t - P_{t-1}) \quad ,$$

where E_t = number of eligibles at the beginning of period t, P_t = number of participants at the end of period t, and β = the proportion of possible contacts which lead to new participants. The probability of participating is given by

$$(3) \qquad (P_t - P_{t-1})/(E_t - P_{t-1}) = \beta P_{t-1},$$

so that the inverse of βP_{t-1} can be interpreted as the mean duration of nonparticipation by an eligible. This equation highlights the two opposing forces at work in the diffusion process. Relative to the number of eligibles, an existing large number of participants means a large storehouse of information about the program; but it also means fewer potential participants from which to draw new members. Note that the model is capable of predicting a downturn in participation (i.e., a negative change). This would occur when the number of participants at t-1 exceeds the number of eligibles at t. It should also be noted that in our formulation (as expressed by (1) and (2)) we implicitly assume that those who lose their eligibility in period t are replaced by former participants who regained their eligibility in period t.

II. SOCIO-ECONOMIC DATA

As in any inherently continuous process which must be modelled discretely, there is a problem of measuring the variables and defining a meaningful period of analysis. We chose to use quarterly values for P_t, E_t, and the exogenous variables FHF_t, U_t, B_t/W_t, S_t. We believe that one year periods are too long for capturing a diffusion phenomenon, but that monthly intervals are apt to be too short given the administrative delays in getting into the program. Hence, we compromised on quarters.

Data on the number of AFDC participant families by state (with the UP segment also reported separately) are available on a monthly basis so that quarterly averages can be readily formed. The primary source is the U.S. Department of Health, Education and Welfare Public Assistance Statistics. Annual data on the number of female-headed families is available in published form for the nation, and for each state in the Census years 1960 and 1970. Since each state's share of the national total is known in 1960 and 1970, we performed a linear interpolation to estimate the shares for the non-Census years. Quarterly values are obtained by assuming a constant rate of growth between successive annual observations. Data on the level of insured unemployment is available for each state on a monthly basis from the U.S. Department of Labor Unemployment Insurance Statistics. Average payments per family are also published monthly in the Public Assistance Statistics. Annual wage rate data in manufacturing is published by the Bureau of Labor Statistics. We assumed constant quarterly levels between annual changes. A preliminary set of consistent estimates of state cost standard data was made available by the Social and Rehabilitation Service, HEW.

III. DIFFUSION MODEL

In order to estimate the rate of diffusion, as expressed by (2), we incorporate the relationship for eligible families, (1), into (2) since data on actual numbers of eligible families are not available:

$$(4) \qquad P_t - P_{t-1} = bP^2_{t-1} + c_o P_{t-1} + c_1 P_{t-1} FHF_t + c_2 P_{t-1} D_t FHF_t + c_3 P_{t-1} U_t + c_4 P_{t-1} B_t / W_t + c_5 P_{t-1} S_t + v_t,$$

where $b = -\beta$, $c_i = a_i\beta$, $i = 0,\ldots,5$, and v_t is an error term assumed to have a zero mean and variance proportional to the square of $P_t - P_{t-1}$. This is consistent with the assumption that in a model purporting to explain the level of participation, the corresponding error term (due to measurement and specification errors) increases with the level, let's say is proportional to the square of the level. Hence, the error variance in a model explaining changes in the level would be proportional to a difference of squares.

The model is exactly identified in the sense that the a_i's and the β can be uniquely inferred from the c_i's so that the number of eligible families is predicted by

$$(5) \qquad E_t = c_o/-b + (c_1/-b)FHF_t + (c_2/-b)D_t FHF_t + (c_3/-b)U_t + (c_4/-b)B_t/W_t + (c_5/-b)S_t.$$

In any period, accordingly, we predict a rate of participation P_t/E_t.

To eliminate the assumed heteroscedasticity of the error term in (4), we rewrite the equation and estimate a percentage change rather than a level difference:

$$(6) \qquad (P_t - P_{t-1})/P_{t-1} = bP_{t-1} + c_o + c_1 FHF_t$$

$$+ c_2 D_t FHF_t + c_3 U_t + c_4 B_t/W_t + c_5 S_t + \varepsilon_t,$$

where ε_t has constant variance. Moreover, the suspected presence of seasonal fluctuations in the dependent variable beyond those accounted for by the insured unemployment variable suggests that we introduce seasonal dummies into the equation, D_I, D_{II}, D_{III}, D_{IV} where they are constructed to satisfy $c_6 D_{I,t} + c_7 D_{II,t} + c_8 D_{III,t} + c_9 D_{IV,t} = 0$. For convenience the dependent variable is also transformed to express annual percentage changes, although the underlying behavioral period is still the quarter. The final form of the estimating equation is

$$(7) \qquad 4\ln(P_t/P_{t-1}) = bP_{t-1} + c_o + c_1 FHF_t + c_2 D_t FHF_t$$

$$+ c_3 U_t + c_4 B_t/W_t + c_5 S_t + c_6 D_{I,t} + c_7 D_{II,t} + c_8 D_{III,t} + c_9 D_{IV,t} + \varepsilon_t,$$

where the reader may recall from the calculus that $(P_t - P_{t-1})/P_{t-1} = \Delta P/P \cong dP/P$ for small changes, and that $dP/P = d\ln(P) = \ln P_t - \ln P_{t-1} = \ln(P_t/P_{t-1})$.

IV. PARAMETER ESTIMATES AND STRUCTURAL CHANGE

In estimating (7) using ordinary least squares methods, we found that the measure of the relative attractiveness of welfare, B/W, was highly correlated with FHF so that precise estimates of both parameters was not possible. These high correlations are consistent with the formation of program induced female-headed families reported by Honig in cross-sectional work. The cost standard and FHF variables were also collinear and, moreover, this apparently produced negatively signed coefficients for S. This could also indicate that use of the cost standard variable may reflect welfare policy reaction to the growth in the number of participants rather than measuring the effect of increasing cost standards upon participation. In view of the multicollinearity just described, B/W and S were eliminated from the estimating equation.

We also tested the effects of alternative assumptions regarding the lags with which participation responds to eligibility in (2) and with which eligibility responds to the socio-economic variables in (1). In general, however, estimates based on the alternatives were insignificant or of the incorrect sign. Thus we conclude that E responds without lag to FHF and U, while growth in participation is the result solely of current interaction of participants and eligible nonparticipants.

Parameter estimates are presented in Table 1 for the twenty largest AFDC states. In sixteen states, both the growth of eligible families and the diffusion of information played important roles as indicated by positive coefficients significantly different from zero on one or more of the eligibility variables -- FHF, DFHF, U -- and a significantly negative coefficient on the lagged participation variable, implying a positive coefficient of diffusion. Female-headed families and the unemployment level were significant explanations of eligibility growth in all but two of the sixteen, while in ten of them the assumed structural change brought about by the "man-in-the-house" families was of the expected direction and statistically significant.

TABLE 1

PARAMETER ESTIMATES
ANNUAL PERCENT CHANGE IN AFDC FAMILIES
1960I-1973I

STATES	b (P_{t-1})	c_o	c_1 (FHF_t)	c_2 ($D_t FHF_t$)	c_3 U_t	c_6 D_I	c_7 D_{II}	c_8 D_{III}	c_9 D_{IV}	DEP. MEAN	S.E.E.	R^2	D.W.
	number of families				# of people					percent/100			
Alabama	-.22E-4***	- .14	.92E-5	.69E-5***	.77E-5*	.24E-1	.10E-1	-.30E-1	-.04E-1	.060	.138	.41	.92
California	-.27E-5**	- .61	.30E-5	.65E-6***	.12E-5*	.53E-1	.35E-1	-.76E-1	-.11E-1	.137	.122	.46	.63
Florida	-.16E-4***	-1.11***	.17E-4***	.12E-5**	.53E-5**	-.34E-1	.32E-2	-.18E-1	.50E-1	.098	.089	.57	.69
Georgia	-.18E-4***	-1.59***	.29E-4***	.48E-5***	.49E-5**	.33E-1	-.42E-3	-.82E-1	.49E-1	.143	.084	.75	1.14
Illinois	-.54E-5***	-1.07***	.89E-5***	.46E-5	.42E-5***	-.56E-1	.33E-1	.16E-1	.07E-1	.135	.121	.58	.77
Kentucky.a	-.90E-5	-1.91***	.53E-4***	—	.66E-5***	-.56E-1	.29E-1	.10E-2	.26E-1	.036	.086	.43	1.37
Kentucky.b	-.12E-5	.33***	-.25E-5	—	+.39E-5	.40E-1	-.45E-2	-.26E-1	-.10E-1	.095	.041	.44	1.57
Louisiana	-.25E-4***	- .64	.23E-4**	.66E-5***	.10E-5	.60E-2	.36E-1	-.53E-1	.12E-1	.078	.141	.35	1.59
Maryland	-.24E-4***	-1.10***	.34E-4***	-.17E-5*	.56E-5***	.46E-1	-.56E-1	-.24E-1	.34E-1	.150	.082	.54	1.01
Massachusetts.a	-.17E-4	-1.80**	.35E-4**	—	.19E-5	-.18E-1	.28E-1	-.17E-2	-.08E-1	.124	.071	.34	.86
Massachusetts.b	-.12E-5	1.00***	-.12E-4*	—	.27E-5**	-.18E-1	-.40E-1	.80E-2	.30E-1	.158	.045	.85	1.82
Michigan	-.49E-5***	- .93***	.10E-4***	.13E-6	.24E-5***	-.70E-1	.83E-1	-.55E-1	.42E-1	.144	.140	.52	1.89
Mississippi	-.15E-4*	- .62**	.29E-4**	.21E-5**	-.87E-7	-.14E-1	.25E-1	-.14E-1	.04E-1	.069	.060	.52	1.40
Missouri	-.14E-4***	- .83***	.19E-4***	.19E-5***	.59E-5***	-.50E-1**	.35E-1	.14E-1	.01E-1	.074	.060	.76	1.32
New Jersey.a	-.42E-4***	-1.39***	.32E-4***	—	.25E-5**	-.59E-1	.58E-1	.11E-1	-.09E-1	.134	.070	.67	.71
New Jersey.b	.13E-5	1.51	-.11E-4	—	-.20E-5	.12	.68E-1	-.21	.20E-1	.225	.223	.48	1.75
New York.a	.14E-5	-1.02	.26E-5	—	.18E-5**	-.96E-3	-.52E-1	.29E-1	.24E-1	.141	.086	.56	1.38
New York.b	-.65E-5***	.23	.43E-5***	—	.19E-5***	-.52E-1	.70E-2	.23E-1	.22E-1	.107	.043	.86	.91
North Carolina	-.13E-4***	- .06	.40E-5	.24E-5***	.52E-5***	.31E-1	.46E-1	-.13	.57E-1	.043	.071	.79	1.86
Ohio	-.97E-5***	-1.23***	.14E-4***	-.15E-6	.18E-5***	.16E-1	.32E-1	-.36E-1	-.11E-1	.132	.082	.59	1.74
Pennsylvania	-.81E-5***	- .78*	.86E-5**	.23E-5***	.18E-5***	.62E-1	-.11E-1	-.39E-1	-.12E-1	.098	.133	.57	1.07
Tennessee	-.23E-4***	-1.08***	.28E-4***	.33E-5***	.77E-5***	-.62E-1	-.37E-1	-.12E-1	.12E-1	.073	.084	.60	.48
Texas	-.16E-4***	-2.51***	.20E-4***	.11E-5*	.61E-5**	.26E-1	-.14E-1	-.57E-1	.45E-1	.134	.157	.59	1.54
Washington	-.60E-4***	-1.43**	.50E-4**	.51E-5	.13E-4***	.93E-1	-.10E-2	-.15	.62E-1	.110	.253	.36	1.29

Note: Asterisks denote that coefficients are significantly different from zero at the ten(*), five(**), and one (***) percent levels.

In the states of Massachusetts, New Jersey, New York, and Kentucky, diffusion of information did not play a significant role (and in New York it even operated insignificantly in the "wrong" direction), nor in general did growth in the number of eligible families (with the exception of significant structural change in Massachusetts and New Jersey). In fact the unemployment level appeared with the "wrong" sign in all four states. The possibility of more pervasive structural changes in these states than that envisioned and incorporated into the DFHF variable was investigated. The explosive growth in AFDC from approximately 1968-1970 may have been the result of compounding factors. The drive to get people onto the rolls, eligibility for "man-in-the house" families, and state policies which produced falling rejection rates all came together around 1967-1968. Since specification of the effects of all these interactions was not feasible, we resorted to a more general hypothesis -- that of a change in the parameters in the underlying eligibles and diffusion equations as between two periods rather than different variable specifications in each period. The observation period was, therefore, divided in 1968 and the parameter estimates for each subperiod are also reported in Table 1 for these states with the earlier period denoted by "a" and the later period by "b".

In Massachusetts, the diffusion of information did not play a statistically significant role in either period. Growth in the number of eligible families did play a significant role through FHF in both periods and U in the later period, although female-headed families acted in the wrong direction in the later period and testing did not reveal this to be a case of multicollinearity. Partitioning the observation period in New Jersey produced significant coefficients of the correct sign for all the variables in the earlier period and wrong signs on all the coefficients in the later period. Evidently diffusion and eligibility growth, as portrayed, were quite important into 1968 at which time a structural shift occurred which the model cannot explain. To determine if the brief experimentation with the UP program in 1969-71 was a factor, we excluded these UP families and tested the model; the results were not altered. In New York, just the reverse occurred. In the later period, diffusion and both eligibility variables were significant factors, whereas in the earlier period only the unemployment level was significant (and, in fact, the diffusion parameter has the incorrect sign). Exclusion of the UP segment in the earlier period did not alter the results. Hence, there appears to have been a structural shift in 1968 which more nearly corresponds to our view of the AFDC growth process than the relationships which prevailed in the earlier period. In Kentucky, diffusion of information did not play a significant role in either period, while growth in eligibility was significant in only the earlier period.

The proportion of variance explained, R^2, ranges from a low of 34 percent to a high of 86 percent, with thirteen states between 50 and 70 percent. In thirteen of the states, the standard error of the estimate (S.E.E.) is smaller than the mean of the dependent variable. The Durbin-Watson (DW) values indicate significant positive serial correlation in half of the states and therefore large variance of the coefficient estimates though the estimates are unbiased. Testing indicated that the autoregressive scheme was not a simple first-order one, and so we hesitated to correct for the serial correlation in view of the fact that so little is known about the resulting estimates -- there is no guarantee that the result is any better than the uncorrected estimates.[5] Finally, it should be noted that the seasonal dummies were statistically significant as a group at the five percent level in only five states. Apparently the insured unemployment variable accounted for any seasonal variation in the number of participating families.

Having estimated growth in participation as a function of past participation and the number of eligible nonparticipants, we can infer the underlying relationship between eligibility, female-headedness, man-in-the-house families, and unemployment as described in (5) using the parameter estimates of (7). In Table 2 these derived coefficients of (1) are reported along with their confidence intervals. Confidence intervals for these parameters derived from the ratios $c_i/-b$ depend upon the variance of the numerator and denominator as well as the covariance between them. Typically the intervals will be asymmetrical around the point estimate. By the very nature of the variables we might expect that increases in FHF, DFHF, and U would be associated with less than proportionate changes in the number of eligibles. Indeed, this is the case for DFHF and U. We found, however, that in most states (exceptions were Alabama, Louisiana, New Jersey a., New York b., North Carolina, and Washington) an increase in the FHF variable is associated with a relatively larger increase in the number of eligibles. An explanation which is supported by the multicollinearity previously discussed is that the FHF variable reflects not only growth in female-headed families but also increasing benefit levels, expanding state cost standards, and other variables which undoubtedly contributed to eligibility growth but were not included in the model as estimated.

V. HISTORICAL ANALYSIS

Rather than describe and plot the observed and predicted values of the dependent variable over the period 1960-73, we thought it would be more valuable and a better test of the model to track the observed values. The difference

ELIGIBILITY EQUATION
DERIVED PARAMETER ESTIMATES
WITH CONFIDENCE INTERVALS**

	a_0	a_1 (THF_t)	a_2 $(U_t THF_t)$	a_3 (U_t)
Alabama	- .64E+4 (-3.46E+4, 1.01E+5)	.38 (-1.81, .94)	.32 (.18, 1.05)	.36 (-.06, .83)
California	- 2.26E+5 (-3.64E+5, 7.10E+6)	1.09 (-1.55, 1.51)	.24 (.09, 1.14)	.44 (-.10, 1.16)
Florida	- 6.64E+4 (-8.68E+4, -4.56E+4)	1.06 (.84, 1.25)	.07 (.01, .17)	.33 (.09, .52)
Georgia	- 8.88E+4 (-1.08E+5, -6.11E+4)	1.64 (1.28, 1.90)	.27 (.18, .39)	.27 (.03, .45)
Illinois	- 1.96E+5 (-2.82E+5, -1.16E+5)	1.65 (1.03, 2.23)	.08 (-.09, .35)	.77 (.51, 1.26)
Kentucky.a	- 2.12E+5)-5.31E+4, 1.16E+5(	5.94)-2.38, 1.93(	—	.73)-.33, .13(
Kentucky.b	2.79E+5)-2.27E+4, 9.30E+3(	- 2.15).77, .77(	—	- 3.36).30, .30(
Louisiana	- 2.56E+4 (-4.88E+4, 2.74E+4)	.94 (.17, 1.35)	.25 (.14, .58)	.04 (-.64, .31)
Maryland	- 4.55E+4 (-5.86E+4, -3.44E+4)	1.41 (1.22, 1.72)	- .07 (-.17, .01)	.23 (.11, .48)
Massachusetts.a	- 1.05E+5)-6.03E+4, 5.19E+4(	2.06)-.33, 1.46(	—	.11)-.05, -.37(
Massachusetts.b	8.45E+5)-2.09E+5, 4.92E+4(	- 9.83).03, 3.16(	—	2.29)-.12, .34(
Michigan	- 1.88E+5 (-3.15E+5, -9.66E+4)	2.12 (1.21, 3.32)	.03 (-.28, .49)	.49 (.24, 1.09)
Mississippi	- 4.10E+4)-2.17E+5, 2.70E+6(	1.90)-97.4, 1.38(	.14)-28.8, .03(	- .01)1.23, 1.48(
Missouri	- 6.13E+4 (-7.94E+4, -4.15E+4)	1.42 (1.05, 1.74)	.14 (.04, .32)	.44 (.30, .61)
New Jersey.a	- 3.29E+4 (-4.63E+4, -2.26E+4)	.76 (.67, .88)	—	.06 (.01, .14)
New Jersey.b	- 1.14E+6)-1.37E+5, -6.74E+4(	8.33)1.22, 1.22(	—	1.58).58, .58(
New York.a	7.40E+5)-4.07E+5,-9.16E+4(	- 1.88).95, 2.22(	—	-1.31)-.19, .17(
New York.b	.35E+5 (-.11E+5, 1.33E+5)	.66 (.32, .83)	—	.29 (.20, .44)
North Carolina	- .48E+4 (-2.74E+4, 5.46E+4)	.30 (-.72, .63)	.18 (.07, .63)	.39 (.22, .85)
Ohio	- 1.27E+5 (-1.60E+5, -9.74E+4)	1.47 (1.23, 1.76)	- .02 (-.09, .08)	.18 (.11, .31)
Pennsylvania	- .97E+5 (-1.63E+5, 2.42E+4)	1.06 (.12, 1.56)	.28 (.15, .58)	.22 (.14, .36)
Tennessee	- 4.70E+4 (-6.23E+4, -2.69E+4)	1.20 (.84, 1.48)	.14 (.06, .28)	.33 (.19, .47)
Texas	- 1.55E+5 (-1.90E+5, -1.03E+5)	1.25 (.93, 1.50)	.07 (-.01, .20)	.38 (.03, .59)
Washington	- 2.37E+4 (-3.61E+4, -3.86E+3)	.84 (.34, 1.15)	.08 (-.02, .26)	.22 (.13, .33)

**

See Z. Griliches, "Distributed Lags: A Survey," Econometrica, 35, no. 1, January 1967, pp. 32-33, for a short discussion of the calculation. We report 95 percent confidence intervals. For Kentucky a. and b., Massachusetts a. and b., Mississippi, New Jersey b., and New York a. the estimate of b $(=-\beta)$ is not significantly different from zero at the five percent level. Accordingly, the confidence intervals obtained for the $c_1/-b$ parameters are not closed so that confidence "non-intervals" are really obtained. These are indicated with outward opening parentheses. For example, in Mississippi the chances are 95 to 5 that a_1 is less than -97.4 and greater than 1.38.

between this and calculating predicted values is that in tracking the number of participants in period t, P_t, we use the value of P_{t-1} predicted by the model rather than the observed P_{t-1} as would be done in calculating predicted values and residuals from the regression model. In other words, our ability to track P_t is a function of our ability to track P_{t-1}, which in turn depends upon our ability to track P_{t-2}, etc. This can be seen by deriving from (7) an expression for P_t:

$$(8) \qquad P_t = P_{t-1}\exp\{.25(bP_{t-1} + c_o + c_1FHF_t + c_2D_tFHF_t + c_3U_t + c_6D_{I,t} + c_7D_{II,t} + c_8D_{III,t} + c_9D_{IV,t})\}$$

The system is initialized by setting P_{1960I} equal to the observed value and tracking from 1960II through 1973I. The value of P predicted in period t is inserted on the right side of the equation when tracking in period t+1. For the four states in which the observation period is split, the tracking is reinitialized in 1968III.

In a comparison of the observed number of AFDC families and the tracked values the model does an excellent job. Errors are rarely more than a few thousand. Moreover, the model does a good job tracking downturns. As mentioned earlier, it accomplishes this by producing a negative $(E_t - P_{t-1})$ which when multiplied by β gives a negative percentage change from period t-1 to t. For example, consider the model's capability to track in a representative state such as Illinois. In Table 3 we present data for the observed number of participating families, the tracked values, the derived number of eligibles (from (5)), an estimated rate of participation, and an estimate of the average elapsed time between eligiblity and participation. Negative estimates of

TABLE 3 **Illinois**

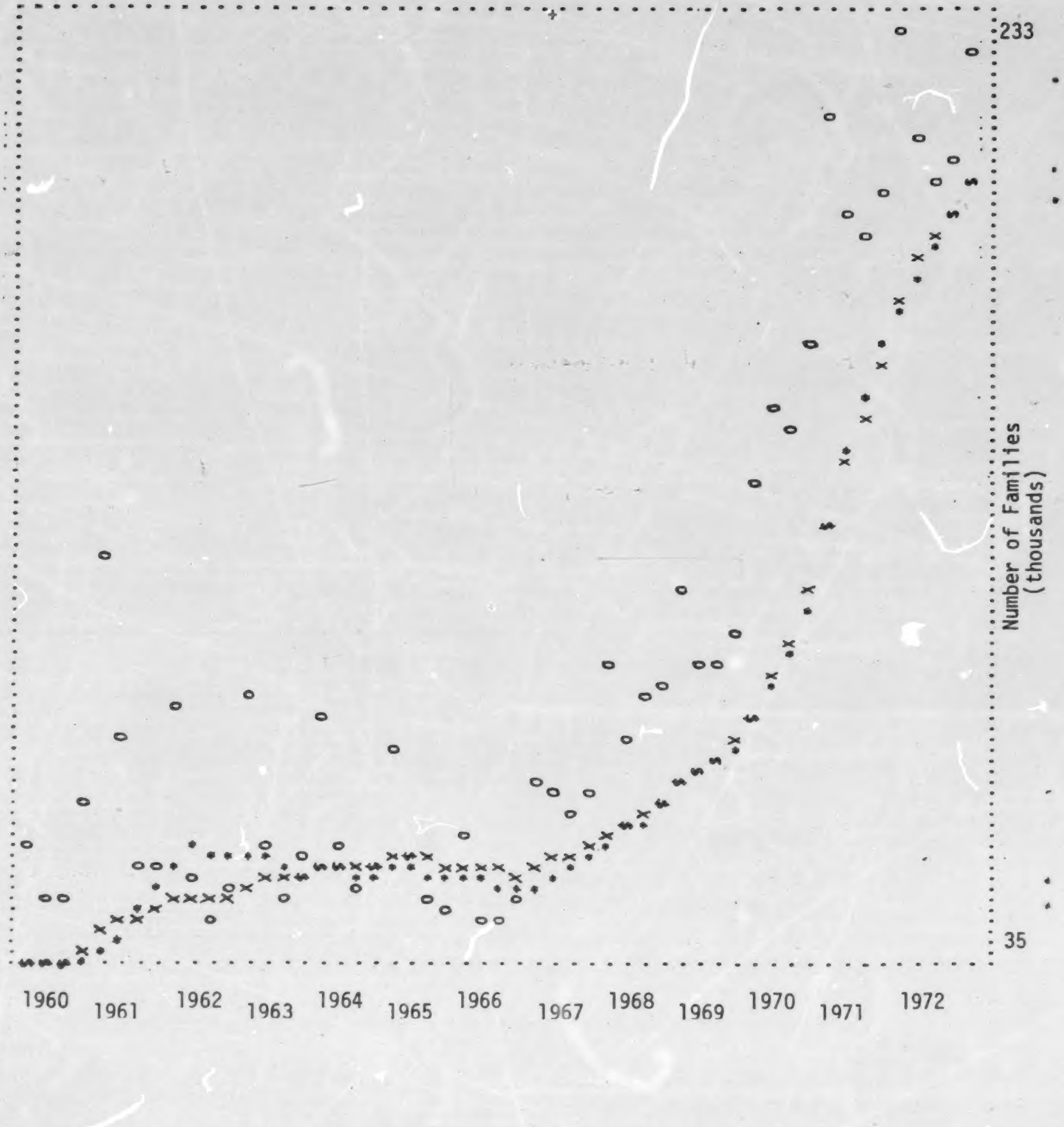
QUARTER	ACTUAL PARTICIPATION	TRACKED PARTICIPATION	DERIVED ELIGIBLES	PARTICIPATION RATE	MEAN NONPARTICIPATION TIME
19601	34973.	34973.	60375.	.58	*******
19602	35441.	35976.	49030.	.72	5.27
19603	35911.	36004.	49820.	.74	5.13
19604	36291.	36579.	70118.	.55	5.01
19611	37671.	42609.	122291.	.35	4.78
19612	41416.	45381.	83111.	.55	4.33
19613	46995.	46323.	57592.	.80	4.06
19614	51105.	47118.	57430.	.82	3.98
19621	57117.	49311.	91089.	.54	3.92
19622	51641.	50110.	55157.	.91	3.74
19623	59576.	49933.	44555.	1.12	3.68
19624	55131.	50242.	53047.	.95	3.69
19631	58762.	52422.	91978.	.57	3.67
19632	57595.	53441.	60617.	.88	3.52
19633	55536.	53433.	50392.	1.06	3.45
19634	55067.	53997.	59737.	.90	3.45
19641	56330.	55721.	87571.	.64	3.42
19642	56516.	56632.	61694.	.92	3.31
19643	55271.	56481.	51715.	1.09	3.26
19644	54771.	56641.	57122.	.99	3.27
19651	56133.	57804.	82030.	.70	3.26
19652	56371.	58304.	58160.	1.00	3.19
19653	55355.	57843.	49499.	1.17	3.16
19654	53960.	57145.	47437.	1.20	3.19
19661	53960.	56802.	63101.	.90	3.23
19662	53559.	56432.	45972.	1.23	3.25
19663	52750.	55749.	44499.	1.25	3.27
19664	52457.	55409.	49790.	1.11	3.31
19671	53133.	56125.	75285.	.75	3.33
19672	54500.	57675.	72815.	.79	3.29
19673	55767.	58934.	68272.	.86	3.19
19674	53467.	60204.	73212.	.82	3.13
19681	51857.	62657.	100078.	.62	3.06
19682	54600.	65042.	84216.	.77	2.94
19683	66207.	67803.	92757.	.73	2.84
19684	69367.	70509.	95232.	.74	2.72
19691	73633.	73891.	115479.	.64	2.62
19692	75833.	77223.	100429.	.77	2.50
19693	79557.	80022.	100551.	.80	2.39
19694	82567.	83199.	107298.	.78	2.31
19701	87800.	88329.	137749.	.64	2.22
19702	94200.	97328.	153911.	.63	2.09
19703	101933.	105027.	150555.	.70	1.90
19704	112000.	114436.	167227.	.68	1.76
19711	129000.	129440.	215452.	.60	1.61
19712	145657.	142581.	194781.	.73	1.43
19713	156718.	152565.	189576.	.80	1.29
19714	167546.	162871.	199352.	.82	1.21
19721	175775.	175501.	232566.	.76	1.13
19722	181934.	136369.	210639.	.88	1.05
19723	188542.	190897.	201137.	.95	.99
19724	194393.	195112.	205566.	.95	.97
19731	202434.	201585.	229587.	.88	.95

eligibility, participation rates, and mean nonparticipation times are theoretically impossible and are replaced by asterisks when they occur. The same information is presented graphically in Figure 1 to illustrate the logistic nature of the AFDC growth process. Actual participation is plotted by "*", tracked participation by "X", and derived eligibles by "0"; coincidence of values is plotted by "$".[6]

For Illinois in the 1960-73 period there are 12 downturns in AFDC participation -- 2 occur in 1962, 3 in 1963, 2 in 1964, 2 in 1965, and 3 in 1966. Of these 12, the model successfully tracks the downturn in 8 cases. To work through one case, participation fell from 56,516 families in 1964II to 55,271 families in 1964III. Correspondingly we find that the predicted number of eligibles fell in 1964III presumably from a decrease in unemployment so that $E_{1964III}$ is less than the predicted number of participants in 1964II, i.e., the pool of eligible nonparticipants is negative: 51,715 - 56,632; accordingly, the predicted number of families falls from 56,632 to 56,481 in 1964III.

The derived number of eligible families is that estimate defined or constrained by (1) which most closely simultaneously supports the participation behavior posited in (2). The estimates of E_t are reasonable for all the states with the exception of New Jersey in the later period and New York in the earlier period. In these cases one or more of the estimated parameters in (7) are of the incorrect sign and sufficiently important (relative to the other coefficients) to produce estimated E_t's which are negative. However, these estimates best explain participation and so we find that the corresponding tracked values are quite close to the observed number of participating families. In many of the states we note that there is a sizeable jump in E at observation 35 (1968III). This corresponds to the assumed increase in the number of eligible

FIGURE 1 - ILLINOIS



families brought about by the "man-in-the house" families and signalled by $D_t FHF_t$.

Unless there is fraud, estimates of the rate of participation should not exceed unity, but as mentioned already when following downturns the pool of eligible nonparticipants becomes negative ($E_t - P_{t-1} < 0$) so that we may find the corresponding rate of participation (P_t/E_t) larger than unity. Accordingly, in order to obtain an overview of the trends, if any, in participation rates over the period, we must exclude periods during which participation has been falling and also discount the fall in participation rates which corresponds to the jump in E in 1968III.

There are four basic patterns discernible. In California, New Jersey, Pennsylvania, and Maryland rates of participation have unambiguously increased in the period. In ten of the states (Florida, Louisiana, Massachusetts, New York, Alabama, Mississippi, Missouri, North Carolina, Tennessee, Washington) the rate of participation is constant to increasing. In Georgia, Michigan and Texas the rate of participation declines and subsequently increases during the period. Illinois, Ohio, and Kentucky are characterized by fluctuating rates without a dominating trend. These patterns, along with the estimated parameter results, suggest that not only was diffusion of information a factor in explaining the growth in participation levels but also in many of the states it manifested itself in increasing rates of participation.

Finally, the last column of Table 3 reports the mean nonparticipation time, i.e. $(\beta P_{t-1})^{-1}$, in years in Illinois. It should be noted that we observe a falling series in all states[7] which reflects rising participation levels and hence increasing probabilities of contacts between participants and eligible nonparticipants.

VI. CONCLUSIONS

The significant growth in the number of families participating in the AFDC program during the 1960's has been attributed to growth in both the number of families eligible for the program and the rate of participation by those who are eligible. In this paper, we have developed a two equation model of AFDC participation which allows for both change in the number of eligibles and in the participation rate over time. The number of families eligible for AFDC in a state is assumed ultimately to be a function of the number of female-headed families and the level of unemployment. Participation is then assumed to be the result of the diffusion of information regarding the existence of the program from those participating to those who are not but who are eligible for the program.

Since the number of eligibles is not directly observable, the eligibles equation was substituted into the diffusion equation and the parameters of the resulting equation estimated by ordinary least squares. Separate parameters were estimated for twenty states, which account for 85 percent of AFDC participation, using quarterly time-series data for the period 1960 through 1973. With a few exceptions, growth in the number of eligible families and the diffusion of information played statistically significant roles in explaining the growth of AFDC in each state.

The model was evaluated by using the estimated equation for each state to predict AFDC participation (during the period of estimation) given only the actual values of the variables in the implicit eligibles equation and an initial value in 1960I of AFDC participation in the state. For almost every state, the model does an excellent job, tracking not only upturns but also downturns -- when they occur -- as well. Moreover, the model not only predicts participation during the period, but also estimates eligibility as well. For one group of states we observed an increasing rate of participation over time; for a second group, a constant to increasing rate; for a third group a rate that declined and subsequently increased; and for a fourth group a rate that fluctuated. Changes in the participation rate influenced the level of participation in most of the states and we conclude, therefore, that the diffusion of information along with the growth in eligibility has played an important role in explaining the growth of AFDC.

We believe that the model would be significantly improved if eligibility was derived not from a single reduced form equation but from a system of structural equations describing the interaction between AFDC eligibility and labor supply. Nevertheless, the time-series model in its existing form both demonstrates the importance of the diffusion of information regarding a government program in determining its effect and does a better job of explanation than previous cross-section models.

FOOTNOTES

*This research was supported by funds from the Office of Research and Development, Manpower Administration; and the Office of the Assistant Secretary for Policy Evaluation and Research, U.S. Department of Labor; and the Social and Rehabilitation Service, U.S. Department of Health, Education and Welfare.

Opinions expressed are those of the authors and do not necessarily represent the views of the Urban Institute or its sponsors. We thank Linda Royster and Carola Pedone for programming assistance.

1. The top twelve states are: California, Florida, Georgia, Illinois, Louisiana, Massachusetts, Michigan, New Jersey, New York, Ohio, Pennsylvania, and Texas. The remaining eight are: Alabama, Kentucky, Maryland, Mississippi, Missouri, North Carolina, Tennessee and Washington.

2. The states - among the twenty analyzed - which adopted a UP program are Illinois (1961-), Massachusetts (1961-), New York (1961-), Pennsylvania (1961-), Maryland (1961-), Washington (1961-62, 1963-), North Carolina (1962-63), California (1964-), Michigan (1964-), Ohio (1964-), Missouri (1968-71), New Jersey (1969-71).

3. See the conceptual and empirical analyses by Brehm and Saving [3] and Kasper. [9].

4. Change and the diffusion of information is an important concept in the social sciences. In an early econometric study Griliches [6] employed logistic growth functions (essentially the integral of the rate of change formulation used in this paper) to study past and current rates of the use of hybrid corn in various districts in the U.S. In sociology, Hamblin Jacobsen, and Miller [7] describe the forms of social adaptation, diffusion, and innovation processes which characterize social change and empirically test their propositions. Bernhardt and Mackenzie [1] formulate diffusion models to study the marketing of new products.

5. See R. Wonnacott and T. Wonnacott, Econometrics, New York: John Wiley and Sons, 1970, pp. 132-143, for a discussion of problems arising from serial correlation.

6. Tables and figures analogous to Table 3 and Figure 1 are available for all twenty states upon request from the authors.

7. In Kentucky and Massachusetts there is a jump between the "a" and "b" periods because $\beta(a)>\beta(b)$.

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THE PERSONAL INCOME TAX AND FAMILY LABOR SUPPLY

by C. Duncan MacRae and Anthony M.J. Yezer

Abstract

This paper demonstrates that labor supply effects of a progressive income tax on a multi-worker family can be analyzed as a combined wage tax and income transfer specific to each tax bracket. The wage tax equals the marginal rate paid on family income while the income transfer equals net savings from not having to pay tax at this high marginal rate on all earnings. At intervals where tax rates change the family departs from its reduced form supply equation entirely. These results limit the implications of survey research and suggest modifications in procedures for estimation and simulation of supply relationships.

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7.1

THE PERSONAL INCOME TAX AND FAMILY LABOR SUPPLY*

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The traditional theory of family labor supply based on utility maximization subject to a budget constraint implies that there is a reduced form relationship between hours supplied by family members and the market wages of individual workers and family non-labor income. Using this theory, Kosters [11] has analyzed effects of the personal income tax on family labor supply by converting the tax into a change in wage rates at the marginal tax rate. This technique is appropriate for a proportional tax but not for a progressive income tax in which average and marginal rates diverge.

Using other approaches to the determination of labor supply behavior, Hall [7] and Wales [12] have developed appropriate procedures for dealing with the effects of a personal income tax. Hall develops a model in which labor supply is a function of primary and secondary worker wages and family whole income, where whole income is the sum of non-labor income and family earnings calculated under the assumption that each worker is employed full time. Hall adjusts for the presence of the personal income tax by interpreting the tax as a combined wage tax at the marginal tax rate and lump sum tax reducing whole income. The lump sum tax is equal to the total tax that would be paid if gross income were equal to whole income and if the prevailing marginal tax rate remained constant for all higher levels of income. Following Cooper [5], Wales [12] handles a progressive income tax by maximizing an explicit utility function for a single worker family subject to a budget constraint that includes the tax schedule. He rejects the reduced form estimation approach, because "it is not clear how the after-tax wage rate and the gross wage rate would be incorporated in an _ad hoc_ reduced form regression analysis."

The purpose of this paper is to present the generalized analysis of reduced form labor supply functions in the presence of a progressive income tax. We demonstrate that within income intervals for which the marginal tax rate is constant the family supplies labor as if it were subject to a wage tax combined with an income transfer. The wage tax is equal to the marginal tax rate on family income while the income transfer is equal to net tax savings from not having to pay tax at this marginal rate on all earnings. At boundaries between intervals, where there is a discontinuous change in marginal tax rates, the income tax cannot be translated into a change in wage rates or unearned income because family labor supply is not based on the equality of marginal conditions for utility maximization. Instead, the utility-maximizing labor supply is that which maintains constant family income at the boundary.

In the first section of the paper, single worker families are analyzed. The results are extended to the case of a multi-worker family in the second section. Finally implications of the theoretical analysis for empirical estimation and simulation of labor supply functions are drawn.

I. LABOR SUPPLY OF THE SINGLE-WORKER FAMILY

The family labor supply model is based on a family utility function with total income and hours of leisure of all family members as arguments. In general not all family members can be regarded as potential workers. Child labor laws, other institutional restrictions, and the prevailing structure of wage rates make participation by some family members extremely unlikely. The analysis in this section considers a family with only one potential worker. The family utility function implicitly assumes a social welfare function interna to the family unit weighting utility of all family members so that we can write:

$$U = U\{L,Y\}, \tag{1}$$

where L is leisure of the single potential worker and Y is real family income. The family utility function generates indifference curves in (L,Y) space, as shown in Figure 1.

The family is assumed to supply labor so as to maximize utility subject to a budget constraint specifying attainable combinations of income and leisure:

$$Y = W(K-L) + Y^n = WH + Y^n, \tag{2}$$

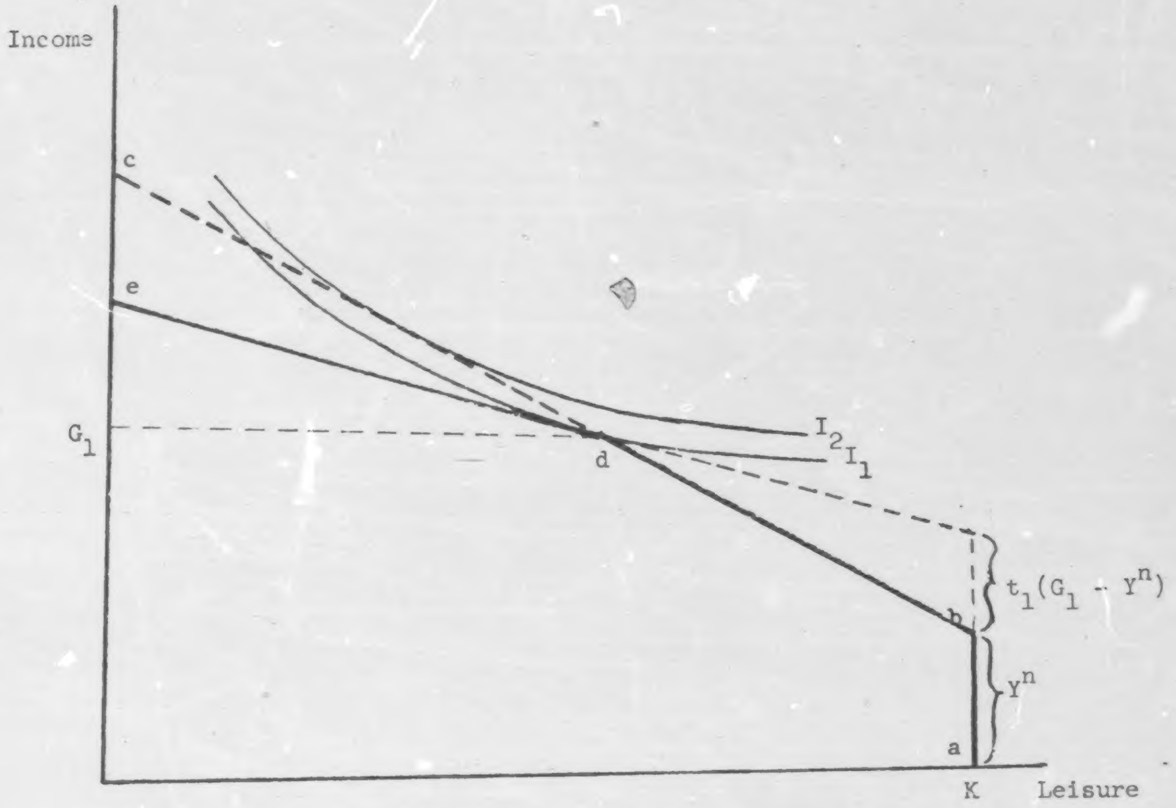
where W is the worker's market wage, H is the number of hours worked, K is the number of potential hours available for work and leisure, and Y^n is unearned income. Equation (2) describes the family budget constraint under the implicit assumption that each worker faces a single market wage known with perfect certainty and the level of work effort is completely flexible. Workers may have some difficulty choosing the hours they work. But over a period of one year they can always adjust their average level of work effort through periodic withdrawal from the labor force so that the actual number of hours worked is flexible. Housework and other home labor services are regarded as a component of leisure activity increasing family utility in a manner analogous with other uses of leisure time. The process of maximizing family utility subject to a budget constraint such as (2) is illustrated in Figure 1 by the tangency of indifference curve I_2 with budget line abc.

$$H = K-L = H\{W,Y^n\}. \tag{3}$$

Figure 1

Budget Constraint Of A Single-Worker Family Gross

And Net Of A Progressive Income Tax


Income
c
e
G_1
d
I_2
I_1
b
$t_1(G_1 - Y^n)$
Y^n
a
K
Leisure

The influence of wages on labor supply consists of the familiar combination of a compensated wage rate effect known a priori to be positive and an income effect based on changes in earnings presumed to be negative if leisure is not an inferior good. Empirical estimates of equations such as (3) indicate that the uncompensated wage rate effect often varies inversely with wages, becoming negative at high wage rates and, hence, producing labor supply curves that bend backward at high wage rates.

Now consider the imposition of an income tax whose marginal tax rates increase with gross income. If t_i is the marginal tax rate associated with gross income G in the interval between G_i and G_{i+1} for $i = 0,1,\ldots$, we can express total tax liabilities, T_i, associated with this level of gross income as:

$$T_i\{G\} = t_i(G - G_i) + \sum_{j=1}^{i-1} t_j(G_j - G_{j-1}) = t_i(G - G_i) + T_{i-1}\{G_i\}, \tag{4}$$

where $T_i\{G\}$ is the total tax liability associated with gross income G, $T_{i-1}\{G_i\}$ is the total tax liability on gross income up to G_i and G is gross income such that $G_{i+1} > G > G_i$ and $T_0 = G_0 = 0$.

Although no explicit attention will be given to exemptions, deductions or adjustments in gross income important in the calculation of taxable income, such adjustments can be incorporated in the analysis by appropriate choice of marginal tax rates and gross income intervals to which they apply. For example, exemptions can be interpreted as a marginal tax at a zero rate on the interval of gross income equal to the value of exemptions.

In the presence of an income tax, families maximize utility of leisure and after-tax income. The relevant budget constraint is a piecewise-linear curve with each linear segment corresponding to a range of gross income over which marginal tax rates are constant. Thus for a family whose gross income, G, is in the range where marginal tax rate t_i is applicable, $G_{i+1} > G > G_i$, the budget constraint becomes:

$$(5) \qquad Y = (1-t_i)(G-G_i) + (G_i - T_{i-1}\{G_i\}).$$

Note that $T_{i-1}\{G_i\} = \sum_{j=1}^{i-1} t_j(G_j - G_{j-1})$, which is the sum of tax payments at the marginal rates in lower income brackets.

Since gross income is the sum of earnings and non-labor income, (5) can be rewritten in the form usually associated with a budget constraint:

$$(6) \qquad Y = (1-t_i)WH + Y^n + t_i(G_i - Y^n) - T_{i-1}\{G_i\}.$$

In (6) the income tax is shown clearly to consist of a wage or earnings tax at the marginal tax rate, t_i, and an income transfer equal to the difference of taxes that would have been paid if the rate t_i applied to all earnings, $t_i(G_i - Y^n)$, and taxes actually paid, $T_{i-1}\{G_i\}$. This is a surrogate income transfer in the sense that, over the ith gross income range, the family acts as if non-labor income was increased by $t_i(G_i - Y^n) - t_{i-1} G_i$. The after-tax budget line for an income tax with marginal rate equal to zero for gross income below G_1, and equaling t_1 elsewhere is illustrated by line abde in Figure 1. The equivalent wage tax and income transfer for families with gross income above G_1 are t_1 and $t_1(G_1 - Y^n)$, respecitvely.

First order conditions for a maximum of utility with respect to leisure and after tax income generate labor supply functions incorporating the wage tax and income transfer appropriate for each level of gross income. Thus for the interval of gross income for which equation (6) gives the appropriate budget constraint, labor supply is given by:

$$(7) \qquad H_i = H\{(1-t_i)W,\ Y^n + t_i(G_i-Y^n) - T_{i-1}\{G_i\}\}$$

Following (7) both linear segments bd and de of budget line abde in Figure 1 generate labor supply functions

$$(8) \qquad H_{bd} = H\{W, Y^n\}$$

and

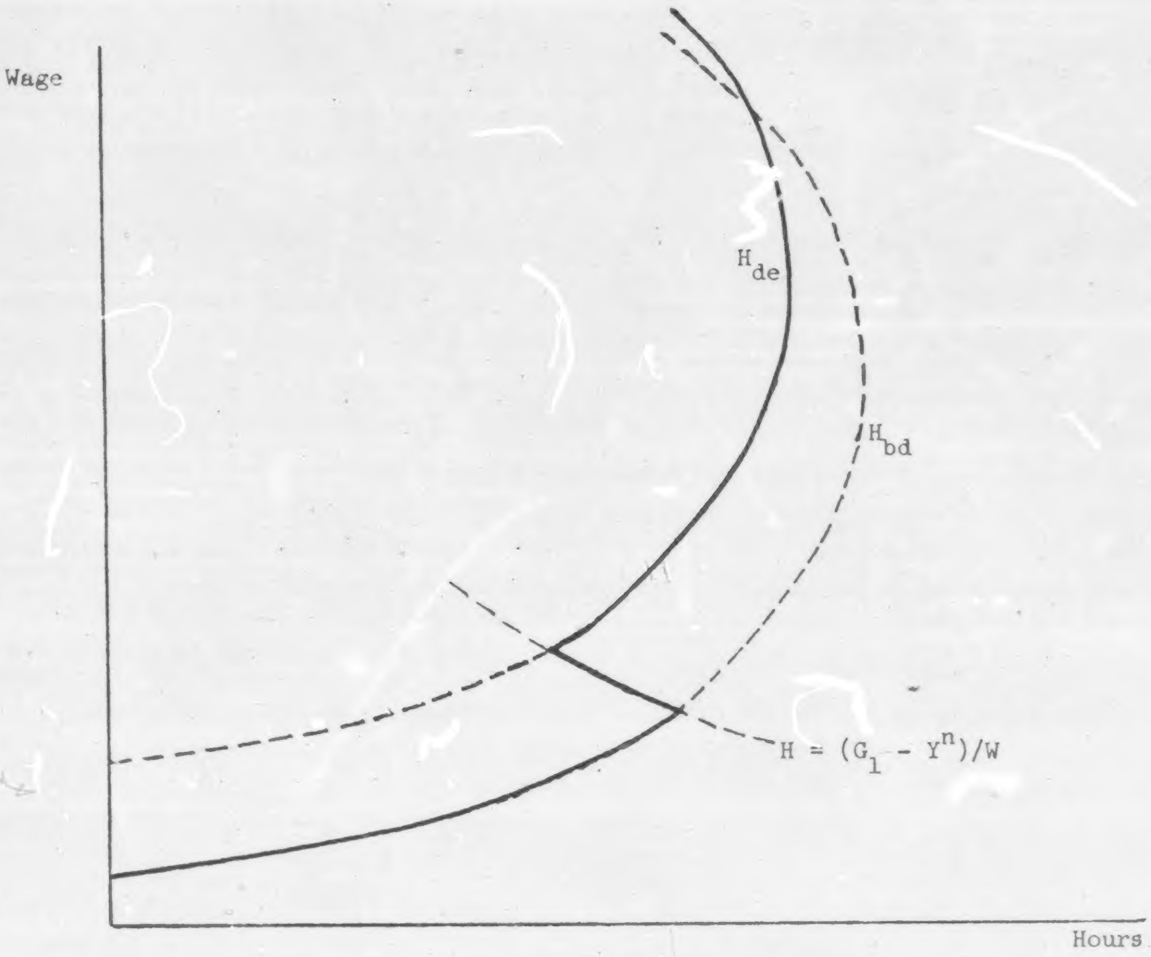
$$(9) \qquad H_{de} = H\{1-t_1)W,\ Y^n + t_1(G_1-Y^n)\}.$$

The supply function associated with tangency solutions on the line segment bd is shown by equation (8) to be equivalent to the initial labor supply relationship given by equation (3). In contrast, equilibria on the upper segment of the budget line, de, are shown by equation (9) to fall along a supply function incorporating a wage tax at rate t_1 and an income transfer of $t_1(G_1-Y^n)$. Figure 2 shows that the manhours, H_{de}, associated with equation (9) are displaced from H_{bd} by an uncompensated wage rate effect that shifts the supply curve vertically upward and an income transfer shifting the curve to the left, if leisure is not an inferior good.

At point d on budget line abde, where gross income is equal to G_1, the derivative of after-tax income with respect to leisure is not defined. Any number of indifference curves whose slope varies from $-W$ to $-(1-t_1)W$ may be tangent to the budget constraint at the kink where gross income equals G_1. Marginal conditions for a maximum of utility will not be satisfied by such tangencies resulting in a range of wages over which the family is not on a supply curve of the form implied by equations (8) or (9). At these wage rates the family will

Figure 2

Labor Supply Of A Single-Worker Family Subject To A Progressive Income Tax



$H_{bd} = H\{W, Y^n\}$

$H_{de} = H\{(1 - t_1)W, Y^n + t_1(G_1 - Y^n)\}$

find an equilibrium labor supply where gross income equals G_1, producing a constant earnings labor supply relationship:

(10) $$H = (G_1 - Y^n)/W.$$

Equation (10) is shown in Figure 2 to be a rectangular hyperbole connecting the two supply curves H_{bd} and H_{de}. Thus after-tax labor supply consists of a segment along H_{bd} until earnings, WH_{bd}, equal $(G_1 - Y^n)$; followed by a "jog" along the constant earnings supply curve $H = (G_1 - Y^n)/W$; and then by a segment along H_{de} beginning at the wage rate where WH_{de} equals $(G_1 - Y^n)$. We have illustrated the effect of a two-internal income tax, but it should now be obvious that the effect of a multiple-interval tax follows in like manner with a succession of segments joined by constant earnings jogs formed by rectangular hyperbolas.

II. LABOR SUPPLY OF THE MULTI-WORKER FAMILY

The multi-worker family has more than one potential worker. We shall consider the case of two potential workers, one primary worker and one secondary worker, which can be generalized to include additional workers. The algebraic analysis of family labor supply is already well established but the diagrammatic exposition presented here is most useful for interpretation of the effects of a progressive income tax. The family utility function has the same basic properties as in the single worker family analysis except that utility is maximized with respect to leisure of both the primary and secondary worker, L^P and L^s respectively:

(11) $$U = U\{L^P, L^s, Y\}.$$

The family utility function generates a family of indifference surfaces in (L^p, L^s, Y) space.

Family utility is maximized subject to a budget constraint which in the absence of a tax sets income equal to the sum of earnings from all family members and non-labor income:

$$Y = (K-L^p)W^p + (K-L^s)W^s + Y^n, \tag{12}$$

where K is total hours available for work and leisure, W^p and W^s are wage rates of primary and secondary workers respectively, and Y^n is non-labor income. The budget constraint defines a two dimenstional plane in L^p, L^s, Y space whose slope is $-W^p$ in the (L^p,Y) plane and $-W^s$ in the (L^s,Y) plane. When L^p and L^s are both equal to K, the plane passes through (K,K,Y^n) and when $L^p = L^s = 0$ it passes through $(0,0,W^pK^p + W^sK^s + Y^n)$.

Figure 3 shows the budget plane along with an equilibrium tangency with an indifference surface in (L^p,L^s,Y) space. First order conditions for a maximum of utility for L^p, L^s, and Y yield a familiar expression for manhours worked by primary and secondary workers:

$$H^p = K-L^p = H^p\{W^p,W^s,Y^n\} \tag{13}$$

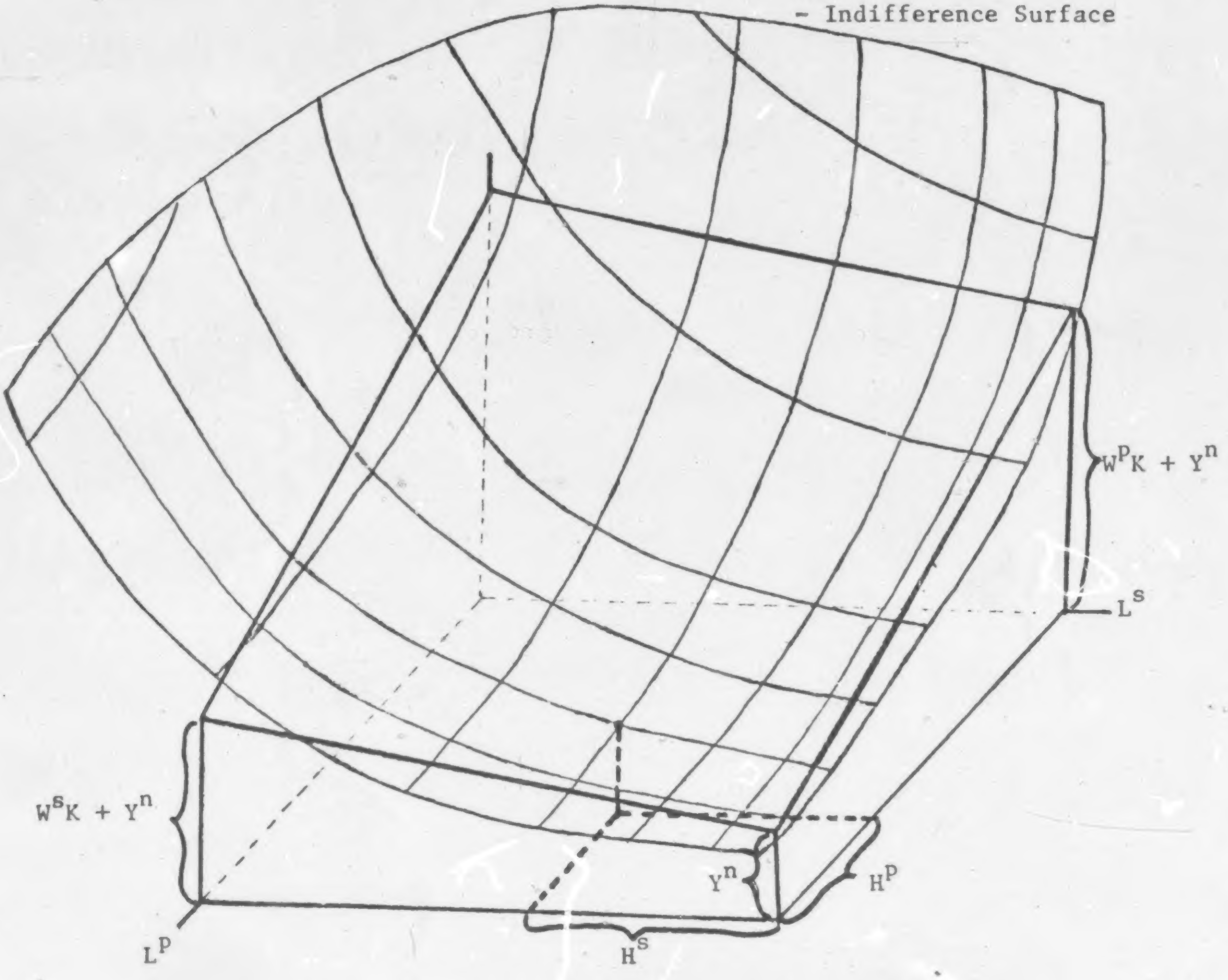
and

$$H^s = K-L^s = H^s\{W^s,W^p,Y^n\}. \tag{14}$$

The wage terms in (13) and (14) influence labor supply through both a compensated wage rate effect along an indifference surface and an income effect based on earnings of each individual, equivalent to the effect of non-labor income on

Figure 3

Equilibrium Tangency Of A Budget Constraint And An Indifference Surface Of A Multi-Worker Family


- Indifference Surface
$W^pK + Y^n$
L^s
$W^sK + Y^n$
Y^n
H^p
L^p
H^s

Equation Of The Budget Plane:

$$Y = W^pH^p + W^sH^s + Y^n$$

labor supply.

The introduction of a progressive income tax greatly modifies the income-leisure possibilities facing the family by making net wages endogeneous. The budget surface is composed of a series of planes, each of which is uniquely related to a particular tax bracket in which the marginal tax rate is constant. These individual planes are displaced from one another by a wage tax at the marginal tax rate and an income transfer equal to the difference between total tax payments that would be forthcoming if the marginal tax rate applied to all income and taxes actually paid.

Consider the budget plane associated with marginal tax rate t_i, which applies to gross income levels in the interval between G_i and G_{i+1}. Letting G represent any level of gross income such that $G_{i+1}>G>G_i$, we can express after-tax income, Y, as

$$Y = (1-t_i)(G-G_i) + G_i - T_{i-1}\{G_i\}, \tag{15}$$

which, of course, is the same as (3).

Gross income can be written as the sum of earnings of both workers and nonlabor income; (15) can be rewritten in the form usually associated with a budget constraint.

$$Y = [(1-t_i)W^p]H^p + [(1-t_i)W^s]H^s + [Y^n + t_i(G_i-Y^n) - \sum_{j+1}^{i-1} t_j(G_j-G_{j-1})] \tag{16}$$

In (16) the wage tax is shown clearly as $(1-t_i)W^p$ and $(1-t_i)W^s$ while the income transfer necessary to generate this particular budget plane is the difference of taxes that would have been paid if the rate t_i applied to all earnings $t_i(G_i-Y^n)$, and taxes actually paid, $\sum_{j+1}^{i-1} t_j(G_j-G_{j-1})$. Equation (16) defines the plane whose slope is $-(1-t_1)W^P$ in the (L^P,Y) plane and $-(1-t_1)W^S$ in the (L^S,Y) plane and whose height when $L^P=L^S=K$ is the sum of non-labor income and the income transfer. This is but one of a series of budget planes each associated with the range of income in which a given marginal tax rate applies.

Figure 4 illustrates the budget surface of a family facing a two-interval income tax with a zero marginal tax rate on gross income below G_1 and a positive marginal tax rate of t_1 otherwise. At levels of gross income below G_1 the budget surface coincides with plane abcd generated by the pre-tax budget constraint:

$$Y = W^P H^P + W^S H^S + Y^n. \tag{17}$$

When gross income exceeds G_1, the relationship in equation (16) can be used to write the after-tax budget constraint in terms of a wage tax and income transfer:

$$Y = [(1-t_1)\ W^P]H^P + [(1-t_1)W^S]H^S + [Y^n + t_1(G_1-Y^n)] \tag{18}$$

The plane wxyz in Figure 4, drawn with a slope of $-(1-t_1)W^P$ in the (L^P,Y) plane and with slope $-(1-t_1)W^S$ in the (L^S,Y) plane, forms the after-tax budget surface relevant for gross incomes above G_1.

Maximizing utility of income and leisure subject to budget constraint (13) yields labor supply functions for the primary and secondary family workers identical to the pre-tax case:

$$H_a^P = K-L_a^P = H^P\{W^P,W^S,Y^n\} \tag{19}$$

and

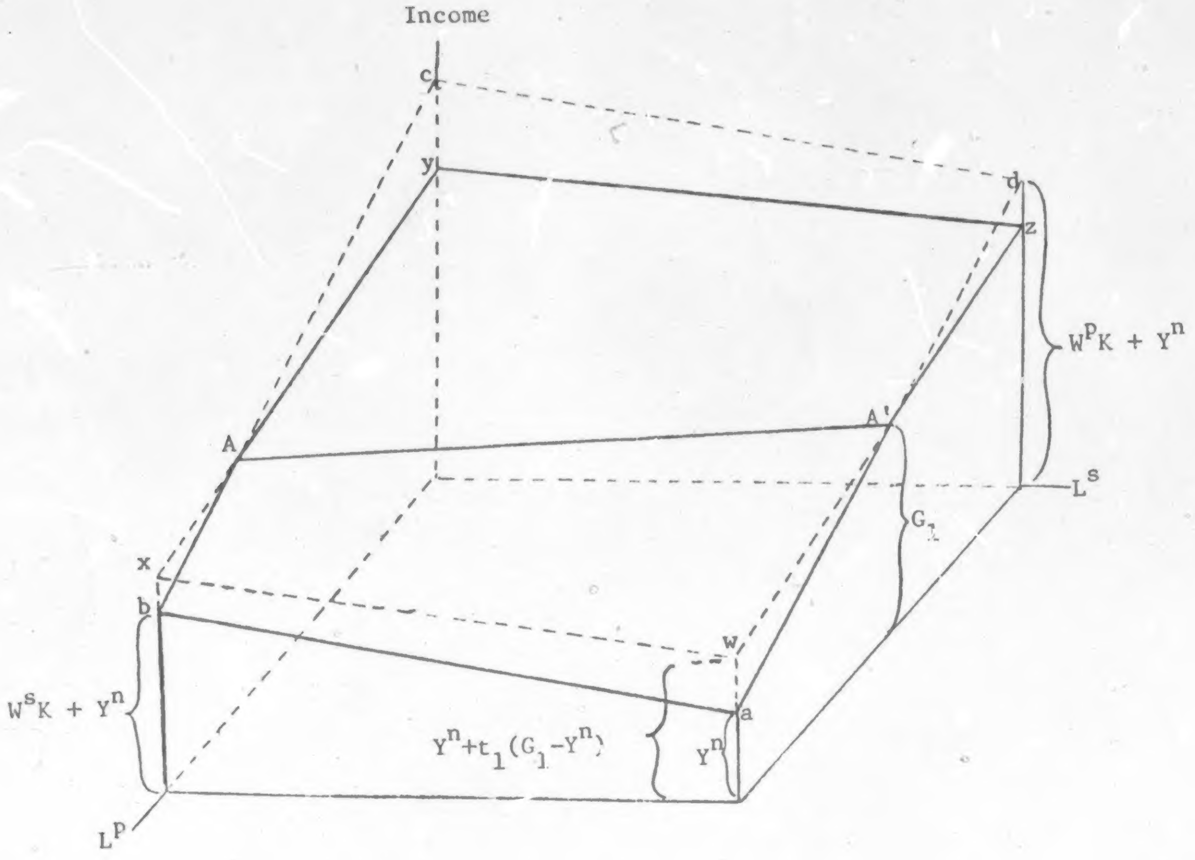
$$H_a^S = K-L_a^S = H^S\{W^S,W^P,Y^n\}$$

The subscript a on H_a^P and H_a^S indicates that labor supply is given by an equilibrium tangency on budget plane abcd. Similarly maximizing utility on budget constraint (18) gives labor supply associated with tangency solutions on plane wxyz:

$$H_w^P = K-L_w^P = H^P\{(1-t_1)W^P,\ (1-t_1)W^S,\ Y^n +t_1(G_1-Y^n)\} \tag{20}$$

Figure 4

Budget Constraint Of A Multi-Worker Family Subject To A Progressive Income Tax


Income
c
y
d
z
$W^PK + Y^n$
A
A'
L^s
G_1
x
b
w
a
$W^sK + Y^n$
$Y^n+t_1(G_1-Y^n)$
Y^n
L^P

Equations Of Budget Planes And The Intersection Line

Plane abcd: $Y = W^PH^P + W^sH^s + Y^n$

Plane wxyz: $Y = (1-t_1)W^PH^P + (1-t_1)W^sH^s + Y^n + t_1(G_1-Y^n)$

Line AA'. $Y = G_1 = W^PH^P + W^sH^s + Y^n$

and

$$H_w^s = L - L_w^s = H^s\{(1-t_1)W^s,\ (1-t_1)W^p,\ Y^n + t_1(G_1 - Y^n)\}$$

For given W^p, W^s, and Y^n, (19) and (20) determine the equilibrium levels of primary and secondary work hours implied by a tangency solution on planes abcd and wxyz respectively. In order to simulate or predict labor supply in the presence of an income tax based on given primary and secondary supply functions it is necessary to calculate gross family income implied by both (19) and (20). If both $W^pH_a^p + W^sH_a^s + Y^n$ and $W^pH_w^p + W^sH_w^s + Y^n$ are less than G_1 then labor supply is given by (15) with a tangency on the portion of the budget surface in plane abcd. However, if both inequalities are reversed, equilibrium is found on the budget surface lying in plane wxyz and labor supply is given by equations (16). Other possible combinations of inequalities indicate equilibria on the line $G_1 = Y$ and raise special problems which will be discussed below.

Within the gross income intervals where marginal tax rates are constant, the after-tax budget surface has been shown to consist of a section of the appropriate budget plane embodying the wage tax and income transfer associated with the marginal tax rate. These budget planes intersect in lines along which the sum of net earnings and non-labor income is equal to the after-tax income where marginal tax rates change, $G_i - T_{i-1}\{G_i\}$. Figure 4 illustrates such an intersection between budget planes generated by an income tax. The line AA' in Figure 6, along which after-tax income is $G_1 - T_0\{G_i\}$ or G_1, is a locus of points at which the after-tax budget surface is not differentiable. Marginal conditions for a maximum of utility for family leisure and income are not satisfied at the intersection of after-tax budget planes. All planes whose slope varies from $-(1-T_1)W^p$ to W^p in the (L^p, Y) plane and from $-(1-t_1)W^s$ to W^s in the (L^s, Y) plane could pass through (contain) the line AA' and form an equilibrium tangency with an indifference surface along AA'. There will be a range of wage rates for which tangency solutions occur on such planes containing AA' but which do not coincide with any of the planes that are part of the budget surface. Labor supply over such a range of wages is given by a constant-earnings supply-relationship:

$$G_1 = W^pH^p + W^sH^s + Y^n. \tag{21}$$

Of course, (21) is the equation of the line AA'.

Marginal conditions for a maximum of utility used to generate labor supply functions such as (19) and (20) are not operative along the constant earnings supply relationship. The range of wage rates for which labor supply follows equation (21) is determined by eliminating cases in which an equilibrium could occur on the portion of a plane which is part of a budget surface. But if $W^pH^p_a + W^sH^s_a + Y^n$ is greater than or equal to G_1 and $W^pH^p_w + W^sH^s_w + Y^n$ is less than or equal to G_1, neither tangency occurs on the section of the relevant planes within the budget surface. For wage rates and non-labor income producing such inequalities, family labor supply is given by a constant-earnings supply relationship.

Along a constant-earnings labor supply relationship all family members are off their behavioral labor supply curves defined by first-order conditions for a maximum of utility. The income tax produces a programmatic interaction between labor supply decisions of family members, provided that joint returns are filed. A complete specification of the family utility function is necessary to determine the mix of primary and secondary workers hours long the constant earnings relationship. In the absence of such knowledge, only the weighted sum of manhours is determinant, with weights proportional to relative wages and earnings constrained to a constant value.

III. IMPLICATIONS FOR ESTIMATION AND SIMULATION

Empirical inquiry into labor supply in the presence of existing progressive income taxes has taken two forms. Survey research has been used to determine individual perceptions of incentive effects. Econometric estimates of labor supply functions have been performed with data on workers subject to an income tax.

Survey research from the classic study by Break [3] to the recent work of Fields and Stanbury [6], Boskin [2], and Holland [8] interprets the incentive and disincentive effects in terms of the respective income and substitution effects of a wage tax at the marginal tax rate. The ratio of disincentive to incentive effects is often found to rise with income and hence with the marginal tax rate. This is analyzed as the result of a rise in the substitution effect relative to the income effect associated with a fall in the price of leisure. However, the analysis presented above illustrates that high marginal tax rates are associated with large income transfer effects, given the structure of progressive income taxes. The income transfer produces a disincentive effect of its own which reinforces the compensated wage rate effect and contributes to the tendency for the ratio of disincentive to incentive effects to rise with income. The relationship between gross income and the size of the income transfer is totally dependent on the exact structure of marginal tax rates. Thus the survey approach does not, as if often claimed, yield information on the effect of high marginal tax rates on work incentive. It only gives an insight into the effect of high marginal rates in the context of the particular progressive income tax structure to which the respondent is subject. To the extent that incentive and disincentive effects appear to be offsetting when both income and marginal tax rates are high, our analysis suggests that families are on the backward bending portion of their supply curves so that a wage tax raises hours supplied while the associated income transfer lowers hours. The net effect may well be to leave after-tax hours equal to labor supplied in the absence of an income tax.

An extensive literature of econometric estimates of reduced form family labor supply functions has developed in recent years including the work of Kosters [10, 11], Kalachek and Raines [9], Cohen, Rea, and Lerman [4], and Ashenfelter and Heckman [1]. With the exception of an ex post test for the magnitude of bias by Cohen, Rea, and Lerman [4], the effects of wage taxes and income transfers implied by the tax structure are not taken into account. The effect of ignoring the wage tax and income transfer adjustments, both of which increase at higher marginal tax rates, is to have gross wages overstate after-tax wages and non-labor income understate the sum of non-labor income and the appropriate income transfer by increasing amounts as wages and income rise. Such an omission would bias the absolute value of estimates of uncompensated wage rate effects downward while the income effect based on non-labor income would be biased upward. These sources of bias can be eliminated by transforming wage and income data for families to incorporate the appropriate wage tax and income transfer based on gross income before the estimation. Such transformations are possible because families reaching equilibrium solutions on budget surfaces formed by planes such as abcd or wxyz have the same behavioral labor supply functions, distinguished only by taxes and income transfers.

Fundamental problems for the estimation of underlying labor supply functions from data on families subject to an income tax are raised by the presence of constant earnings supply relationships. These difficulties arise because there is no combination of wage taxes and income transfers which can transform the arguments of the labor supply function based on marginal conditions to reflect behavior when families are not satisfying these conditions. In the absence of a transformation which can adjust observed data to return families to a common supply function, unbiased estimation requires that observations of families with gross income at or near level where marginal rates change be dropped from the sample.[1] Observations on families with gross income well within intervals where marginal rates are constant should be modified to include the appropriate wage tax and income transfer.

The analysis presented above indicates that estimation of labor supply relationships in the presence of an income tax requires extensive transformation of wage and income data. However, even assuming that an estimate of the true parameters of the supply function is obtained, simulation of supply responses to a reform in the income tax requires special attention.[2] The labor supply function associated with each interval of gross income in which marginal rates are constant must be generated by noting the appropriate wage tax and income transfer effects. Thus to simulate multi-worker family supply response to the personal income tax one would first have to generate the supply functions in equations (19) and (20). Then the gross income associated with hours supplied by family members facing given W^p, W^s, and Y^n must be calculated using each pair of supply equations. This calculation gives the gross family income associated with an equilibrium tangency on each plane which comprises the budget constraint. But we must determine whether the equilibrium is on a segment of the plane which makes up the budget surface or if the tangency occurs elsewhere on the plane. There is a simple test for the relationship between the point of tangency with each plane and the budget surface. The tangency solution is on the budget surface if and only if gross income defined by the tangency solution and calculated from the associated primary and secondary supply equations lies within the interval of gross income for which that plane forms the relevant constraint on after-tax income.

There are two possible outcomes of the simple test for gross income implied by each pair of supply equations. If one pair of equations gives gross income consistent with the interval for which the wage tax and income transfer effects in the equations is relevant, labor supply is determined by the single pair of equations that pass the test. A second possibility is that no pair of equations can be found which yield gross income which is consistent with the interval for which the equations are appropriate. This indicates the presence of a tangency solution on a line which generates a constant earnings supply relationship.

The existence of constant earnings supply relationships implies a fundamental limitation on the ability to simulate family work effort in the presence of an income tax. Along a constant earnings relationship, the mix of primary and secondary worker hours is determined only through detailed knowledge of the family utility function. In practice only the weighted sum of hours can be simulated, with weights determined by relative wages and the sum constrained to equal total earnings. This reflects a fundamental difference between the single and multi-worker family labor supply models. Programmatic interaction in the labor supply decision introduced by an income tax makes the mix of multi-worker hours interdependent along constant earnings supply relationships.

IV. CONCLUSIONS

The diagrammatic analysis of family labor supply developed here provides a useful vehicle for analyzing the special effects of personal income taxes. A progressive income tax is translated into a series of wage taxes and income transfers in order to predict labor supply response of families whose gross income falls in a region where marginal tax rates are constant. Labor supply depends on the structure as well as the size of marginal tax rates. Survey research on work effort in the presence of an income tax does not separate the supply effects of the marginal tax rate from those of the income transfer implied by the prevailing rate structure. At levels of family income, where marginal tax rates change, family labor supply falls on "constant earnings" supply relationships along which earnings are fixed but the mix of hours supplied by various family members can only be determined from the family utility function. In contrast to results for gross income intervals where marginal rates are constant, analysis of "constant earnings" supply relationships demonstrated that there was no transformation combining a wage tax and income transfer which could describe labor supply in terms of behavioral labor supply function based on marginal conditions for utility maximization.

The theoretical points made here have significant implications for econometric estimation and simulation of family labor supply in the presence of the personal income tax. A series of special adjustments in the data, including omission of some observations and transformation of wage and non-labor income observations, is necessary for unbiased estimation of reduced form labor supply functions. None of the previous studies surveyed have attempted such adjustments.

Simulation of family supply responses to new or existing income taxes requires that wage tax and income transfer effects be incorporated in the analysis. In some ranges family labor supply is interdependent so that only detailed knowledge of the underlying family utility function can separate the mix of primary and secondary worker hours. The final effect of given marginal tax rates on labor supply depends on the entire structure of the personal income tax. It is this structure which determines the income transfer effects associated with any given marginal tax rate.

FOOTNOTES

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1. Wales (12) found it necessary to exclude observations near boundaries from his sample. His observation that such exclusion may not be necessary for reduced form models is not accurate.

2. Kosters (10) simulates labor supply effects of the personal income tax as a wage tax at the marginal tax rate. Other authors are not so explicitly concerned with the personal income tax but they do discuss effects of wage taxes at alternative marginal tax rates.

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WORK EFFORT, HUMAN INVESTMENT AND THE INCOME TAX

by C. Duncan MacRae and Elizabeth C. MacRae

Abstract

A model of individual human capital accumulation is applied in this paper to the analysis of the effects of a progressive income tax with a given level of income exempted from tax and a constant marginal tax rate. It is shown that the effects of changes in the tax parameters can be described in terms of intertemporal substitution and income effects. Given an increase in the tax rate, if the intertemporal substitution effect dominates, then the individual substitutes income in the early stages for income later in the life cycle. If the intertemporal income effect dominates, however, the individual spreads the life cycle income reduction over all stages. Similarly, given an increase in the exemption level, if the intertemporal income effect dominates, the increase in total income is spread over all stages; if the intertemporal substitution effect dominates, income just before the taxable stages is substituted for income in all other stages.

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WORK EFFORT, HUMAN INVESTMENT, AND THE INCOME TAX*

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The traditional analysis of the effect of the personal income tax on work effort is based on the static income-leisure model of behavior. The individual is assumed to allocate his time between labor and leisure so as to maximize the utility of income and leisure subject to a budget constraint which is determined by the individual's wage rate and unearned income. The income tax is then analyzed by translating it into an effective change in the budget constraint.[1] As useful as this analysis is, it ignores the alternative to work effort of allocating time to human capital accumulation and, thereby, increasing the wage available to the individual. Indeed this alternative may be more important in determining work effort than is the allocation of time to leisure activities.

The purpose of this paper is to apply a dynamic human capital model[2] to the analysis of the effect of the income tax on work effort and human investment. The individual is assumed to allocate his time between earning income and accumulating capital so as to maximize the utility of income over the life cycle subject to a budget constraint relating income and investment to human capital through the wage rate and a production function for human capital. The amount of time allocated to leisure is assumed to be constant. Life-cycle behavior under the income tax is then analyzed with the aid of optimal control theory. The paper concludes with a comparative dynamics analysis of the effects of a change in the marginal tax rate or the level of income exempted from tax.

I. WORK-INVESTMENT MODEL

In this section a model of human capital accumulation by an individual subject to an income tax is developed. We begin by stating the time allocation problem over his life cycle and then characterize its solution with the aid of the maximum principle.

Time Allocation Problem

By definition an individual's gross income, G, is equal to the sum of his earned income $W\ell$, and his unearned income, Y^n:

$$G = W\ell + Y^n \,, \tag{1.1}$$

where W is the individual's wage rate, and ℓ is the proportion of non-leisure time allocated to work ($0<\ell<1$); both Y^n and the amount of time allocated to leisure are assumed to be given and constant over time.[3] The wage rate is not taken as given but is assumed to be the product of the market rental on human capital, ω, and the stock of capital possessed by the individual, K:

$$W = \omega K, \tag{1.2}$$

where ω is assumed constant throughout the life cycle.

The personal income tax is applied on earned and unearned income without distinction but subject to an exemption E so that net income, Y, is given by

$$Y = G - T\{G\} \tag{1.3}$$

where the tax function $T\{G\} = 0$ for $G<E$ and $T\{G\} = \alpha(G-E)$ otherwise, where α is the given constant marginal tax rate. This piecewise linear tax function corresponds to the typical progressive income tax save only that there are more segments in the typical tax.

Human capital can be acquired through formal schooling or on-the-job-training.[4] In either case, the production of human capital is assumed to occur under constant returns to scale so that gross investment, I, is given by

$$(1.4)\qquad I = (1-\ell)\gamma K,$$

where $(1-\ell)$ is the proportion of non-leisure time devoted to investment and γ is a constant rate of productivity.[5] Assuming a constant rate of depreciation, μ, in the stock of human capital, net investment is then

$$(1.5)\qquad dK/dt = I - \mu K,$$

where $K\{0\} = K^{o}$, the individual's initial endowment of capital. Note that dK/dt can be negative, if capital depreciation (μK) exceeds gross investment (I).

Finally, the individual is assumed to have an intertemporal utility function, J, on income and leisure over his life cycle; since leisure time is assumed to be constant, J can be expressed solely as a function of income:

$$(1.6)\qquad J = \int_{0}^{N} U\{Y\}e^{-\delta t}dt,$$

where N is the individual's horizon, $\delta > 0$ is his given time preference rate, and U has the properties that $U' > 0$ and $U'' < 0$, corresponding to positive but diminishing marginal utility from income.[6] The time allocation problem, therefore, is to choose ℓ over the life cycle so as to maximize utility (J) subject to the income identity (1.1), the wage equation (1.2), the budget equation (1.3), the production function (1.4), and the accumulation equation (1.5), given the initial endowment K^{o}.

Maximum Principle Solution

The solution to the time allocation problem can be characterized with the aid of the maximum principle.[7] If the path of ℓ is to maximize J in the long run, then in the short run ℓ must be chosen to maximize the Hamiltonian

$$(1.7)\qquad H = U\{Y\}e^{-\delta t} + \Psi(I-\mu K),$$

given K and Ψ, where the paths of K and Ψ over time are determined by the differential equations

$$(1.8)\qquad dK/dt = \partial H/\partial\Psi$$

and

$$(1.9)\qquad d\Psi/dt = -\partial H/\partial K$$

with boundary conditions $K\{0\} = K^o$ and $\Psi\{N\} = 0$. Note that (1.8) is equivalent to (1.5). The auxiliary variable Ψ is the present discounted value of human capital to the individual measured in utility units, so that the Hamiltonian H may be interpreted as discounted short-run utility. The zero value for $\Psi\{N\}$ reflects the fact that human capital has no value to the individual at the end of his working life.

We can simplify the notation by rewriting equations (1.7) and (1.9) using current values rather than present discounted values. To do this, let P be the current value of capital in utility units, i.e.,

$$(1.10)\qquad P = \Psi e^{\delta t}\ .$$

Then in the short run, ℓ must maximize the undiscounted Hamiltonian

$$(1.11) \quad H = U\{Y\} + PI,$$

with K and P taken as given. In the long run P is determined by

$$(1.12) \quad dP/dt = - \partial H/\partial K + (\mu+\delta)P$$

with $P\{N\} = 0$, as can be seen by differentiating (1.10) with respect to time and substituting the result into (1.9). Therefore, the solution to the long-run time allocation problem is completely characterized by equation (1.5) with an initial condition and equation (1.12) with a terminal condition where ℓ is chosen so as to maximize H in the short-run subject to the constraints (1.1)-(1.4).

II. LIFE-CYCLE BEHAVIOR

In this section we begin by describing the short-run behavior of an individual subject an income tax, when the value and stock of human capital are taken as given. A comparative statics analysis of changes in the market rental, in the value and stock of capital, and in the tax parameters is then performed. The results of this analysis will be used in the next section. Finally, we analyze the behavior of the individual in the long run, when the value and stock of human capital are changing over the life cycle.

Short-Run Behavior

The solution to the short-run time allocation problem can best be seen geometrically in Figure 1. Gross investment (I) is measured on the positive horizontal axis, net income (Y) is measured on the positive vertical axis, and work effort (ℓ) is measured on the negative vertical axis. The individual's budget frontier begins on the horizontal axis at γK where according to

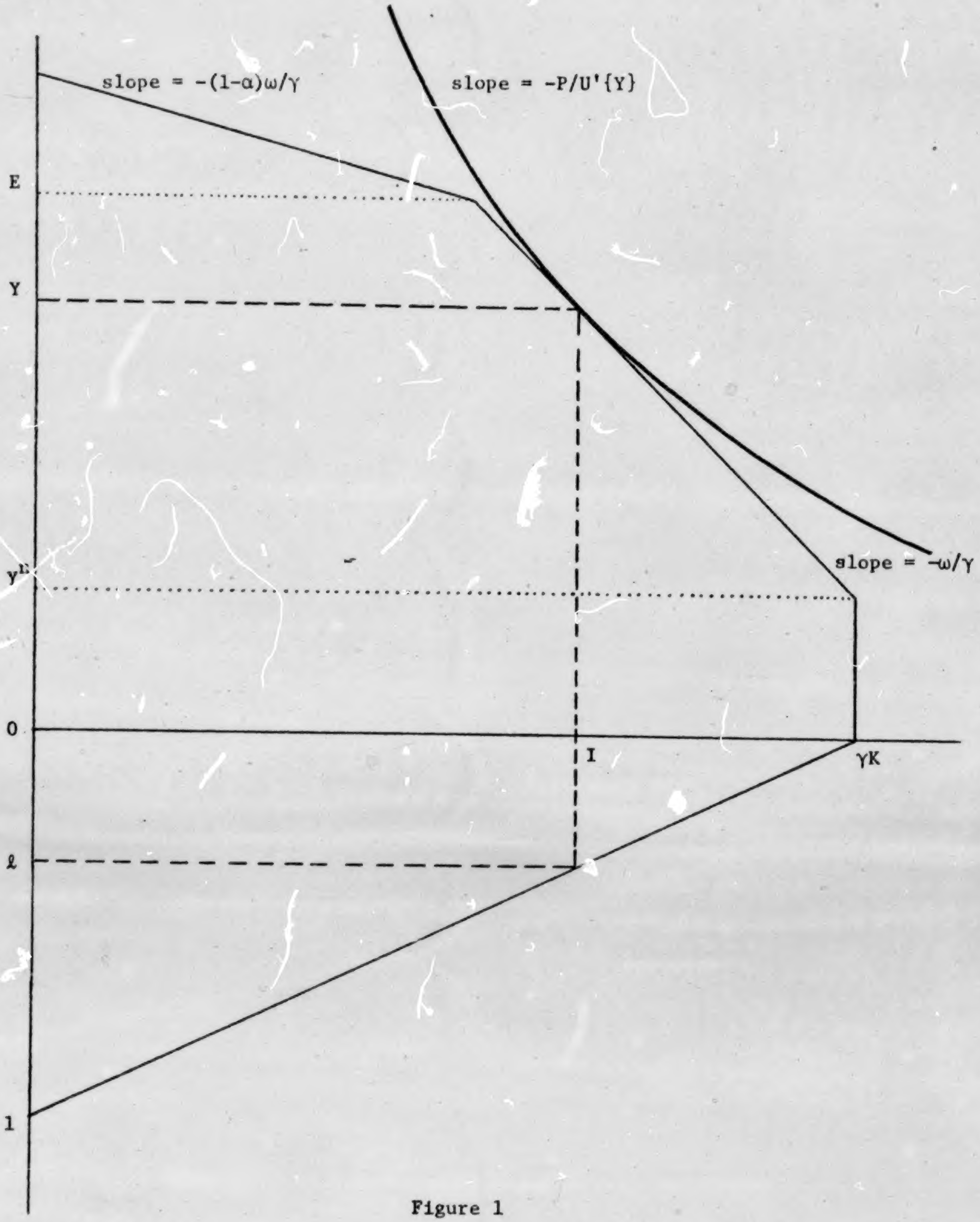
slope = -(1-α)ω/γ
slope = -P/U'{Y}
E
Y
slope = -ω/γ
Y^L
0
I
γK
ℓ
1

Figure 1

Hamiltonian Map and Income - Investment Constraint Set

(1.4) gross investment is at a maximum and $\ell=0$. Then, assuming for interest that E is greater than Y^n, the budget frontier is vertical at $\ell=0$ up to the point where $Y = Y^n$; even with no labor devoted to work the individual still has some unearned income. From there, as can be seen from (1.3) and (1.4), the frontier has a slope of $-\omega/\gamma$ for Y (and G) below E, then a lower slope of $-(1-\alpha)\omega/\gamma$ in the range of taxable income. If G is less than E, the individual pays $(1-\ell)\omega K$ in foregone current income to receive $(1-\ell)\gamma K$ increments of human capital. Therefore, the opportunity cost of a unit of investment is ω/γ. In the taxable range, however, he pays only $(1-\alpha)(1-\ell)\omega K$ so that the opportunity cost, depicted by the lower negative slope of the budget frontier, is $(1-\alpha)\omega/\gamma$. Finally, note that the slope does not depend upon the level of K. Thus, increases in the individual's human capital stock simply shift the entire frontier to the right parallel to the horizontal axis, so that both potential investment and potential income are increased.

The Hamiltonian, as defined by (1.11), represents a map of short-run indifference curves for income and investment, similar to the income-leisure utility map in static analysis, and displays the usual properties of an additive utility function. The marginal rate of substitution between investment and income, $-P/U'$, increases as Y increases, but is not infinite at $I=0$ so that the indifference curves cross the vertical axis. Since the marginal utility of investment in the Hamiltonian is constant for given P, the slope of the indifference curves does not depend upon I.

As in the static income-leisure model, the solution to the short-run time allocation problem can be obtained by finding the point on the budget frontier which lies on the highest indifference curve; this corresponds to maximizing H subject to $0\leq\ell\leq1$. We can see from Figure 1 that there are five possible types of solution dependong on K and P: two interior solutions and three corner solutions. The two interior solutions are those for which $\partial H/\partial \ell = 0$, and either $G<E$ or $G>E$, where

$$(2.1) \qquad \partial H/\partial \ell = U'\{Y\}\omega K(1-T') - P\gamma K,$$

and the three corner solutions occur where $\ell=0$, $\ell=1$ or $G=E$.

The five possible types of solution to the short-run problem occur in five different stages of the life cycle. These stages are portrayed in Figure 2. Stage I, where all effort is devoted to investment ($\ell=0$) corresponds to all points in (K,P) space above the horizontal line

$$(2.2) \qquad P = U'\{Y^n\}\omega/\gamma,$$

which is obtained by setting both $\partial H/\partial \ell$ and ℓ equal to 0 in (2.1). Stage II, where $\ell>0$ but no tax is paid, lies below the line given by (2.2) but above the horizontal line.

$$(2.3) \qquad P = U'\{E\}\omega/\gamma \ ,$$

and to the right of Stage V ($\ell=1$) which will be defined below. The lower boundary of Stage III ($G=E$) is also a horizontal line,

$$(2.4) \qquad P = U'\{E\}(1-\alpha)\omega/\gamma$$

and the left-hand boundary is the edge of Stage V. Stage IV, where $\ell>0$ and tax is paid lies below Stage III and above the boundary of Stage V given by

$$(2.5) \qquad P = U'\{(\omega K + Y^n) - \alpha(\omega K + Y^n - E)\}(1-\alpha)\omega/\gamma,$$

which is obtained by setting $\partial H/\partial \ell = 0$ and $\ell=1$ in (2.1), and is negatively

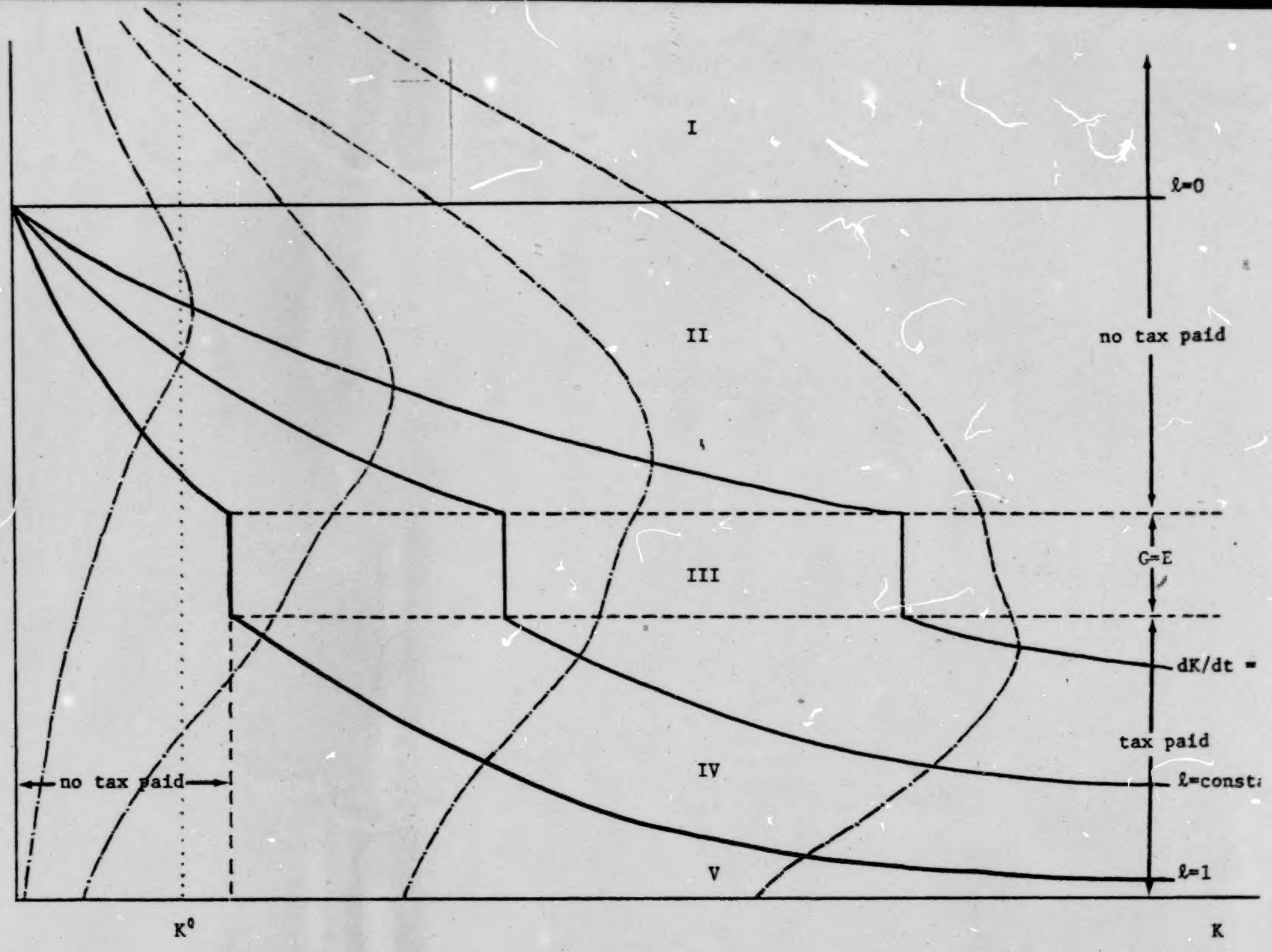
I
ℓ=0
II
no tax paid
G=E
III
dK/dt =
tax paid
IV
ℓ=const
V
ℓ=1
no tax paid
K⁰
K

Figure 2

Optimal Paths in (K,P) Space

sloped since $U''<0$. The other two boundaries, between Stage V and Stages II and III, are defined by the negatively sloped line

$$(2.6) \qquad P = U'\{\omega K + Y^n\}\omega/\gamma$$

and by the vertical line

$$(2.7) \qquad K = (E - Y^n)/\omega$$

respectively. Note that all constant ℓ lines are similar in shape to the $\ell=1$ line which forms the upper boundary of Stage V.

Comparative Statics

In the short run the market rental (ω) plays the role in the dynamic human capital model that the wage rate plays in static income-leisure analysis. Given K and P, the individual's work effort (ℓ) is a function of ω, as illustrated in Figure 3. In general, the higher is the market rental, the higher is ℓ. However, there are exceptions. One is when the individual is at the kink in the income-investment constraint depicted in Figure 1, corresponding to Stage III in Figure 2. Then lower work effort is associated with higher market rentals so as to maintain G=E.[8] The other exception is if the income effect dominates the substitution effect of a higher ω.

To calculate the short-run effects of alternative market rentals, we set $\partial H/\partial \ell = 0$ in (2.1) and then differentiate implicitly with respect to ω to get the partial derivative

$$(2.8) \qquad \partial\ell/\partial\omega = -(1 + \varepsilon_{W\ell})\ell/(\omega\varepsilon_{W\ell}), \qquad \text{Stages II and IV,}$$

where the net income elasticity of marginal utility is given by

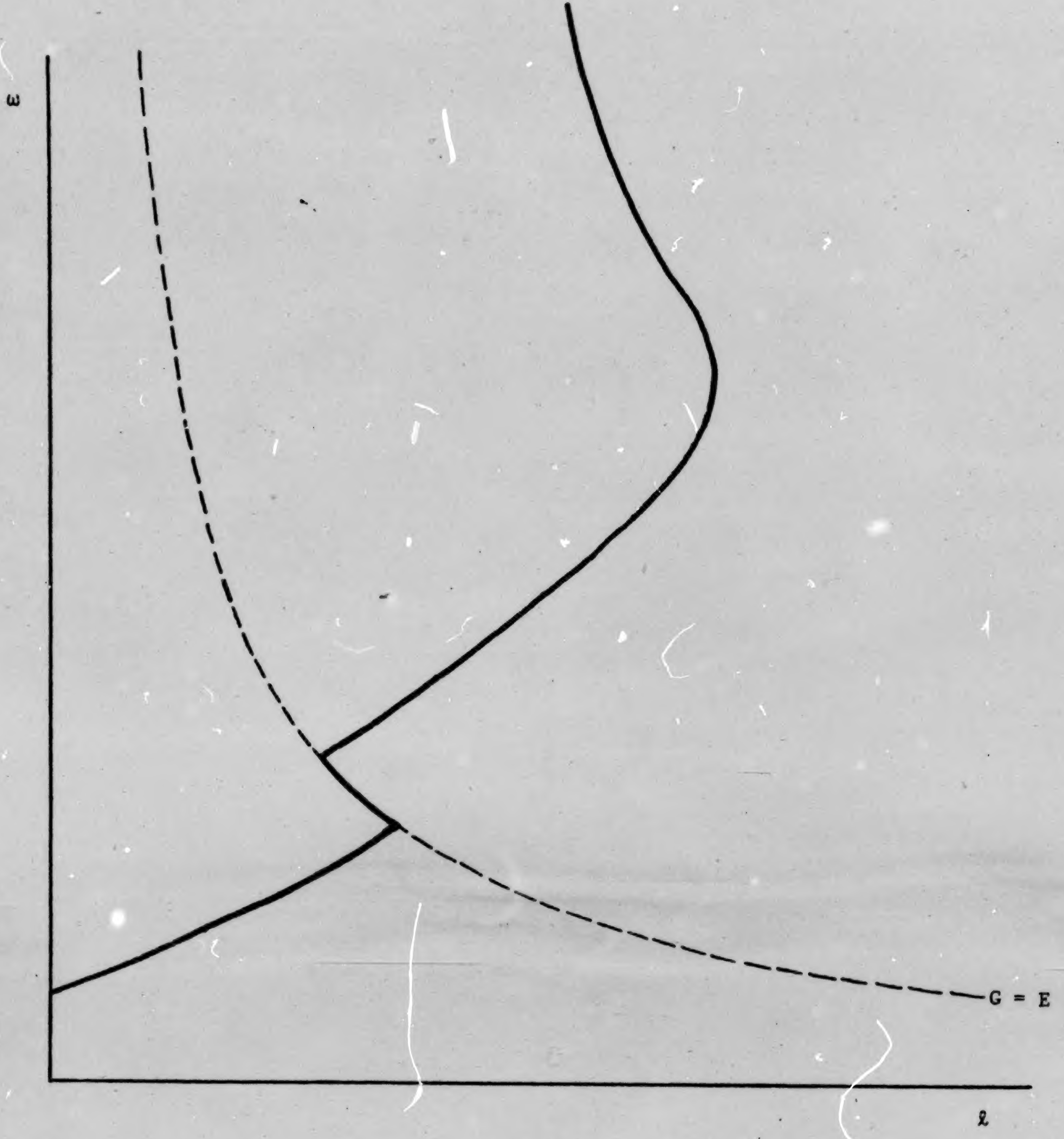

Figure 3

Work Effort as a Function of Market Rate of Return in the Short Run (K, P constant)

8.11

$$(2.9) \qquad \varepsilon_Y = (dU'/dY)(Y/U') = U''Y/U'$$

and the earned income elasticity of marginal utility, defined analogously, is

$$(2.10) \qquad \varepsilon_{W\ell} = ((1-T')W\ell/Y)\varepsilon_Y, \qquad \text{Stages II and IV.}$$

Only if the negative income effect, $-\ell/\omega$, dominates the positive substitution effect, $-\ell/(\omega\varepsilon_{W\ell})$ will a lower level of work effort be associated with a higher market rental, corresponding to a backward-bending labor supply. As can be seen from (2.8), this can only occur if $|\varepsilon_{W\ell}| > 1$. Since $|\varepsilon_{W\ell}| < |\varepsilon_Y|$, a backward-bending labor supply requires that the schedule of marginal utility with respect to net income be sufficiently elastic to offset the effects of unearned income and the income tax.

Changes in the stock (K) and value (P) of human capital which occur over the life cycle bring about shifts in the short-run labor supply. An increase in K moves the individual's budget constraint to the right parallel with the horizontal axis in Figure 1. Since the marginal short-run utility of investment is constant, the level of Y is unchanged by the shift so that work effort must decrease, as is shown in Figure 4, to offset the increase in K and hence, in W:

$$(2.11) \qquad \partial\ell/\partial K = -\ell/K, \qquad \text{Stages II, III, and IV.}$$

An increase in P simply makes the indifference curves steeper so that work effort is lower, as is illustrated in Figure 5, corresponding to a movement to the right along the budget constraint:

$$(2.12) \qquad \partial\ell/\partial P = \gamma/(\omega^2(1-T')^2KU''), \qquad \text{Stages II and IV,}$$

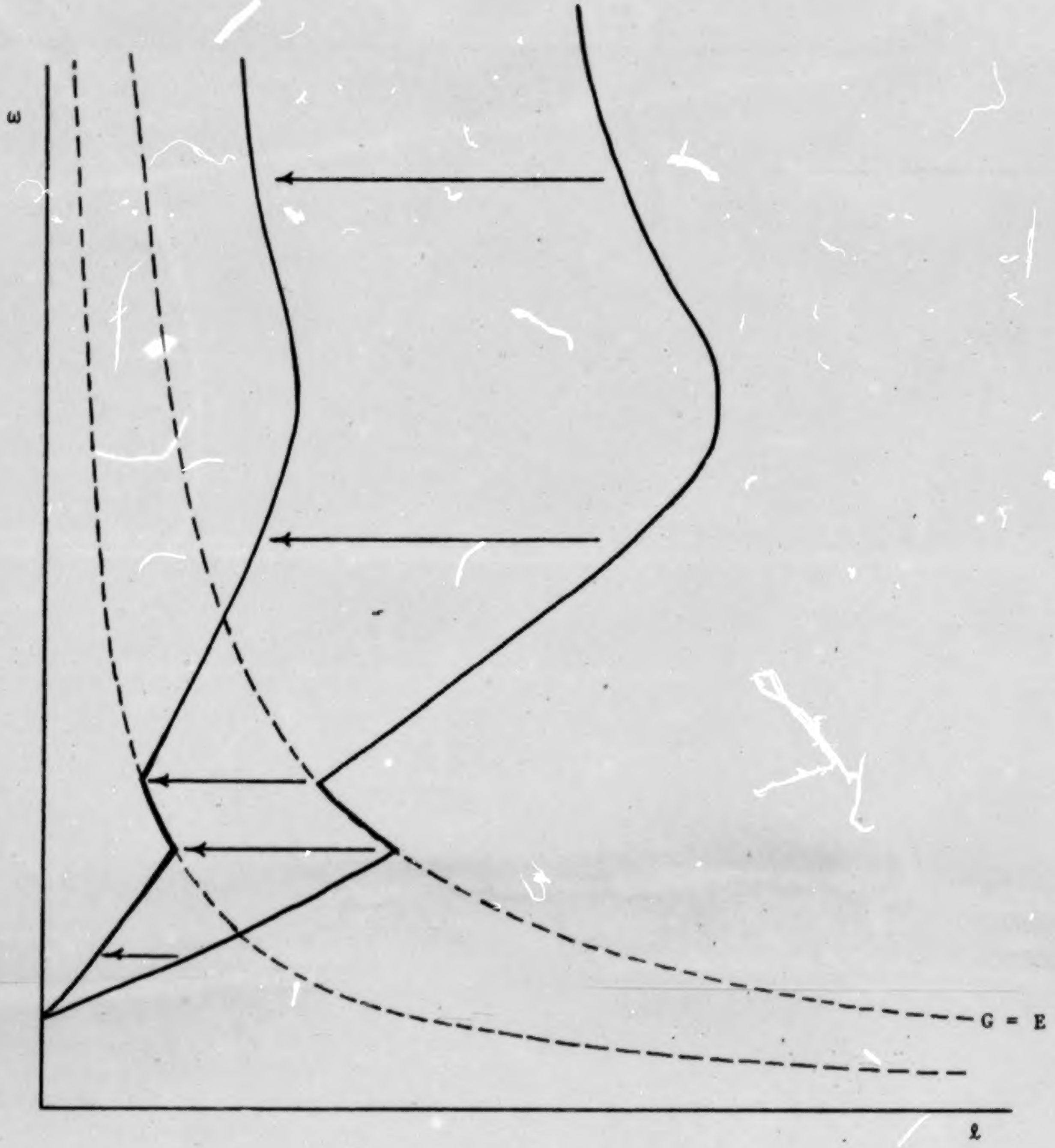

Figure 4

Effect of Increase in K

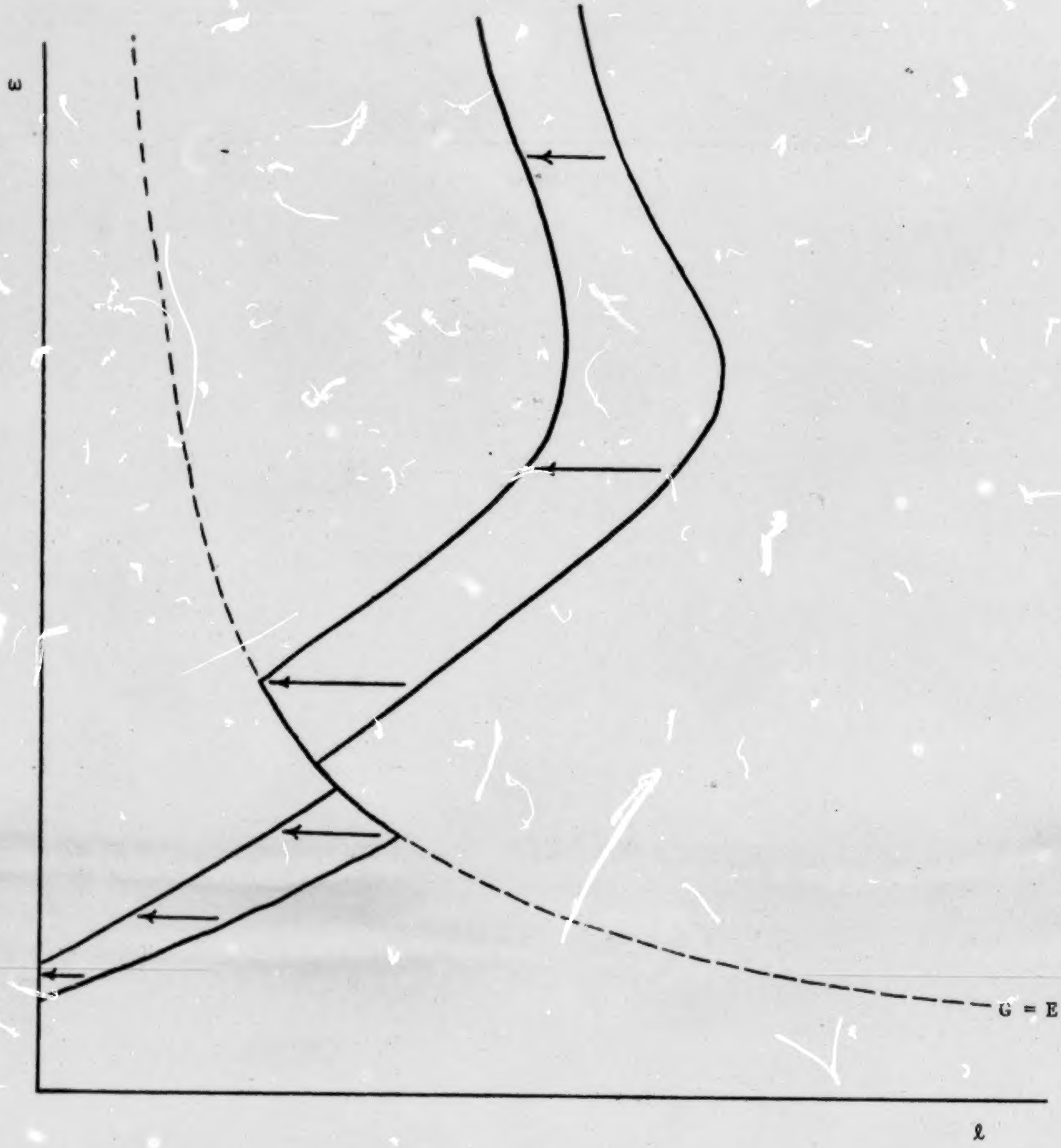

Figure 5
Effect of Increase in P

which is derived by differentiating $\partial H/\partial \ell = 0$. Of course, a change in K or P will have no effect on work effort in Stages I and V, and a change in P will have no effect on ℓ in Stage III.

Changes in the marginal tax rate (α) and in the exemption level (E) will also cause shifts in the labor supply as shown in Figures 6 and 7. In the taxable Stage IV, a change in α will have both substitution and income effects:

$$(2.13) \quad \partial \ell/\partial \alpha = (1 + \varepsilon_{G-E})\ell/((1-\alpha)\varepsilon_{W\ell}) \qquad \text{Stage IV,}$$

where the gross taxable income elasticity of marginal utility is given by

$$(2.14) \quad \varepsilon_{G-E} = ((1-\alpha)(G-E)/Y)\varepsilon_{Y} = ((G-E)/(G-Y^{n}))\varepsilon_{W\ell}$$

On the one hand, an increase in α will motivate the individual to substitute human investment for current income. On the other hand, an increase in the marginal tax rate will also motivate him to increase work effort so as to maintain net income in face of the tax increase. As can be seen from (2.13), the relative magnitudes of these two effects depend on the elasticity of the schedule of marginal utility with respect to gross taxable income (ε_{G-E}). In particular, it follows from (2.14) that unless $|\varepsilon_{W\ell}|$ is sufficiently greater than unity to offset the effect of E on ε_{G-E}, i.e., unless the individual is well onto the backward-bending portion of his labor supply, then the negative substitution effect, $1/((1-\alpha)\varepsilon_{W\ell})$, will dominate the positive income effect, $\varepsilon_{G-E}\ell/((1-\alpha)\varepsilon_{W\ell})$, and work effort will be decreased by an increase in α.

Finally, an increase in the exemption level has only an income effect in Stage IV.

$$(2.15) \quad \partial \ell/\partial E = -\alpha/(\omega K(1-\alpha)) \qquad \text{Stage IV.}$$

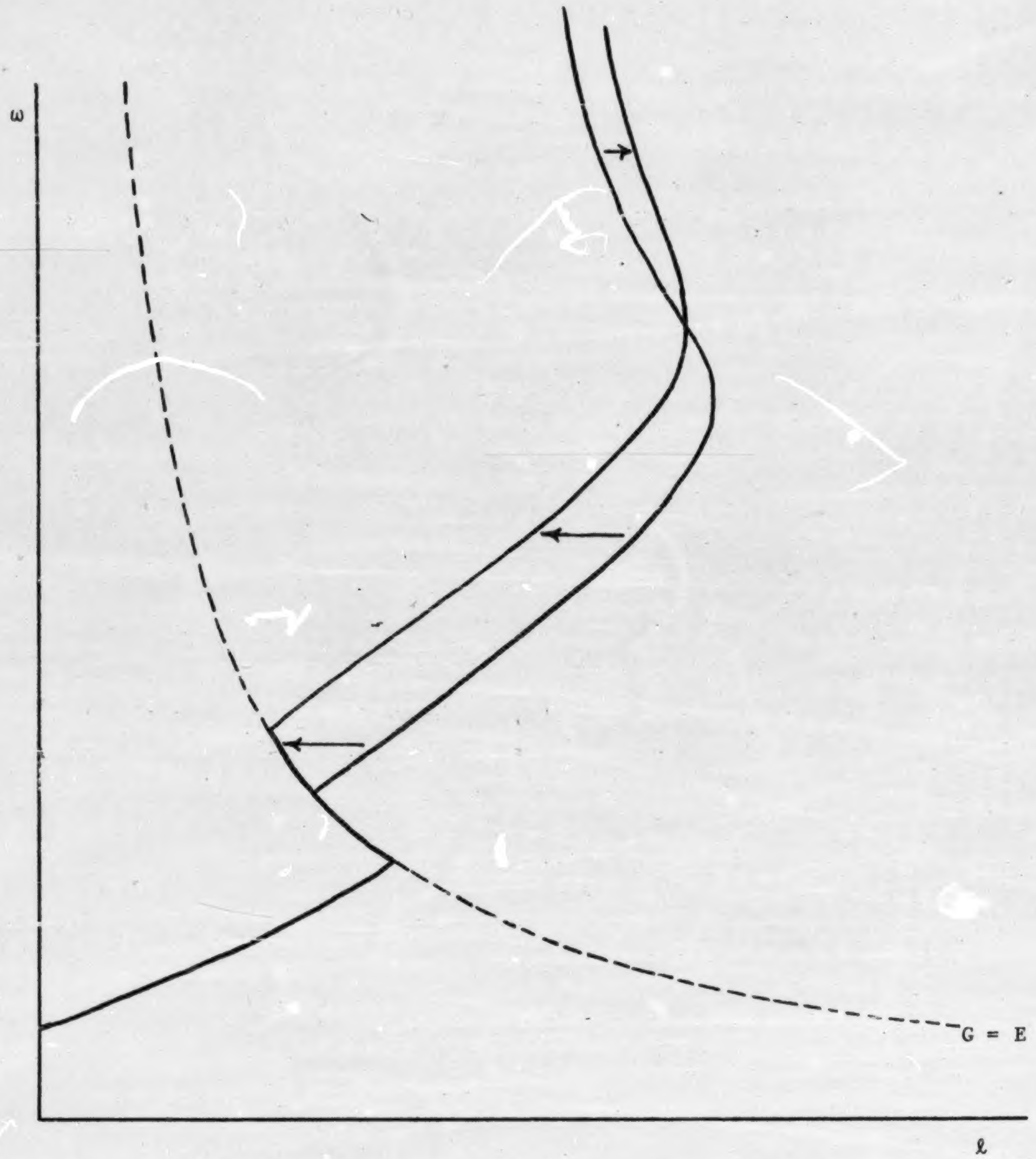

Figure 6
Effect of Increase in α

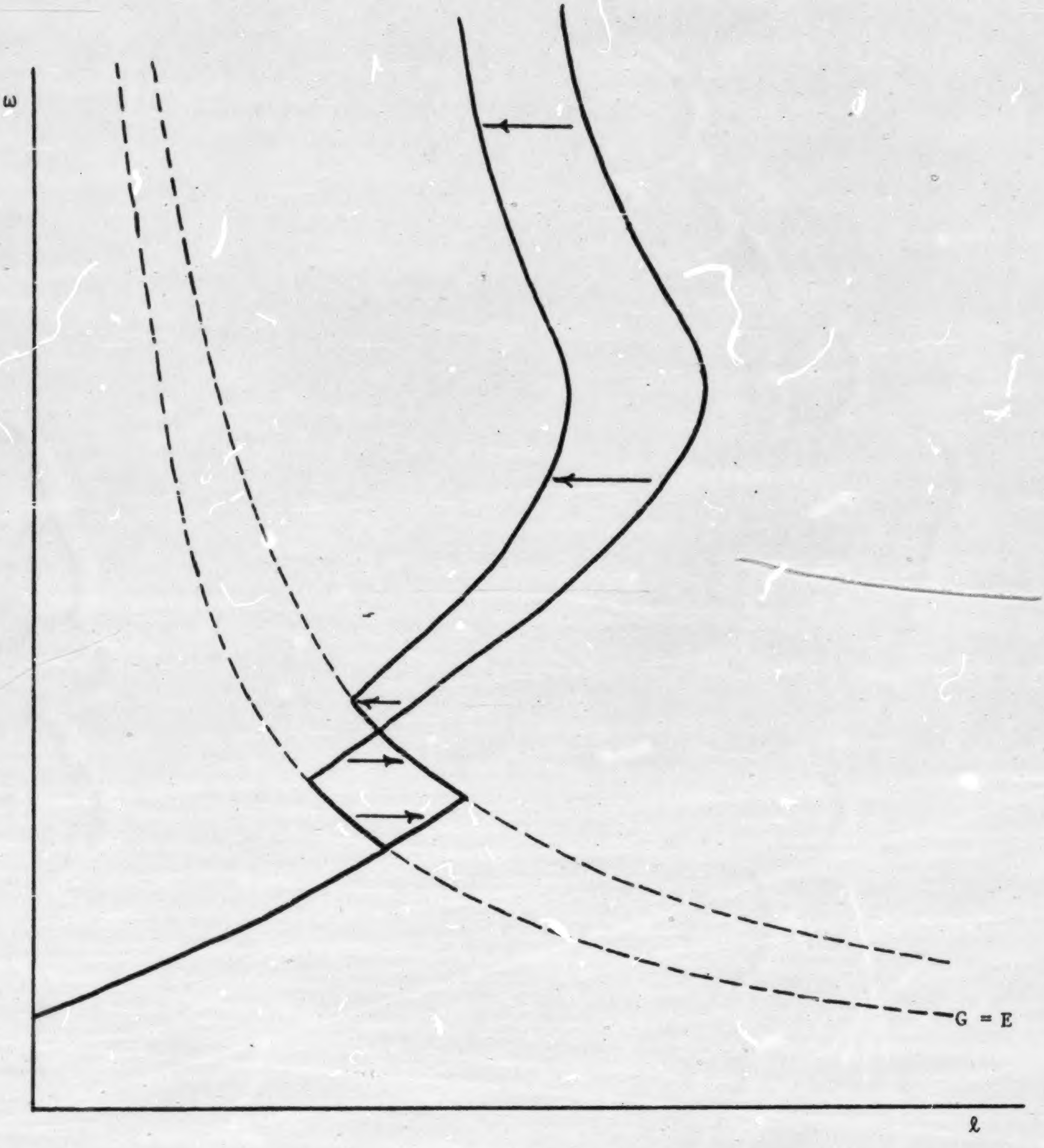

Figure 7
Effect of Increase in E

Both income and investment are increased by the increase in the exemption level with investment being increased by a decrease in work effort. But unlike α, an increase in E also affects the labor supply of those who are not actually paying any tax but who are in Stage III with incomes exactly at the exemption level:

$$(2.16) \qquad \partial \ell/\partial E = 1/(\omega K)$$

Therefore, an increase in E will have a quasi-substitution effect by increasing work effort so that income is substituted for investment.

Long-Run Behavior

In the long run, the stock (K) and value (P) of human capital are not given. However, now that the optimal level of work effort (ℓ) has been described as a function of K and P, the optimal paths of these two quantities over time are completely determined by the differential equations (1.5) and (1.12) given the boundary conditions $K(0) = K^0$, and $P(N) = 0$.

To describe the behavior of K, we note from (1.5) that dK/dt is either positive or negative depending on whether ℓ is either less than or greater than $1-\mu/\gamma$, assuming that the individual's human capital can increase, i.e., $\gamma>\mu$. Therefore, the evolution of K is particularly simple to portray in Figure 2, since the dK/dt=0 line is one of the constant ℓ lines. Above this line, the stock of human capital is increasing; below the line, it is decreasing.

To describe the optimal behavior of P, we evaluate $\partial H/\partial K$ in (1.12) with P held constant but with optimal ℓ considered as a function of K, yielding:

$$(2.17) \qquad dP/dt = -\,\partial H/\partial K - (\partial H/\partial \ell)(\partial \ell/\partial K) + (\mu+\delta)P$$

$$= -\,(\partial H/\partial \ell)(\ell/K + \partial \ell/\partial K) - (\gamma-\mu-\delta)P.$$

In Stages I, II and IV, the term $(\partial H/\partial \ell)(\ell/K + \partial \ell/\partial K)$ is zero, since either $\ell=0$ (and $\partial \ell/\partial K = 0$) or $\partial H/\partial \ell = 0$. In Stage III the partial derivative $\partial H/\partial \ell$ (and $\partial H/\partial K$ as well) is either a left or right hand derivative because of the kink in the tax function at G=E. In either case, the product of $\partial H/\partial \ell$ with $(\ell/K + \partial \ell/\partial K)$ in Stage III is always zero since, from (2.11), $\partial \ell/\partial K = - \ell/K$. Thus, in Stages I-IV, assuming that the individual's discounted human capital can increase, i.e., $\gamma>\mu+\delta$, P is always decreasing and is given by

(2.18) $$dP/dt = -(\gamma-\mu-\delta)P$$ Stages I-IV.

Finally, in Stage V, $\ell=1$, $\partial \ell/\partial K=0$ and $\partial H/\partial \ell>0$, so that P continues to decrease:

(2.19) $$dP/dt = -U'\{Y\}(1-T')\omega + (\mu+\delta)P$$ Stage V.

The actual trajectory that the individual takes is determined by the initial condition $K\{0\} = K^0$, represented by a vertical line in Figure 2, and the terminal condition $P\{N\} = 0$. The initial level of P on the K^0 line at time 0 is determined so that at time N the terminal condition is satisfied. For given K^0, the larger is N, then the higher must be the initial value of P as will be demonstrated in the next section. The nature of the trajectory is now immediately apparent. In general, K rises to a maximum when $\ell = 1-\mu/\gamma$ and then declines, while P always declines. However, there are certainly horizons so short that it never pays for the individual to accumulate human capital and, hence, the initial $P\{0\}$ is low and K is always declining.

The behavior of the individual's wage (W) follows immediately from the path of K as shown in Figure 8. When $\ell=0$, the wage grows at the maximum rate of $100(\gamma-\mu)$ percent. The growth then declines when $\ell>0$ until the wage is maximum when $dK/dt = 0$. From then on, the wage declines, reaching a maximum

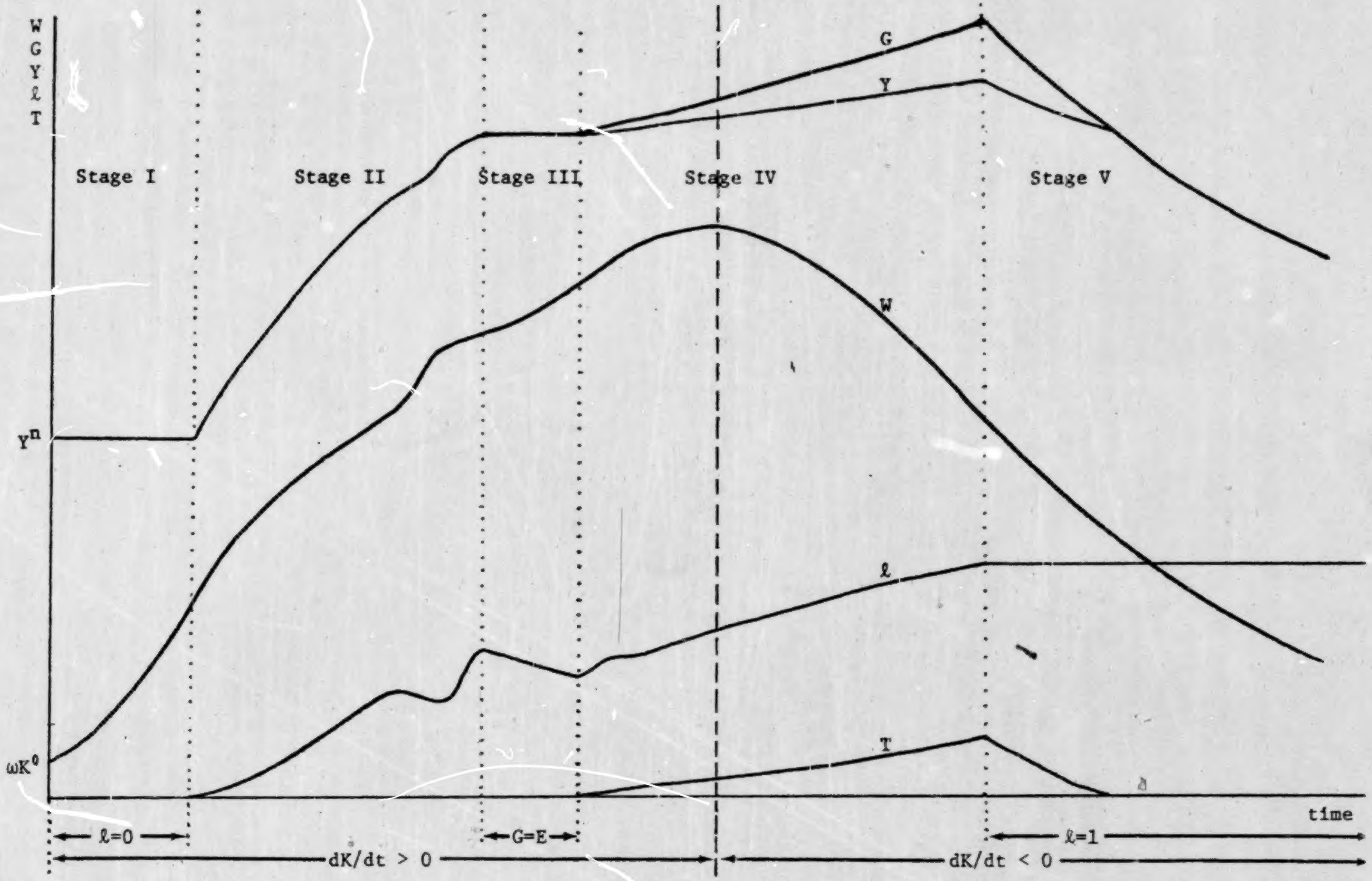

Figure 8
Life-Cycle Pattern of Wages, Income, Work Effort and Taxes*

* Scale for work effort differs from scale for other quantities.

rate of decrease of μ when $\ell=1$.

The evolution of individual earnings ($W\ell$), gross income (G), and net income (Y) in Stages II and IV can be seen from the first-order condition that $\partial H/\partial \ell = 0$. This implies, from (2.1), that

$$(2.20) \qquad P = U'(1-T')\omega/\gamma;$$

differentiating both sides of (2.20) with respect to time yields

$$(2.21) \qquad dP/dt = U''(1-T')(dY/dt)\omega/\gamma = U''(1-T')^2(dG/dt)\omega/\gamma, \qquad \text{Stages II \& IV.}$$

Since we know that dP/dt is negative, Y, G and $W\ell$ must increase over time in Stages II and IV. In Stages I and III all three quantities are obviously constant since either $\ell=0$ or G=E. In Stage V, all three quantities must decline since $\ell=1$ and W is declining. Since taxes (T) are a non-decreasing function of gross income, they behave in a similar manner, except of course that T is constant at the zero level until G reaches E, and may be zero again in Stage V if G falls below E. Note that it is possible for an individual to have such a low initial level of human capital that he never earns enough income to pay taxes.

In contrast to the straightforward evolution of the individual's wage, earnings, income, and taxes, the behavior of his work effort (ℓ) and human investment (I) is relatively complex. In general, ℓ increases over the life cycle, eventually reaching a value of 1. The increase need not be monotonic, however. In Stage III, where G=E, ℓ either decreases or increases depending on $dK/dt \gtrless 0$, since income is maintained constant in this region. In Stages II and IV, also, the course of ℓ need not be monotonic. Since the optimal ℓ is a function of K and P, the behavior of ℓ over time can be described by

(2.22) $d\ell/dt = (\partial\ell/\partial K)(dK/dt) + (\partial\ell/\partial P)(dP/dt)$.

From (2.11) and (2.12), $\partial\ell/\partial K$ and $\partial\ell/\partial P$ are both negative, and dP/dt is always negative. Thus, when dK/dt is negative, ℓ must increase and increasing ℓ insures that K continues to decrease. However, when the level of work effort is below $1-\mu/\gamma$, so that dK/dt is positive, ℓ may be either increasing or decreasing depending on the relative magnitudes of the quantities in (2.22). Thus, while the individual is in the process of increasing his stock of human capital, the level of his work effort may oscillate. Since every optimal path must eventually have $\ell=1$, so that $\tilde{P}\{N\}$ may reach zero, it is clear that $d\ell/dt$ can be negative for only a finite amount of time.

Finally, let us analyze the behavior of human investment over the life cycle. Of course, the proportion of capital devoted to investment $(1-\ell)$ is immediately determined by the course of work effort. When one increases, the other decreases and vice-versa. Similarly, the level of net investment (dK/dt) follows from the previous discussion. To determine the path of the level of gross investment we differentiate I with respect to time and get

(2.23) $dI/dt = (\gamma/\omega)(dW/dt - dG/dt)$,

since $I = (W-G+Y^n)\gamma/\omega$. We know that $dG/dt > 0$, except in Stage V, so that gross investment decreases monotonically when K, and hence W, is decreasing and reaches a level of zero at the start of Stage V. Gross investment generally increases when K is increasing, but not necessarily monotonically. Nonmonotonic behavior of I may occur even if ℓ is monotonically increasing.

III. COMPARATIVE DYNAMICS

In this section we analyze the effects of marginal changes in the tax parameters on individual life-cycle behavior. The effect of a tax parameter change is expressed in terms of derivatives over time which show the effect of the change on the stock and value of human capital at each moment of the individual's life cycle. These paths of derivatives are characterized as the solution to a two-point boundary value problem which is solved using a technique developed by Oniki.

Variational Differential Equations

It is well known[9] that the effect on the solution $X\{t\}$ of a system of differential equations

$$(3.1) \qquad dX/dt = F\{X\}$$

of a marginal change in a parameter Θ of the system is given by the solution of a system of non-autonomous linear differential equations in the partial derivatives of X with respect to Θ:

$$(3.2) \qquad dX_{\Theta}/dt = F_X\{X\{t\}\}X_{\Theta} + F_{\Theta}\{X\{t\}\},$$

where the boundary conditions for (3.2) depend upon the effect of Θ on the boundary conditions for the original differential equations (3.1). In particular, if $X\{t\}$ is the solution to a two-point boundary value problem, then also $X_{\Theta}\{t\}$, is a solution to a two-point boundary value problem.

In our case we are interested in the effects of changes in the tax parameters (α,E) on the individual life-cycle behavior described by the long-run solution to the equations (1.5) and (2.18)-(2.19) in the stock (K) and value (P) of human capital with $K\{0\} = K^0$ and $P\{N\} = 0$. Therefore, in system (3.1),

$X \equiv [K,P]$, and Θ is either α or E. The required matrices and vectors for (3.2), with $\partial\ell/dP$, $\partial\ell/\partial K$, and $\partial\ell/\partial\Theta$ given by the comparative statics results (2.11)-(2.16), are:

$$(3.3) \qquad F_X = \begin{bmatrix} \gamma-\mu & 0 \\ 0 & -(\gamma-\mu-\delta) \end{bmatrix} = \begin{bmatrix} + & 0 \\ 0 & - \end{bmatrix} \text{ and } F_\Theta = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \text{Stage I;}$$

$$(3.4) \qquad F_X = \begin{bmatrix} \gamma-\mu & -\gamma K\partial\ell/\partial P \\ 0 & -(\gamma-\mu-\delta) \end{bmatrix} = \begin{bmatrix} + & + \\ 0 & - \end{bmatrix} \text{ and } F_\Theta = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \text{Stage II;}$$

$$(3.5) \qquad F_X = \begin{bmatrix} \gamma-\mu & 0 \\ 0 & -(\gamma-\mu-\delta) \end{bmatrix} = \begin{bmatrix} + & 0 \\ 0 & - \end{bmatrix} \text{ and } F_\Theta = \begin{bmatrix} -\gamma K\partial\ell/\partial\Theta \\ 0 \end{bmatrix}, \qquad \text{Stage III;}$$

$$(3.6) \qquad F_X = \begin{bmatrix} \gamma-\mu & -\gamma K\partial\ell/\partial P \\ 0 & -(\gamma-\mu-\delta) \end{bmatrix} = \begin{bmatrix} + & + \\ 0 & - \end{bmatrix} \text{ and } F_\Theta = \begin{bmatrix} -\gamma K\partial\ell/\partial\Theta \\ 0 \end{bmatrix}, \qquad \text{Stage IV;}$$

$$(3.7) \qquad F_X = \begin{bmatrix} -\mu & 0 \\ -\omega^2(1-\alpha)^2U'' & \mu+\delta \end{bmatrix} = \begin{bmatrix} - & 0 \\ + & + \end{bmatrix}$$

$$\text{and} \quad F_\Theta = \begin{bmatrix} 0 \\ \omega(\partial\alpha/\partial\Theta)U' - \omega(1-\alpha)(\partial U'/\partial\Theta) \end{bmatrix} \qquad \text{Stage V.}$$

Since neither $K\{0\}$ nor $P\{N\}$ are affected by a change in α or E, the initial and terminal conditions for system (3.3)-(3.7) are $K_\Theta\{0\} = P_\Theta\{N\} = 0$.

Oniki has demonstrated[10] that if there are no discontinuities in dX/dt when crossing a boundary between regions in X, then there are also no discontinuities in X_Θ between regions. Since there are no jumps in dK/dt or dP/dt when K and P cross the boundaries between any of the five stages, there are therefore no jumps in K_Θ or P_Θ. Thus the effect of a parameter change on an optimal path is found by solving the two-point boundary value problem characterized by (3.2) with matrices F_X and F_Θ given by (3.3)-(3.7) and with $K_\Theta\{0\} = 0$ and $P_\Theta\{N\} = 0$.

Oniki has developed a method for converting the above two-point boundary value problem into an initial value problem which is more readily solved. The first step is to determine the effect of an increase in the initial value of capital, P^0, with Θ held constant. In particular, we need to know whether a path with higher initial P^0 has a higher or lower final value of P at terminal time N, i.e., whether $P_{P^0}\{N\}|\Theta$ constant < 0. The next step is to determine the effect of a change in parameter Θ without allowing any change in P^0. It is easy to see that, in general, the new path with changed Θ will not reach the $P=0$ axis at time N if it starts from the same initial point as the original path. Therefore, in order for the individual's horizon N to remain the same under a marginal change in parameter Θ, there must be a marginal change in the initial value of P. Oniki demonstrates that this marginal change must be given by

$$(3.8) \qquad P_\Theta\{0\} = -(P_\Theta\{N\}|_{P^0 \text{ constant}})/(P_{P^0}\{N\}|_{\Theta \text{ constant}}).$$

In addition to the above technique for determining the effect of parameter changes on an optimal path, Oniki has also presented a formula for determining the effect of a parameter change on the time when an optimal path crosses a boundary.[11] If a boundary in X-space is described by the equation

$$(3.9) \qquad h\{X\} = 0,$$

and if an optimal path crosses this boundary at time s, then the effect on s of a change in parameter Θ is given by

$$(3.10) \qquad s_\Theta = -[\partial h/\partial X \cdot X\{s\} + \partial h/\partial\Theta]/[\partial h/\partial X \cdot dX\{s\}/dt]$$

where $\cdot$ indicates inner product. For our model the functions h are given by equations (2.2) through (2.7).

Tax Rate Effects

Our analysis of the effects of changes in α and E begins by determining the effect of a change in P^0 in order to compute the denominator of (3.8). The differential equations which characterize the paths of $K_{P^0}\{t\}$ and $P_{P^0}\{t\}$ are specified by matrices F_X given by (3.3)-(3.7), and by $F_\Theta = F_{P^0} = 0$ in all regions. The initial conditions for the two paths are $K_{P^0}\{0\}(= K^0_{P^0}) = 0$ and $P_{P^0}\{0\}(= P^0_{P^0}) = 1$. From the signs of the elements in F_X, it can readily be seen that P_{P^0} is positive for Stages I through IV, and that K_{P^0} is positive in Stages II through IV. Both must, therefore, remain positive through Stage V, and in particular $P_{P^0}\{N\} > 0$.

We now consider the effect of α on an optimal path with P^0 held constant. The initial calculations are aimed at finding the paths of K_α and P_α with initial conditions $K_\alpha\{0\} = P_\alpha\{0\} = 0$. From (3.3) through (3.5), it is clear that K_α and P_α remain zero through Stage III since $\partial \ell/\partial \alpha = 0$. In Stages IV and V, the effect of α depends upon the elasticity of marginal utility with respect to taxable income, (ε_{G-E}) given by (2.14). Evaluating F_Θ with $\Theta = \alpha$ in (3.6) and (3.7), we obtain

$$(3.11) \qquad F_\alpha = \begin{bmatrix} -(1+\varepsilon_{G-E})W\ell\gamma/((1-\alpha)\omega\varepsilon_{W\ell}) \\ 0 \end{bmatrix} = \begin{bmatrix} + \text{ or } - \\ 0 \end{bmatrix} \qquad \text{Stage IV;}$$

$$(3.12) \qquad F_\alpha = \begin{bmatrix} 0 \\ (1+\varepsilon_{G-E})\omega U' \end{bmatrix} = \begin{bmatrix} 0 \\ + \text{ or } - \end{bmatrix} \qquad \text{Stage V.}$$

If $|\varepsilon_{G-E}|$ is always less than unity, i.e., if the short-run substitution effect of α is always dominant, then the non-zero elements of F_α are positive in both Stages IV and V. Thus K_α becomes positive in Stage IV and hence P_α becomes unambiguously positive in Stage V so that $P_\alpha\{N\} > 0$. Now although $|\varepsilon_{G-E}|$ is necessarily less than unity at the beginning of Stage IV where G=E, it is possible, if the individual is sufficiently far up on the backward-bending portion of his labor supply, for the short-run income effect of α to become dominant at some point in Stage IV, and hence for F_α to become negative. However, only if the short-run income effect becomes dominant sufficiently early in Stage IV will it be possible for P_α to be driven negative in Stage V and, hence, for $P_\alpha\{N\}$ to be negative.

Knowing both the denominator and numerator in (3.8), we are now able to describe the total effect of an increase in the tax rate on value and stock of capital over the individual's life cycle. There are two possible patterns, depending upon the balance of short-run income and substitution effects during Stage IV, i.e. depending upon whether the individual is sufficiently far up on the backward-bending portion of his labor supply curve or not during the time when he is paying the tax.

According to (3.8), if the short-run substitution effect dominates in Stage IV, then $P_\alpha\{0\} < 0$. That is, the effect of an increase in the tax rate is to decrease the value of capital at the beginning of the individual's life cycle. It then follows from (3.3)-(3.6) that P_α remains negative through Stage IV. Since P_α must be zero at the end of the working life, it will in general remain negative through Stage V until it reaches zero at time. It should be noted, however, that for certain types of utility functions, a decrease in capital stock ($K_\alpha < 0$) may allow P_α to become positive in Stage V before it finally reaches zero.

If the value of capital is reduced, the short-run utility attached to investment is also reduced, motivating the individual to substitute income for investment in the non-taxable stages. By definition, in Stage I all non-leisure time is devoted to investment, but as is shown in Figure 9 and confirmed by (3.3)-(3.5) the stock of capital is reduced in Stages II and III. In Stages IV and V, however, the tax rate increase also reduces the opportunity cost of investment in terms of foregone income. Thus, it is possible for K_{α} to be positive or negative in these taxable stages, as can be seen from (3.6), (3.11) and (3.7). If the reduction in short-run utility dominates the reduction in opportunity cost, an increase in the tax rate will decrease the capital stock; otherwise, an increase in α will increase K in the last two stages.

If the short-run income effect dominates the substitution effect of a tax rate increase sufficiently early in Stage IV then $P_{\alpha}\{0\} > 0$, and the effect of α on the value and stock of capital is essentially opposite to the effect when P_{α} is negative as is shown in Figure 9. The increase in the value of capital motivates the individual to increase investment in nontaxable Stage II so as to maintain income in the taxable stages. As can be seen from (3.3)-(3.7), K_{α} is positive at least in the second and third stages and also in the last two stages unless the reduction in short-run utility dominates the reduction in opportunity cost in the taxable stages. Therefore, if the income effect dominates, the individual substitutes investment for income in Stage II so as to spread out over all stages the reduction in income brought about by tax increase.

As suggested above, an increase in the tax rate produces two distinct patterns of effects on net income over the life cycle, depending upon whether the short-run substitution or income effect is dominant. These two patterns are confirmed by totally differentiating Y with respect to α at each point in the life cycle. Of course, $Y_{\alpha} = 0$ in Stage I, assuming as we have that $Y^{n} < E$.

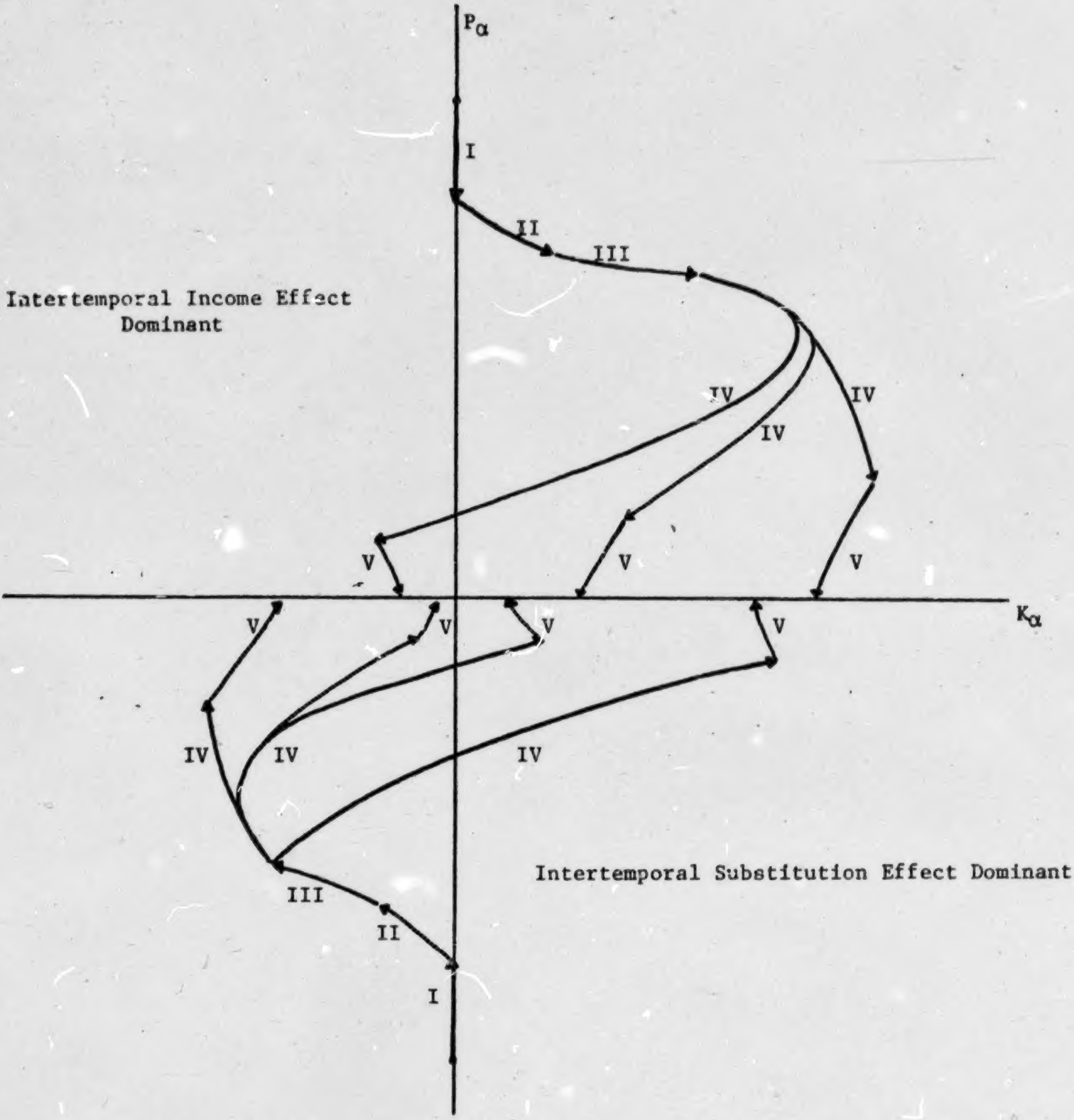
P_α
I
II
III
Intertemporal Income Effect Dominant
IV
IV
IV
V
V
V
K_α
V
V
V
V
IV
IV
IV
Intertemporal Substitution Effect Dominant
III
II
I

Figure 9

Effect of an Increase in α on K and P

In Stage II (and IV), Y_α is calculated by setting (2.1) equal to zero and totally differentiating with respect to α, yielding

$$(3.13) \qquad Y_\alpha = \gamma P_\alpha/(\omega U'') \qquad \text{Stage II,}$$

so that the sign of Y_α depends on the sign of P_α. If the short-run subsitution effect dominates, then $P_\alpha<0$ and net income is increased in Stage II; otherwise, income is decreased in the nontaxable stage so that capital can be increased to maintain income in the latter stages. Again, in Stage III, $Y_\alpha = 0$ since $Y=G=E$, but

$$(3.14) \qquad Y_\alpha = (\gamma P_\alpha + \omega U')/(\omega(1-\alpha)U'') \qquad \text{Stage IV,}$$

so that if the income effect dominates and, hence, $P_\alpha>0$, then net income is unambiguously decreased in this taxable stage. However, if the substitution effect prevails ($P_\alpha<0$), then there is the possibility that $Y_\alpha>0$ in Stage IV. Finally, the effect of α on Y in Stage V is obtained by substituting (1.1) and (1.2) into (1.3) and totally differentiating the result with respect to α:

$$(3.15) \qquad Y_\alpha = (1-\alpha)\omega K_\alpha - (G-E) \qquad \text{Stage V.}$$

If $K_\alpha<0$ then net income is unambiguously decreased by the tax increase, but if $K_\alpha>0$, as we know it can be, then the effect of α on Y in this stage is indeterminate. While we cannot determine in which taxable stage net income is reduced, we do know that it must be reduced somewhere in these stages.

Although the effect of a tax rate increase has been discussed thus far in terms of short-run income and substitution effects on the choice between income and investment, it may also be characterized in terms of what may, by analogy, be called intertemporal substitution and income effects involving the choice between income early or later in the life cycle. An increase in the tax rate both alters the marginal rate of substitution between early (nontaxable) and late (taxable) income, and reduces the total life cycle income. As we have seen, when the short-run substitution effect dominates in Stage IV (the taxable years), income in early years is substituted for later income, i.e. the intertemporal substitution effect also is dominant. The intertemporal income effect, which tends to reduce income throughout the entire life cycle, dominates only if the short-run income effect is sufficiently strong early in Stage IV. These two patterns of net income response are displayed in Table 1.

The effect of a tax rate change in gross income (G) is, of course, identical to the effects on Y in the first three nontaxable stages. However, from (1.3) we know

$$(3.16) \qquad G_\alpha = (Y_\alpha + (G-E))/(1-\alpha) \qquad \text{Stages IV and V,}$$

so that if $Y_\alpha > 0$, then $G_\alpha > 0$; otherwise the effect of α on gross income is indeterminate. Therefore, since taxes (T) are direct function of gross income, the effect of α on T is given by

$$(3.17) \qquad T_\alpha = \alpha G_\alpha + G-E \qquad \text{Stages IV and V,}$$

so that if $G_\alpha > 0$, then $T_\alpha > 0$; otherwise, again, the sign of T_α is indeterminate.

By now, the effects of a tax rate increase on work effort (ℓ) and human investment (I) should be evident. Of course, in Stages I and V there is no effect, but there will be an effect in the three remaining stages. Since ℓ is a function in these stages of P, K and α:

$$(3.18) \qquad \ell_\alpha = (\partial\ell/\partial P)P_\alpha + (\partial\ell/\partial K)K_\alpha + (\partial\ell/\partial\alpha) \qquad \text{Stages II - IV,}$$

TABLE 1

Effects of Increase in Tax Rate*

	Stage I	Stage II	Stage III	Stage IV	Stage V
P_α	- +	- +	- +	- +	- +
K_α, W_α	0	- +	- +	?	?
Y_α	0	+ -	0	? -	?
G_α	0	+ -	0	?	?
T_α	0	0	0	?	?
ℓ_α	0	+ -	+ -	?	0
I_α	0	- +	- +	?	0
s_α	- +	- +	? +	?	0

---

*First sign holds when intertemporal substitution effect is dominant; second sign is for dominant intertemporal income effect.

From the comparative statics analysis (2.11) and (2.12) we remember that $\partial \ell/\partial P < 0$ and $\partial \ell/\partial K < 0$. Therefore, in Stages II and III if the intertemporal substitution effect dominates (P_α, $K_\alpha < 0$), work effort is increased by a tax rate increase so as both to shift income from the taxable stages to Stage II and to maintain Y=E in Stage III; otherwise work effort is reduced in both stages so as to distribute the reduction in income over all periods. As is not surprising by now, the effect of α on ℓ is indeterminate in Stage IV since the signs of P_α and K_α are indeterminate. Finally, the effect of a change in α on human investment follows immediately from the work effort behavior:

(3.19) $$I_\alpha = -\ell_\alpha \gamma K + (1-\ell)\gamma K_\alpha$$ Stages II - IV.

Since ℓ and K move in opposite directions in Stages II and III, investment unambiguously decreases if the intertemporal substitution effect dominates and increases if the intertemporal income effect prevails.

Let us now conclude the analysis of the effects of a change in the tax rate, the results of which are summarized in Table 1, with a consideration of the effect of α on the time at which a stage is terminated (s_α). While a change in α does not change behavior in Stage I, it does affect the amount of time spent in that stage. This is made apparent by differentiating (2.2) totally with respect to K, P, and α, and then substituting the results into (3.10) to obtain

(3.20) $$s_\alpha = -P_\alpha/(dP/dt)$$ Stages I & II,

where the same result for Stage II is obtained by differentiating (2.3). Since $dP/dt<0$, s_α has the same sign as P_α. If the substitution effect dominates, the individual spends a shorter amount of time specializing in human investment;

if the income effect prevails, he starts to work later. Since s_α is the same for Stage II, the effect of a tax rate increase on the time at which G=E then follows in like manner. Note, however, that we cannot tell the effect on the amount of time spent in the second stage; we can only tell the effect on the time spent in the first stage and the time spent in both stages. To determine the effect of α on the time when the individual first pays taxes, we differentiate (2.4) to get

$$(3.21) \qquad s_\alpha = -(P_\alpha + P/(1-\alpha))/(dP/dt), \qquad \text{Stage III.}$$

If the income effect dominates ($P_\alpha>0$), then the individual unambiguously starts to pay taxes later because he has been allocating his time to increasing his wage rate rather than his earnings. If the substitution effect prevails, however, s_α is indeterminate. Either by virtue of having devoted his time to generating income rather than capital he can start to pay taxes earlier, or by virtue of now having a lower wage rate he can pay taxes later. If the latter is true, then the individual arrives at the exemption level earlier but remains at this level for so long that the time before he pays taxes is actually increased. Finally, just as all else is indeterminate in Stage IV, so also is the time at which the individual leaves this stage. We only know that if the individual spends less time in the nontaxable stages, he must spend more time in the taxable stages and vice-versa.

Exemption Level Effects

Let us now turn our attention to the effects of an increase in E. From the comparative dynamics analysis of the marginal tax rate we already know that $P_{p_0}\{N\}$ in the denominator of (3.8) is positive. Therefore, to determine the effect of an increase in the exemption level on the initial value of capital, we need only determine $P_E\{N\}$ with P^0 held constant. Thus starting with the initial conditions $K_E\{0\} = P_E\{0\} = 0$, it follows from (3.3)-(3.6) that K remains unchanged through Stage II, and P is unchanged through Stage IV. In Stage III, however, E, unlike α, does have an effect on K. Evaluating F_Θ with Θ=E in (3.5) we obtain from (2.16)

$$(3.22) \qquad F_E = \begin{bmatrix} -\gamma/\omega \\ 0 \end{bmatrix} = \begin{bmatrix} - \\ 0 \end{bmatrix} \qquad \text{Stage III.}$$

Hence, K_E must become negative in this stage, where income is substituted for investment. In Stage IV it follows from (3.6) and (2.15) that

$$(3.23) \qquad F_E = \begin{bmatrix} (\gamma/\omega)[\alpha/(1-\alpha)] \\ 0 \end{bmatrix} = \begin{bmatrix} + \\ 0 \end{bmatrix} \qquad \text{Stage IV,}$$

where there is only a pure short-run income effect. Hence K_E can become positive if the income effect is strong enough or if the individual spends a long enough time in Stage IV. Finally, from (3.7) we see that

$$(3.24) \qquad F_E = \begin{bmatrix} 0 \\ -\alpha(1-\alpha)\omega U'' \end{bmatrix} = \begin{bmatrix} 0 \\ + \end{bmatrix} \qquad \text{Stage V,}$$

so that $P_E\{N\}$ will certainly be positive if K_E is positive and can only be negative if K_E is very negative.

Knowing again both the denominator and numerator in (3.8), we can now calculate the total effect of an increase in the exemption level. Since in the short run, an increase in E has both a quasi-substitution effect in Stage III and a pure income effect in Stage IV, an individual's life cycle response must depend upon the balance of these two effects. If the pure income effect is strong enough, then $P_E\{0\} < 0$, i.e., an increase in the exemption level decreases the initial value of human capital (P^0). Since increases in E and α have opposite effects on the level of total life-cycle income, the effect of an increase in E on K and P should be essentially opposite to the effect of an increase in α if the income effects are dominant in both cases. This is confirmed by (3.3)-(3.7) and (3.22)-(3.24) and illustrated in Figure 10. P_E is in general negative throughout the life cycle until it becomes zero at time N. The lowered value of capital induces the individual to decrease investment in Stage II so as to increase income. In Stage III the reduction in P and increase in E work together to make investment less attractive, thereby inducing the individual to reduce investment in favor of income. In fact, K_E remains negative in the last two stages unless the tax saving brought about by an increase in E is large enough to overcome the reduced value of capital, so that the individual is encouraged to invest more while still enjoying higher net income.

If the short-run quasi-substitution effect of an exemption increase in Stage III dominates the short-run income effect in Stage IV, then $P_E\{0\} > 0$. The value of capital remains higher throughout the life cycle, except possibly in Stage V, as can be seen again from (3.3)-(3.7) and (3.22)-(3.24). In Stage II, the higher value for capital causes investment to be increased thereby increasing K. In Stage III, however, the increased value of capital is offset by the increase in the amount of income that may be earned without paying tax; by the end of Stage III, K_E must have become negative, as shown in Figure 10, and must remain negative through the remainder of the life cycle or P_E would not become zero at time N.

As was the case with a tax rate increase, the effect of an increase in the exemption level can be described in terms of intertemporal substitution and income effects. An increase in the exemption level both increases total life-cycle income and alters the marginal rate of substitution between net

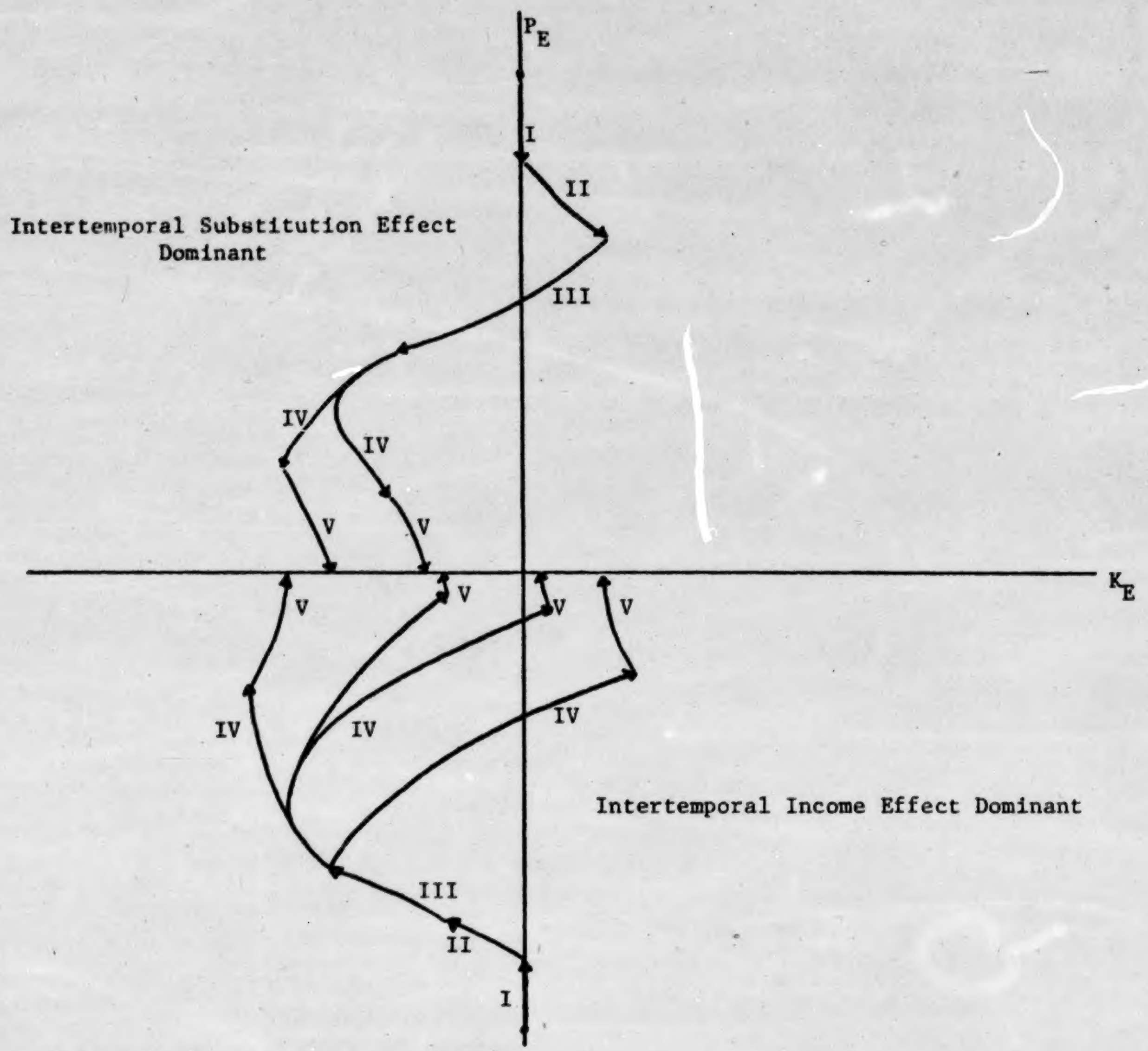
P_E
I
II
III
Intertemporal Substitution Effect Dominant
IV
IV
V
V
K_E
V
V
V
V
IV
IV
IV
Intertemporal Income Effect Dominant
III
II
I

Figure 10

Effect of an Increase in E on K and P

income in Stage III and net income in all other stages, since Stage III now occurs at a gross income level that was taxed before the exemption level increase. If the short-run pure income effect in Stage IV dominates the short-run quasi-substitution effect in Stage III, then the intertemporal income effect is also dominant; the individual will choose to spread his gain in income over all stages of his life cycle. If the opposite case is true, then the intertemporal substitution effect is dominant and the individual will substitute income in Stage III for income in other stages. Although the intertemporal income effect of E and α are essentially opposite, the intertemporal substitution effects are not since the patterns of the changes in marginal rates of substitution are different.

The effect of E on after-tax income (Y) is again confirmed by setting (2.1) equal to zero and totally differentiating with respect to E. Of course $Y_E=0$ in Stage I, and

(3.25) $$Y_E = \gamma P_E/(\omega U'') \qquad \text{Stage II,}$$

so that again the sign of Y_E depends on the sign of P_E. If the intertemporal income effect dominates, P_E is negative and net income is increased. Otherwise income is decreased in this stage so that it can be increased in the next stage where

(3.26) $$Y_E = 1 \qquad \text{Stage III,}$$

which is in contrast to the null effect of α on Y in this stage. Now in Stage IV,

(3.27) $$Y_E = \gamma P_E/[\omega(1-\alpha)U''] \qquad \text{Stage IV,}$$

so that the sign of Y_E is the same as in Stage II. Finally in the last stage, using the same procedure used to obtain (3.15), we get

(3.28) $$Y_E = (1-\alpha)\omega K_E + \alpha$$ Stage V.

If the intertemporal income effect dominates, K_E is indeterminate so that Y_E can be positive or negative. However, if the substitution phenomena dominates, then income is unambiguously decreased.

Again the effect of an exemption level change on gross income (G) is identical to the effects on Y in the first three stages. Similarly from (1.3) we obtain for the fourth stage

(3.29) $$G_E = (Y_E-\alpha)/(1-\alpha)$$ Stage IV,

so that if $Y_E<0$, then $G_E<0$; otherwise G_E is indeterminate. Of course in Stage V the effect on G follows from the effect on K since

(3.30) $$G_E = \omega K_E$$ Stage V.

Now the effect of E on taxes (T) is given by

(3.31) $$T_E = \alpha(G_E-1)$$ Stages IV and V,

so that if $G_E<0$, then $T_E<0$; otherwise T_E is indeterminate.

To analyze the effects on work effort (ℓ) and human investment (I) of an exemption level increase we use the same procedure that was employed to analyze the effects of α in (3.18). Since

(3.32) $$\ell_E = (\partial\ell/\partial P)P_E + (\partial\ell/\partial K)K_E + (\partial\ell/\partial E)$$ Stages II-IV,

$\partial \ell/\partial P<0$ and $\partial \ell/\partial K<0$, we know that if the income effect dominates then work effort is unambiguously increased in Stages II and III so that income can be increased. In Stage IV, however, the sign of ℓ_E is indeterminate since income could be increased instead by an increase in K and, hence, the wage rate. If the substitution phenomena dominates, then work effort is certainly lower in Stage II so that the wage rate can be higher at the beginning of Stage III, when ℓ is also higher. By the end of this stage and throughout Stage IV, however, the effect on work effort is indeterminate since income can be reduced by a decrease in the wage. Finally, the effect of E on human investment follows from the analysis of work effort:

$$(3.33) \qquad I_E = \ell_E \gamma K + (1-\ell)\gamma K_E \qquad \text{Stages II-IV.}$$

Since ℓ and K move in opposite directions in Stage II, investment unambiguously decreases (increases) if the intertemporal income (substitution) effect dominates. We can also be sure that I_E is negative in Stage III if the income effect dominates. All else, however, is indeterminate since ℓ_E and K_E do not necessarily have opposite signs.

We conclude the analysis of the effects of E, which are summarized in Table 2, with an examination of the effect on the time at which a stage is terminated. Using the method to obtain (3.20) we now get

$$(3.34) \qquad s_E = -P_E/(dP/dt) \qquad \text{Stage I,}$$

so that we know the individual spends less time specializing in human investment if the income effect dominates; otherwise, he spends more time. For the second stage, however,

TABLE 2

Effects of an Increase in Exemption Level*

	Stage I	Stage II	Stage III	Stage IV	Stage V
P_E	+ -	+ -	+ -	+ -	+ -
K_E, W_E	0	+ -	? -	- ?	- ?
Y_E	0	- +	+ +	- +	?
G_E	0	- +	+ +	- ?	- ?
T_E	0	0	0	- ?	- ?
ℓ_E	0	- +	? +	?	0
I_E	0	+ -	? -	?	0
s_E	+ -	+ ?	+ ?	?	0

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*First sign is valid when the intertemporal substitution effect is dominant; second sign is for dominant intertemporal income effect.

(3.35) $s_E = (-\gamma P_E + \omega U'')/[\gamma(dP/dt)]$ Stage II,

so that if the substitution phenomena dominates, the individual spends more time in the first two stages; otherwise, the exit time is indeterminate. Similarly, for the third stage

(3.36) $s_E = (-\gamma P_E + \omega(1-\alpha)U'')/[\gamma(dP/dt)]$ Stage III

so that what was true for the previous stage is also true for this stage. In particular, if the substitution phenomena dominates, we see that the total time spent in the nontaxable stages is increased. Finally, just as in the case of α, it can also be demonstrated that the time at which the individual terminates Stage IV and, hence, ceases to carry out any investment at all is also indeterminate whether or not the substitution phenomena dominates.

IV. CONCLUSIONS

In this paper we have applied a model of individual human capital accumulation to the analysis of the effects of a progressive income tax with a given level of income exempted from tax and a constant marginal tax rate. In general, the individual passes through five distinct stages in his life cycle. In the first stage he devotes all his non-leisure time to investment in human capital and, hence, earns no income and pays no taxes. In the second stage he begins to allocate time to earning income as well as to accumulating human capital, but still pays no taxes because his income has not yet reached the exemption level. In the third stage he chooses his levels of work and investment so as to maintain his income exactly at the exemption level, which is the highest income he can enjoy without incurring a tax. In the fourth stage he continues to allocate time to producing both income and investment, but pays taxes since his income is now above the exemption level. Finally, in the fifth stage he devotes all of his non-leisure time to earning income but now, through depreciation, his stock of human capital declines so that his income also declines. Indeed, he may cease paying taxes in this stage if his human capital declines to the point where even full-time work cannot generate income above the exemption level.

We have analyzed the effects of changes in the tax rate and exemption level first in the short-run, when the value and stock of human capital are given. The short-run response of the individual is perfectly analogous to that in the traditional income-leisure model. In the taxable stages a tax rate increase has both substitution and income effects on the allocation of time between earning and investing so that the individual either substitutes investment for income if the short-run substitution effect dominates or chooses to have both reduced by the tax increase if the income effect dominates. An exemption level increase in the taxable stages has a pure income effect, allowing the individual to increase both income and investment. In the third stage, however, it has a substitution effect, motivating those on the verge of paying the tax to substitute additional tax-free income for investment.

Using a technique developed by Oniki we have then analyzed the effects of changes in the tax parameters in the long run. These effects can also be described in terms of substitution and income effects, but with the choice being between income in different stages of the life cycle rather than between income and investment at a single point in time. An increase in the tax rate alters the marginal rate of substitution between income in the early, non-taxable stages and income in the later, taxable stages. It also reduces the total income an individual can enjoy over his entire life cycle. If the intertemporal substitution effect dominates, then the individual substitutes income in the early stages for income later in the life cycle. If the intertemporal income effect dominates, however, the individual spreads the life-cycle income reduction over all stages. Similarly, an increase in the exemption level has both an intertemporal income effect because it increases total income and an intertemporal substitution effect because it alters the marginal rate of substitution between income during the time spent at the exemption level and income in all other stages. If the intertemporal income effect dominates, the increase in total income is spread over all stages; if the intertemporal substitution effect dominates, income just before the taxable stages is substituted for income in all other stages.

FOOTNOTES

*Part of this research was supported by funds from the Office of Research and Development, Manpower Administration, U.S. Department of Labor under Grant No. 21-11-73-09 to the Urban Institute.

Opinions expressed are those of the authors and do not necessarily represent the views of the Department of Labor, the Urban Institute or its sponsors.

1. See G. Cooper, "Taxation and Incentive in Mobilization," *Quarterly Journal of Economics*, LXVI (1942), pp. 43-66, for a development of the static income-leisure analysis of the personal income tax.

2. The model of life-cycle human capital production and labor supply presented in this paper has evolved from the work of Ben Porath. See Y. Ben-Porath, "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, LXXV (1967), pp. 352-65.

3. In *ibid*, unearned income is assumed to be zero.

4. See S. Rosen, "Learning and Experience in the Labor Market," *Journal of Human Resources*, VII (1972), pp. 326-42, for an analysis of the market for learning opportunities.

5. Although Ben-Porath assumes decreasing returns to scale in the production of human capital, he presents empirical evidence of constant returns to scale. See Y. Ben-Porath, "The Production of Human Capital Over Time," in W.L. Hansen, ed., *Education, Income and Human Capital* (New York: Columbia University Press, 1970), pp. 129-54.

6. In *ibid*, the marginal utility of income is assumed to be constant.

7. See L.S. Pontryagin *et al.*, *The Mathematical Theory of Optimal Processes* (New York: Interscience, 1962).

8. In the static income-leisure analysis a discontinuous increase in the marginal tax rate results in an analogous "jog" in labor supply.

9. See L.S. Pontryagin, *Ordinary Differential Equations* (Reading, Mass.: Addison-Wesley, 1962), p. 170ff.

10. See H. Oniki, "Comparative Dynamics in Optimal Control Theory," *Journal of Economic Theory*, VI (1973), p. 275.

11. *Ibid.*, pp. 276-8.

Symbols Used

ℓ	script lower case
ω	Greek omega
γ	Greek gamma
μ	Greek mu
δ	Greek delta
$\mathcal{H}$	script upper case H
Ψ	Greek upper case psi
∂	partial derivative
Θ	Greek theta

LABOR SUPPLY AND THE SOCIAL SECURITY PAYROLL TAX

by C. Duncan MacRae and Elizabeth Chase MacRae

Abstract

The traditional analysis of the effect of the payroll tax for social security on individual labor supply treats the tax as a reduction in the wage rate. This analysis ignores, however, the existence of a ceiling on the earnings subject to the payroll tax. This paper demonstrates that while for individuals with earnings below the ceiling the payroll tax may indeed be analyzed in terms of its effect on the wage rate, for individuals above the ceiling the tax must instead be analyzed in terms of its effect on unearned income. For individuals with earnings below the ceiling the tax acts to reduce their wage rate and, therefore, reduces their work incentive (unless the income effect dominates). However, for individuals with earnings above the ceiling the tax acts solely to reduce their unearned income and, therefore, increases their work incentive (unless leisure is an inferior good). Thus, the introduction of an exemption into the tax would increase labor supply of individuals below the exemption level by increasing their effective wage rate, but would reduce labor supply of those above the exemption level by increasing their effective level of unearned income.

LABOR SUPPLY AND THE SOCIAL SECURITY PAYROLL TAX*

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The traditional analysis of the effect of the payroll tax for social security on individual labor supply treats the tax as a reduction in the wage rate.[1] This analysis ignores, however, the existence of a ceiling on the earnings subject to the payroll tax, which, in recent years, has exempted twenty to thirty percent of covered earnings in the United States.[2] The purpose of this note is to demonstrate that while for individuals with earnings below the ceiling the payroll tax may indeed by analyzed in terms of its effect on the wage rate, for individuals above the ceiling the tax must instead be analyzed in terms of its effect on unearned income. This straightforward but as yet unanalyzed effect of a ceiling is particularly important when considering the effects on labor supply of reforms in the tax.

We begin with a brief review of the income-leisure model of individual labor supply. The payroll tax is then incorporated into the model and the effects on labor supply are analyzed for individuals with earnings both below and above the ceiling. Finally we consider the effect of introducing personal exemptions into the tax, a reform which has been widely proposed.[3]

An individual is assumed to supply labor so as to maximize utility for income and leisure subject to a budget constraint which specifies the trade off between income and leisure given the wage rate and the level of unearned income. Figure 1 shows the boundary of the individual's constraint set as the broken line ABF when Y^n is the level of unearned income and the (negative) slope W of the line AB is the wage rate. For given levels of W and Y^n, the

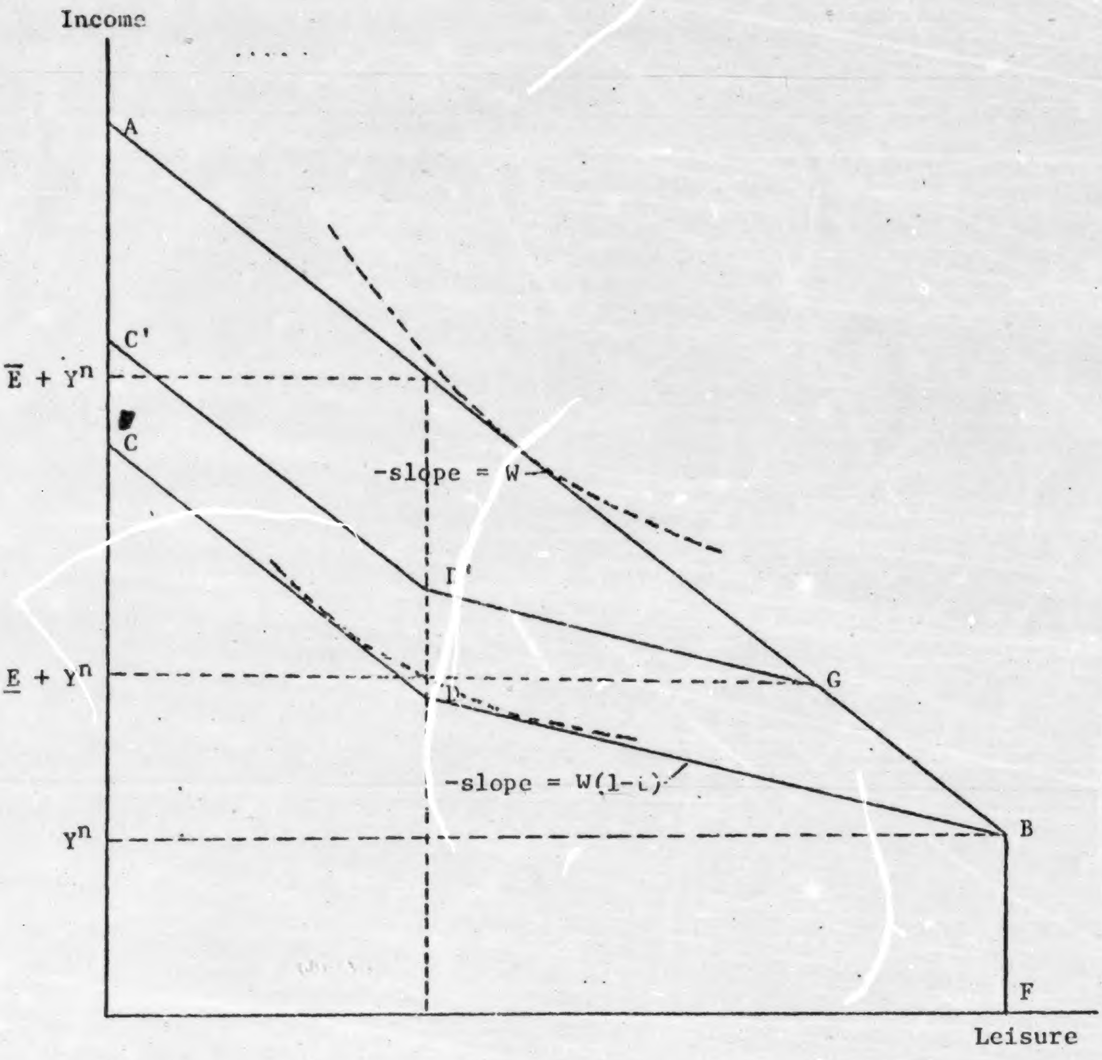
Income
A
C'
$\overline{E} + Y^n$
C
-slope = W
D'
$\underline{E} + Y^n$
G
D
-slope = W(1-t)
Y^n
B
F
Leisure

FIGURE 1

individual will supply a quantity of labor which corresponds to the point on the line AB which lies on the highest possible indifference curve. The individual's labor supply curve, which simply shows the wage-labor combinations for some given level of Y^n, with no payroll tax, appears in Figure 2. Assuming that leisure is not an inferior good, a decrease in Y^n will increase the amount of labor supplied at any wage rate and thus shift the curve to the right, perhaps with a change in shape.

Now let us introduce a payroll tax. As shown by line CDB in Figure 1, the tax corresponds to a percentage reduction in the wage rate of 100t for earnings below the ceiling level of $\overline{E}$ and a constant reduction in income of $t\overline{E}$ for earnings above $\overline{E}$ with no reduction in the wage rate. The lowered slope of the budget line segment DB for earnings below $\overline{E}$ means that a given amount of labor will now be forthcoming only at a higher market wage rate. In Figure 2, this is shown as an upward shift in the labor supply curve in the area below the $WL = \overline{E}$ hyperbola, which will decrease labor supply unless the income effect dominates. The downward shift in the budget line segment CD for earnings above $\overline{E}$ amounts to a fixed reduction in unearned income in the amount of the tax paid, $t\overline{E}$. Consequently, the labor supply curve in Figure 2 is shown shifted to the right in the area above the $\overline{E}$ hyperbola so that labor supply will definitely increase in this area unless leisure is an inferior good.

Since the budget line after the imposition of the payroll tax is no longer convex, there is a "jump" in the labor supply curve in the neighborhood of $\overline{E}$. That is, there is some wage rate at which the individual is indifferent between supplying enough labor to generate earnings above $\overline{E}$ and supplying

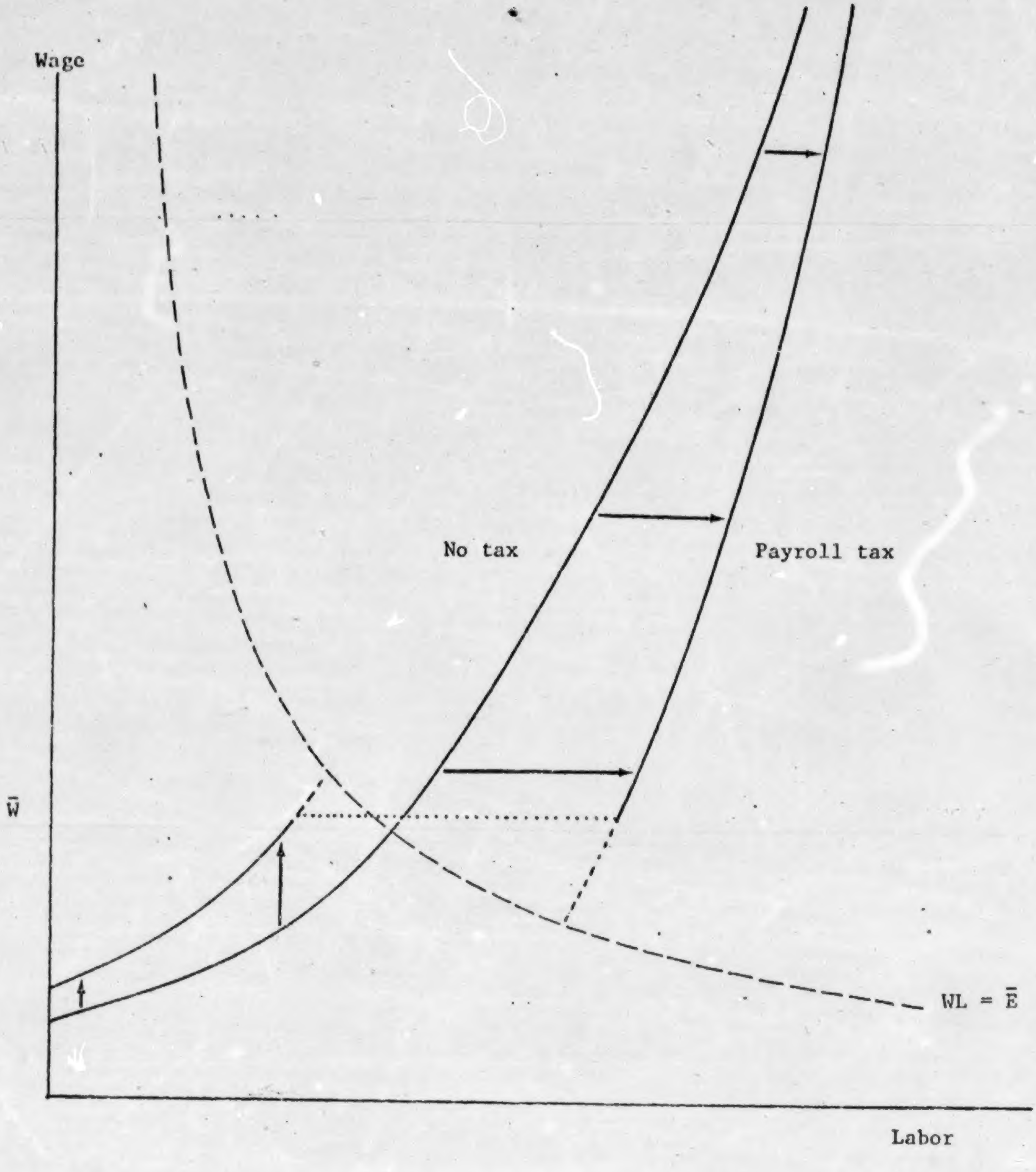

FIGURE 2

less labor to generate earnings below $\overline{E}$. Under the payroll tax, there is no combination of W and Y^n for which the individual will choose to earn exactly the ceiling amount $\overline{E}$, since he can always increase utility by earning more or less than $\overline{E}$.[4]

Finally, we consider the effect of introducing exemptions into the payroll tax, similar to those in the personal income tax. Let $\underline{E}$ be the level of earnings exempt from tax so that only earnings between $\underline{E}$ and $\overline{E}$ are subject to tax. The individual's budget constraint is now given by C'D'GBF in Figure 1. In comparison with a tax which has no exemptions, the higher slope of the segment GB for earnings below $\underline{E}$ means that a given amount of labor will now be forthcoming at a lower wage rate. This is shown in Figure 3 as a downward shift in the "no exemptions" labor supply curve in the area below the WL = $\underline{E}$ hyperbola. This is, of course, exactly the supply that would be forthcoming if there were no payroll tax at all. Individuals in the intermediate earnings range between $\underline{E}$ and $\overline{E}$ do not have their wage rate changed by the introduction of exemptions into the tax. Instead, the effect of exemptions on labor supply in this range of earnings is equivalent to an income subsidy in the amount of the tax saved, $t\underline{E}$, by the exemptions. Consequently, in the area between the $\underline{E}$ and $\overline{E}$ hyperbolas, labor supply with exemptions is shifted to the left of supply without exemptions. Since the budget constraint has a convex kink at earnings level $\underline{E}$, an individual will choose to supply labor so as to earn exactly $\underline{E}$ for a range of wage rates. Thus, as the wage level increases above $\underline{W}$, the level at which he first earns amount $\underline{E}$, the individual decreases his work effort, following along the $\underline{E}$ hyperbola until the wage level is sufficiently high to induce him to move into the area between $\overline{E}$ and $\underline{E}$. This "jog" in labor supply is the result of

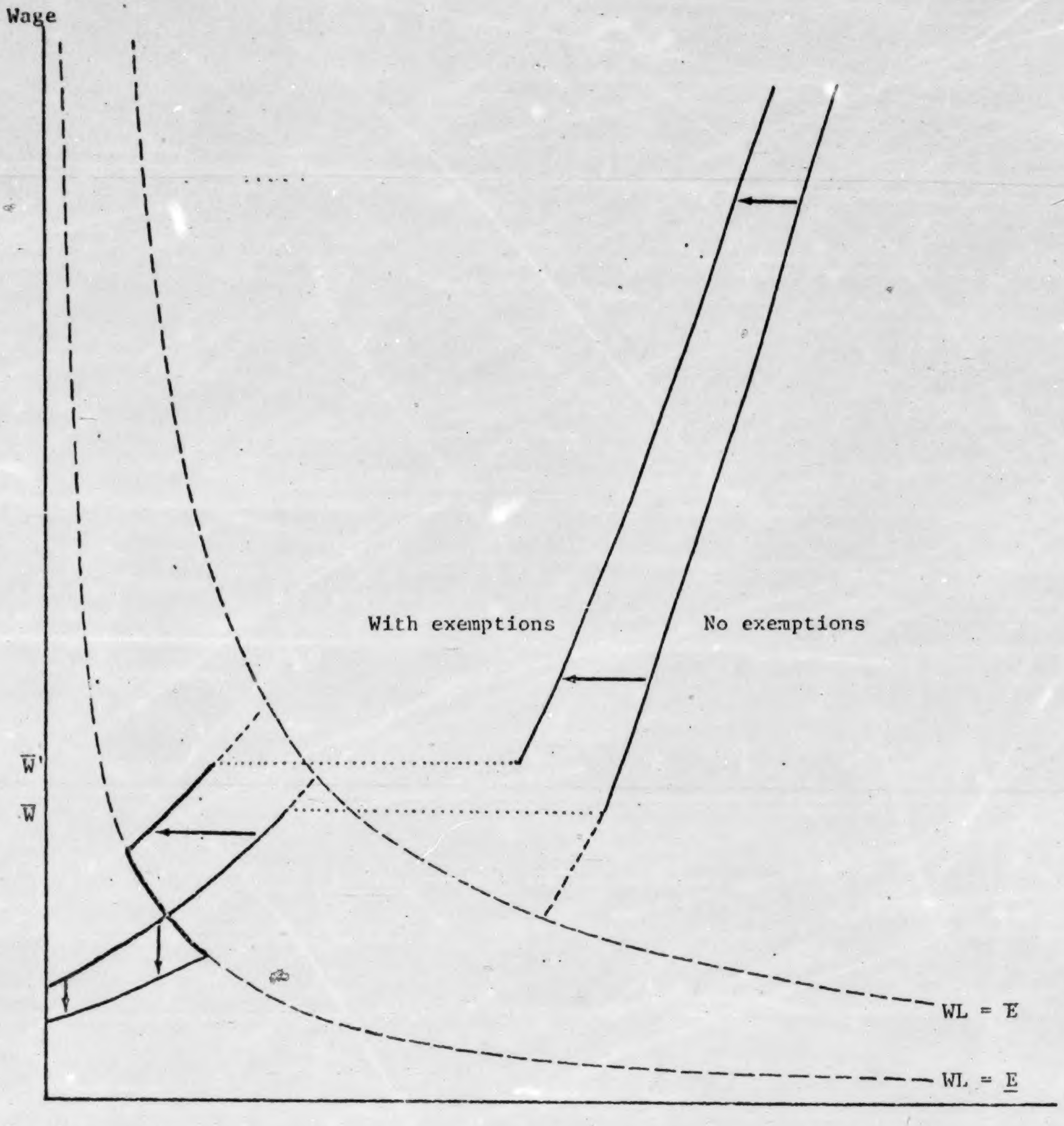

FIGURE 3

a discontinuous increase in the tax rate.[5] Note that there is still a jump in labor supply near the ceiling. The maximum tax paid is reduced, however, by the introduction of exemptions to $t(\overline{E} - \underline{E})$ so that labor supply is shifted to the left in comparison to the supply forthcoming without exemptions.

The conclusion to be drawn from the foregoing analysis is that while for individuals with earnings below the ceiling the payroll acts to reduce their wage rate and, therefore, reduces their work incentive (unless the income effect dominates), for individuals with earnings above the ceiling the tax acts solely to reduce their unearned income and, therefore, increases their work incentive (unless leisure is an inferior good). Thus, the introduction of an exemption into the tax would increase labor supply of individuals below the exemption level by increasing their effective wage rate, but would reduce labor supply of those above the exemption level by increasing their effective level of unearned income.

FOOTNOTES

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1. See, for example, Brittain (1971), Brittain (1972b) and Feldstein.

2. See Table 7-2 in Brittain (1972a).

3. See Chapter V in Brittain (1972a) for a discussion of proposals for personal exemptions and deductions in the social security payroll tax.

4. See Kesselman (1971) for a discussion of the similar jump in labor supply that would result from a negative income tax.

5. The typical piecewise-linear progressive income tax results in analogous jogs in labor supply.

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